College of Science
Statistics and Operations Research Dep.

جامعة الملك سعود
King Saud University
كلية العلوم
قسم الإحصاء وبحوث العمليات

Second Midterm Exam
Academic Year 1440-1441 Hijri- First Semester

Exam Information معلومات الامتحان			
Course name	**Modeling and Simulation النمذجة والمحاكاة**		اسم المقرر
Course Code	**OPER 441**		رمز المقرر
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Exam Time	**01: 00 PM**		وقت الامتحان
Exam Duration	**2 hours**	ساعتان	مدة الامتحان
Classroom No.	**Lecturing Class Room**		رقم قاعة الاختبار
Instructor Name	د. خالد النويبت		اسم استاذ المقرر

SOLUTION KEY

Question # 1 :

Consider a car insurance company collected the following report about their clients:

Age	Male				Female			
	Total #	#accidents	Min Claim	Max Claim	Total #	#accidents	Min Claim	Max Claim
18 – 28	113	284	350	9000	312	435	550	7500
28 – 38	258	314	950	13000	271	361	1050	11300
38 – 48	546	395	1300	6500	687	302	1450	9300
48 – 58	420	150	2000	11000	348	129	2800	11800
58 – 68	302	145	4000	20000	354	159	3700	32000
More 68	93	23	15000	80000	12	43	21000	65000
Total	**1732**	**1311**			**1984**	**1429**		

You are asked to simulation the information of clients based on the above table:

1. List the variables that you will simulate from the table for each client.
2. Write the algorithm of each variable you will simulate.

1. The output of the simulation could be as follows:
    - Age of the client
    - Gender of the client
    - Number of accidents during the insurance period
    - The amount of claim

2. The output of the simulation could be as follows:
    - Gender of the client
        - Use u~U[0,1]
        - If u <= 1732/(1732+1984) then the client is Male
            - Else the client is Female

    - Age of the client if Male
        - Use u_1~U[0,1]
        - If u_1 <= 113/1732 then the age is 18 – 28
            - Use u_2~U[0,1] to generate integer between 18 and 28
        - If u_1 <= 258/1732 then the age is 28 – 38
            - Use u_2~U[0,1] to generate integer between 28 and 38
        - If u_1 <= 546/1732 then the age is 38 – 48

- Use u_2~U[0,1] to generate integer between 38 and 48
- If u_1 <= 420/1732 then the age is 48 – 58
  - Use u_2~U[0,1] to generate integer between 48 and 58
- If u_1 <= 302/1732 then the age is 58 – 68
  - Use u_2~U[0,1] to generate integer between 58 and 68
- If u_1 <= 93/1732 then the age is 68 – 80
  - Use u_2~U[0,1] to generate integer between 68 and 80

- Number of accidents during the insurance period
  - Determine the Gender
  - Determine the Age
  - Chose the distribution for number of accidents per client in a given age
    - Use u~U[0,1]
    - Generate number of accidents using inverse

- The amount of claim
  - Determine the Gender
  - Determine the Age
  - Chose the distribution for claim amount (Min and Max)
    - Use u~U[0,1]
    - Generate number the amount of claim using inverse

Question # 2:

Consider an investment with a monthly return on investment. The monthly percentage of return on investment is a random variable $X\%$ given by the following probability function:

$$f(x)=\begin{cases}\dfrac{|x|}{10}, & -2\le x\le 4\\ 0, & \text{otherwise.}\end{cases}$$

Answer the following:

1) Write the inverse algorithm for generating the monthly percentage of return on investment.

$$f(x)=\begin{cases}\dfrac{-x}{10} & ;\ -2\le x\le 0\\ \dfrac{x}{10} & ;\ 0\le x\le 4\end{cases} \quad => CDF \quad F(x)=\begin{cases}\dfrac{-(x^2-4)}{20} & ;\ -2\le x\le 0\\ \dfrac{4}{20}+\dfrac{x^2}{20} & ;\ 0\le x\le 4\end{cases}$$

The inverse transform is: Let u~ U[0,1]

