King Saud University, Department of Mathematics

Math 280 (Real Analysis)

Midterm(1) 27/03/2022

Question 1.

(a) Show that if $A \subseteq B$ and B is a bounded set then A is a bounded set.

(b) If $A \subseteq B$ and A is an unbounded set. Then B is an unbounded set. Is this statement true or false? give the proof if is it true or give a counterexample if is it false.

Let A and B bounded subset of $\mathbb{R}$.

(c) Let $A \cap B \neq \phi$, What can we say about the connection of

$\sup A,\ \sup B,\ \sup(A \cup B), \sup(A \cap B)$ and $\sup(A \setminus B)$?

Question 2.

(a) Let (a_n) be an sequence. Prove that if $\lim\limits_{n\to\infty} \frac{a_n - 1}{a_n + 1} = 0$, then (a_n) is convergent, and $\lim\limits_{n\to\infty} a_n = 1$.

(b) Show directly from the defnition that if (x_n) and (y_n) are Cauchy sequences, then $(x_n + y_n)$ is Cauchy sequences.

(c) Without any calculation, show that all the following sequences converge to the same limit.

$$\left\{\frac{1}{k^2}\right\}_{k\geq 1}, \left\{\frac{1}{2k}\right\}_{k\geq 1}, \left\{\frac{1}{2k+1}\right\}_{k\geq 1}, \left\{\frac{1}{5k+5}\right\}_{k\geq 1}, \left\{\frac{1}{2^k}\right\}_{k\geq 1}$$

(**Hite:** Note that $\lim\limits_{n\to\infty} \frac{1}{n} = 0$)

Question 3.

(a) If $\sum\limits_{n=1}^{\infty} a_n$ with $a_n > 0$ is convergent, then is $\sum\limits_{n=1}^{\infty} a_n^2$ always convergent? Either prove or give a counterexample.

(b) Using integral test for convergece series to prove that harmonic series is diverges, and $p-$harmonic series is converges for $p > 1$.