King Saud University, Department of Mathematics

Math 280 (Real Analysis)

Midterm(2)   24/04/2022

Question 1.[2+2+2]

Show whether each of the following statements is true or false, and explain or give prove for the false one.

(a) No point of the set of real numbers $\mathbb{R}$ is an interior point.

(b) Every point of the set of real numbers $\mathbb{R}$ is isolated point.

(c) No point at all is an accumulation point of the set of real numbers $\mathbb{R}$.

Question 2.[3+2+3]

(a) Let $f(x) = \frac{|x|}{x}$, $x \neq 0$. Use the sequential definition for limits to find $\lim\limits_{x \to 0} f(x)$ or to prove that the limit does not exist.

(b) Show that $P(x) = 2x^3 - 5x^2 - 10x + 5$ has a root somewhere in interval $[-1, 2]$.

(c) Show that if $f : \mathbb{R} \to \mathbb{R}$ is continuous and such that $\lim\limits_{x \to -\infty} f(x)$ and $\lim\limits_{x \to \infty} f(x)$ are finite, **then** $f(x)$ is uniformly continuous on $\mathbb{R}$.

Question 3.[3+3]

(a) Find the maximum value and the minimum value of the function $f(x) = 3x^4 - 8x^3 + 12x^2 - 48x + 25$ on the closed interval $[0, 3]$.

(b) Let f be differentiable on an interval I. If $f'(x) \geq 0$ for all $x \in I$, then f is nondecreasing on I.

Question 4.[2+2+1]

Let $f : [a, b] \to \mathbb{R}$ be defined by $f(x) = x$.

(i) Choose uniform partition P_n for the interval $[a, b]$ and calculate $U(f, P)$ ana $L(f, P)$

(ii) Prove that $f \in \mathcal{R}(a, b)$

(iii) From (i) and (ii) conclude the value of $\int\limits_{a}^{b} f(x)dx$.