If $-2\le x\le 0$; F(x) = u

$$\frac{-(x^2-4)}{20}=u \;=>\; x^2=4-20u \quad => x=-\sqrt{4-20u}\ \textit{take negative values only}$$

Values of u

$$-2\le x\le 0\to -2\le -\sqrt{4-20u}\le 0 \;\to\; 4\ge 4-20u\ge 0 \;\to\; 0\ge -20u\ge -4 \;\to\; 0\le u\le 0.25$$

If $0\le x\le 4$; F(x) = u

$$\frac{4}{20}+\frac{x^2}{20}=u \;=>\; x^2=20u-4 \quad => x=\sqrt{20u-4}\ \textit{take positive values only}$$

Values of u

$$0\le x\le 4\to 0\le\sqrt{20u-4}\le 4 \;\to\; 0\le 20u-4\le 16 \;\to\; 4\le 20u\le 20 \;\to\; 0.25\le u\le 1$$

- Algorithm:
  - Use u~U[0,1]
  - If u <= 0.25 then $x = -\sqrt{4 - 20u}$
  - If u > 0.25 then $x = \sqrt{20u - 4}$

2) Using simulation and the following U[0,1] numbers, evaluate the results of the investment for one year with initial budget of 100,000 SR.

0.032	0.823	0.865	0.732	0.940	0.618	0.574	0.570	0.910	0.833
0.138	0.776	0.911	0.259	0.458	0.343	0.105	0.940	0.188	0.343
0.623	0.306	0.797	0.238	0.897	0.020	0.434	0.135	0.219	0.328
0.776	0.613	0.623	0.652	0.110	0.813	0.629	0.269	0.077	0.376
0.301	0.120	0.491	0.145	0.448	0.048	0.049	0.846	0.590	0.509
0.691	0.684	0.880	0.963	0.526	0.716	0.495	0.981	0.840	0.467

Mon	u	X	Start Budget	End Budget	Mon	u	X	Start Budget	End Budget
1	0.032	-1.83	100000.00	98166.97	7	0.823	3.53	104755.95	108453.70
2	0.138	-1.11	98166.97	97073.83	8	0.776	3.39	108453.70	112134.74
3	0.623	2.91	97073.83	99897.33	9	0.306	1.46	112134.74	113767.45
4	0.776	3.39	99897.33	103287.95	10	0.613	2.87	113767.45	117037.15
5	0.301	1.42	103287.95	104755.95	11	0.12	-1.26	117037.15	115556.73
6	0.691	3.13	104755.95	108038.68	12	0.684	3.11	115556.73	119152.02

3) From simulation, compute the average and standard deviation of the monthly percentage of return on investment.

Mean = sum (x_i)/12 = 1.75 %

STDEV = 1.94 %

4) From simulation compute the probability that the company will have profit more than 3000 SR.

Mon	Start Budget	End Budget	Δ	Mon	Start Budget	End Budget	Δ
1	100000.00	98166.97	-1833.03	7	104755.95	108453.70	3697.75
2	98166.97	97073.83	-1093.14	8	108453.70	112134.74	3681.04
3	97073.83	99897.33	2823.50	9	112134.74	113767.45	1632.71
4	99897.33	103287.95	3390.63	10	113767.45	117037.15	3269.70
5	103287.95	104755.95	1468.00	11	117037.15	115556.73	-1480.42
6	104755.95	108038.68	3282.72	12	115556.73	119152.02	3595.28

Percentage more than 3000 profit = (# more than 3000 profit)/ 12 = 6/12 = 0.5

5) From simulation output, what is the probability of losing.

Prob. Of losing = (# less than zero profit)/12 = 3/12 = 0.25

Question # 3:

An insurance policy pays for the insured 1000 SR per day spent admitted in a hospital for up to three days. If the insured spend more than three days, the insurance company pays 500 SR per day for each extra day of hospitalization thereafter. The number of days that an insured customer spends in the hospital, X, is a discrete random variable with probability function:

$$P[X=k]=\begin{cases}\dfrac{6-k}{15}, & k=1,2,3,4,5\\ 0, & \text{otherwise.}\end{cases}$$

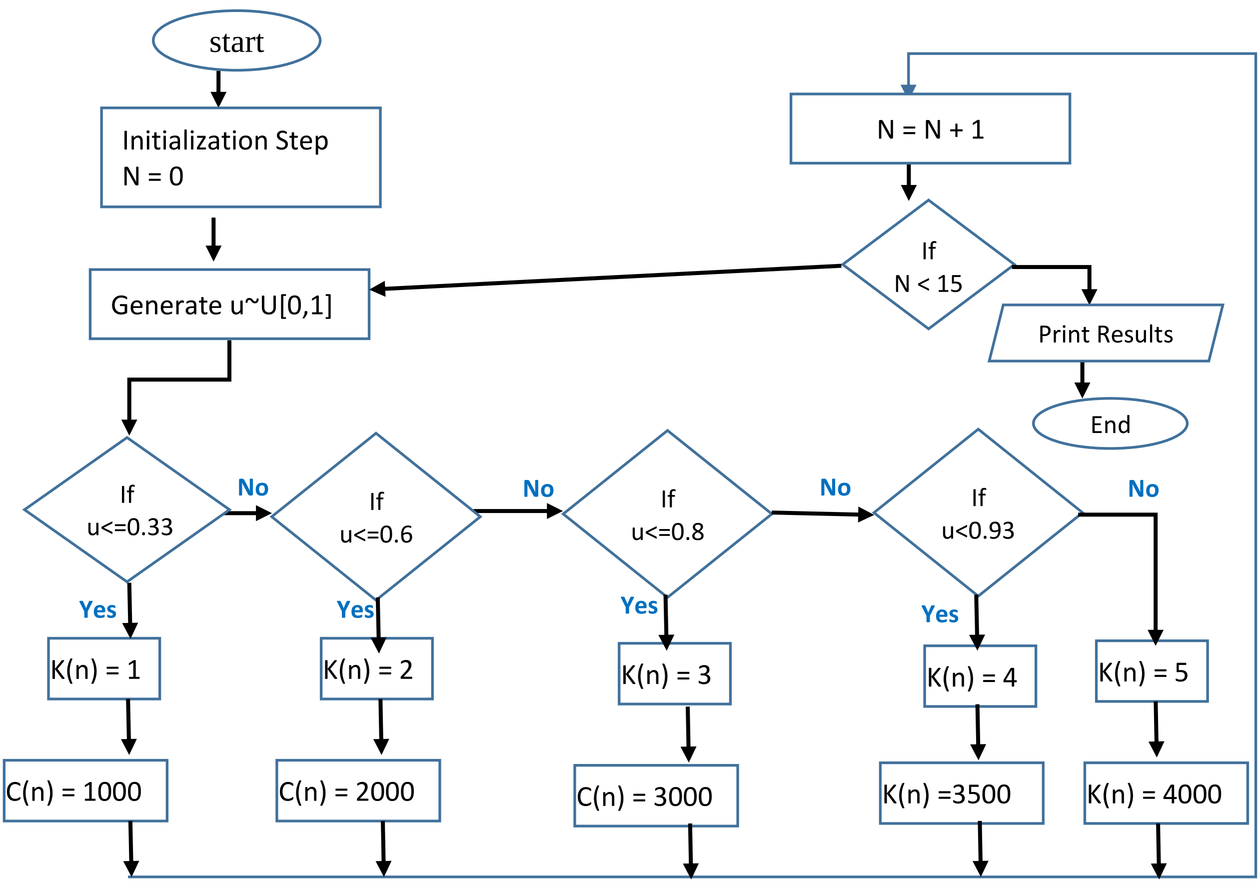
Answer the following:

1) Write the inverse algorithm for generating number of days for the insured clients

K	1	2	3	4	5
P{K}	0.3333	0.2667	0.2000	0.1333	0.0667
CDF{K}	0.333	0.600	0.800	0.933	1.000

- Algorithm:
  - Use u~U[0,1]
  - If 0 < u <= 0.333 then number of days K = 1
  - If 0.333 < u <= 0.6 then number of days K = 2
  - If 0.6 < u <= 0.8 then number of days K = 3
  - If 0.8 < u <= 0.933 then number of days K = 4
  - Else then number of days K = 5

2) Assume that the company wants to analyze the insurance policy using simulation for 15 hospitalized clients. Draw the flow chart for the simulation.



3) Using simulation algorithm and the following U[0,1] numbers, give the results for the 15 insured clients.

u	Days	C(n)	u	Days	C(n)	u	Days	C(n)
0.032	1	1000	0.691	2	2000	0.823	4	3500
0.138	1	1000	0.006	1	1000	0.776	3	3000
0.623	3	3000	0.413	2	2000	0.306	1	1000
0.776	3	3000	0.936	4	3500	0.613	3	3000
0.301	1	1000	0.423	2	2000	0.12	1	1000

4) Compute the average and standard deviation of amount that the insurance company will pay per claim.

Average company pay per claim = **2066.67**

standard deviation company pay per claim = **980.93**

5) Compute the average and standard deviation of number of days that a client spends in the hospital.

Average number of days that a client spends in the hospital = **2.13 day**

standard deviation number of days that a client spends in the hospital = **1.09 day**

6) From simulation compute the probability that the company will pay more than 3000 SR.

Prob company pay more than 3000 = (# more than 3000 cost)/15 = 2/15

Question #4:

Patients arrive to a hospital's emergency room according to a Poisson process with rate 8 patients per hour. Patients come in three different health conditions. The patients are categorized according to their condition as critical, serious, or stable. In the past year, statistics show that:

i. 10% of the emergency room patients were critical; and take random amount of treatment of Erlang with parameters $\alpha = 2$ and $\lambda = 0.5$ per hour

ii. 30% of the emergency room patients were serious; and take random amount of treatment of Exponential with average time of 2 hours.

iii. the rest of the emergency room patients were stable; and take random amount of treatment of integer uniform between 15 min and 30 min.

After treatment at the ER, statistics show that:

- 40% of the critical patients died;
- 10% of the serious patients died; and
- 1% of the stable patients died.

Answer the following:

1. Write the Step for simulation of this process

Random Process 1: Patients Arrival Time PAT(n): Poisson Process with rate $\lambda = 8$ patients/hr

Algorithm:

1. Use u~U[0,1]
2. Generate time between arrivals T(n) ~ Exp (λ=8)

3. $T(n) = -(1/8)\ ln(1-u)$
4. Compute the arrival time $PAT(n) = PAT(n-1) + T(n)$

Random Process 2: The category of the patient CP(n):

Algorithm:

5. Use u~U[0,1]
6. If $0 < u <= 0.1$ then CP(n) = 1 ➔ critical
7. If $0.1 < u <= 0.4$ then CP(n) = 2 ➔ serious
8. Else then CP(n) = 3 ➔ stable

Random Process 3: Patient Treatment Time PTT(n)

Algorithm:

1. If CP(n) = 1 ➔ critical
   i. PTT(n) ~ Er(α = 2 and λ = 0.5 per hour); use convolution method
   ii. Use u_1, u_2 ~U[0,1]
   iii. $PTT(n) = -2(ln(1-u_1) + ln(1-u_2))$
2. If CP(n) = 2 ➔ serious
   i. PTT(n) ~ Exp (mean = 2 hours); use inverse method
   ii. Use u_1 ~U[0,1]
   iii. $PTT(n) = -0.5\ ln(1-u_1)$
3. If CP(n) = 2 ➔ stable
   i. PTT(n) ~ DU (15, 30 min); use discrete inverse method
   ii. Use u_1 ~U[0,1]
   iii. $PTT(n) = 15 + in[(30 - 15 + 1)\ u_1]$

Random Process 4: Patient Exit Conditions PEC(n)

Algorithm:

1. If CP(n) = 1 ➔ critical
   i. PEC(n) ~ Bernoulli (p = 0.4)
   ii. Use u ~U[0,1]
   iii. If u < 0.4 then PEC(n) = Died
      - Else PEC(n) = Lived
2. If CP(n) = 2 ➔ serious
   i. PEC(n) ~ Bernoulli (p = 0.1)
   ii. Use u ~U[0,1]
   iii. If u < 0.1 then PEC(n) = Died
      - Else PEC(n) = Lived
3. If CP(n) = 2 ➔ stable
   i. PEC(n) ~ Bernoulli (p = 0.01)
   ii. Use u ~U[0,1]
   iii. If u < 0.01 then PEC(n) = Died
      - Else PEC(n) = Lived

2. Starting from 6:00 am and using the U[0,1] number below, do the simulation for the ER for 15 patients and show the details of each arrival: arrival time, patient's category, treatment time, and patient's exit condition.

	u	T(n)	PAT(n)	u	CP(n)	u1	u2	PTT(n)	u	PEC(n)
1	0.032	0.2	0.2	0.684	3	0.732		26	0.329	Lived
2	0.138	1.1	1.4	0.73	3	0.259		19	0.575	Lived
3	0.623	7.3	8.7	0.904	3	0.238		18	0.772	Lived
4	0.776	11.2	19.9	0.191	2	0.652		31.67	0.618	Lived
5	0.301	2.7	22.6	0.092	1	0.145	0.963	414.42	0.343	Died
6	0.691	8.8	31.4	0.865	3	0.494		22	0.02	Lived
7	0.006	0.0	31.4	0.911	3	0.849		28	0.813	Lived
8	0.413	4.0	35.4	0.797	3	0.079		16	0.048	Lived
9	0.936	20.6	56.0	0.623	3	0.611		24	0.716	Lived
10	0.423	4.1	60.2	0.491	3	0.94		30	0.974	Lived
11	0.823	13.0	73.2	0.88	3	0.458		22	0.575	Lived
12	0.776	11.2	84.4	0.534	3	0.897		29	0.264	Lived
13	0.306	2.7	87.1	0.12	2	0.11		3.50	0.879	Lived
14	0.613	7.1	94.2	0.072	1	0.448	0.526	160.90	0.574	Lived
15	0.12	1.0	95.2	0.898	3	0.984		30	0.105	Lived

3. What is the probability that any patient enter the Emergency room will live?

probability that any patient enter the Emergency room will live = 14 / 15 = 0.933

4. Given that a patient survived, calculate from simulation the probability that the patient was categorized as serious upon arrival.

probability that the patient was categorized as serious upon arrival | Given that a patient survived = (# serious and lived)/(# lived) = 2 / 14 = 0.1429

Question # 5:

Consider the following probability density function:

$$f(x) = \begin{cases} 0.5003e^{-x/2}, & 0 < x < 15 \\ 0, & \text{otherwise.} \end{cases}$$

Write the algorithm to generate random numbers from *f(x)* using acceptance/rejection method with majorizing function *g(x)* fixed function.

The pdf f(x) is strictly decreasing in the interval [0,15]
Then the max f(x) = f(0) = g(x) = 0.5003 ; x $\in$[0,15]

C= integration of g(x) on [0,15] = 7.5045
W(x) = (0.5003)/(7.5045) = 0.0666 for all x $\in$[0,15]
Then, W^{-1} (u) = 15 u

Algorithm

1. Choose $u_1 \sim U[0,1]$
2. Get W = 15 u
3. Evaluate f(W) and g(W)
4. Get new $u_2 \sim U[0,1]$
5. If f(W) / g(W) >= u_2 ➔ W~f(x)
6. Else, Reject and Go To 1.

n	U1	W	f(W)	U2	g(W)	f(W) / g(W)	f(W)/g(W)>u_2
1	0.280	4.2	0.0612	0.165	0.5003	0.122327	Reject
2	0.318	4.77	0.0461	0.684	0.5003	0.092145	Reject
3	0.270	4.05	0.066	0.768	0.5003	0.131921	Reject
4	0.890	13.35	0.00063	0.667	0.5003	0.001259	Reject
5	0.091	1.365	0.253	0.257	0.5003	0.505697	Accept
6	0.238	3.57	0.084	0.084	0.5003	0.167899	Accept
7	0.611	9.165	0.0051	0.494	0.5003	0.010194	Reject
8	0.772	11.58	0.00153	0.849	0.5003	0.003058	Reject