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الملك سعود
King Saud University

كلية العلوم
قسم الاحصاء وبحوث العمليات

College of Science.
Department of Statistics & Operations Research

First Midterm Exam
Academic Year 1442-1443 Hijri-First Semester

Exam Information معلومات الامتحان			
Course name	Modeling and Simulation	النمذجة والمحاكاة	اسم المقرر
Course Code	OPER 441	441 بحث	رمز المقرر
Exam Date	20-10-2021	14-3-1443	تاريخ الامتحان
Exam Time	12: 00 PM		وقت الامتحان
Exam Duration	2.5 hours	ساعتان ونصف	مدة الامتحان
Classroom No.			رقم قاعة الاختبار
Instructor Name			اسم استاذ المقرر

Student Information معلومات الطالب		
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Section No.		رقم الشعبة
Serial Number		الرقم التسلسلي

General Instructions:

- Your Exam consists of PAGES (except this paper)
- Keep your mobile and smart watch out of the classroom.
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تعليمات عامة:

- عدد صفحات الامتحان صفحة (باستثناء هذه الورقة)
- يجب ابقاء الهواتف والساعات الذكية خارج قاعة الامتحان.
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هذا الجزء خاص باستاذ المادة

This section is ONLY for instructor

#	Course Learning Outcomes (CLOs)	Related Question (s)	Points	Final Score
1	Understanding the processes and steps for building a simulation model			
2	Implement an inverse cumulative distribution function based random variate generation algorithm			
3	Explain and implement the convolution algorithm for random variate generation			
4	Explain and implement the acceptance rejection algorithm for random variate generation			
5	Compute statistical quantities from simulation output			
6	Generate random numbers from any given distribution discrete or continuous			
7	Building simulation models from basic applications			
8				

EXAM COVER PAGE

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Question #1: Answer the following with *True* or *False*:

F	1. Simulation model can be used to evaluate different alternatives and give an optimal solution.
T	2. The sample space in a random experiment is always determined and unique to everyone.
T	3. Simulation modeling is not good if there is less data or no estimates available.
T	4. Validation step is to make sure that the simulation program is running correctly.
F	5. In call center model with two lines it is impossible to lose any incoming call.
T	6. Triangular distribution is used when there is lack of data
F	7. Simulating flight distance for an airplane is a discrete system simulation.
T	8. In Bank simulation, the variable (X = number of customers in waiting) is a state variable for the system.
F	9. In Bank simulation, the variable (X = number of kids with a customer) is a state variable for the system.
T	10. The measures of simulation changes every time a new run of simulation is performed
F	11. Every simulation run for the same model give the same results.
T	12. The Geometric distribution has memory less property
T	13. If a used computer didn't fail for the past 6 months then the probability that it will not fail the next month is always the same as buying a new computer now.
T	14. The normal distribution with parameters μ and σ^2 always cover more than 80% of the distribution within $\pm 2\sigma$
F	15. The Erlang distribution is a special case from exponential distribution.
F	16. We can always get any Erlang distribution with any parameters using Gamma Distribution
T	17. The beta distribution between (0,1) can be rescaled for any real values
F	18. The Erlang distribution always has all parameters positive integer values.
T	19. The random variable with Exponential distribution is always has mean value equals t the standard deviation.
F	20. If the random variable has mean value equals to the variance then it must have a Poisson distribution.
F	21. The number of trials until 1st 2 success is a binomial distribution.

F	22. The Uniform distribution has a single mode value.
T	23. The normal distribution always has the mean equals to the median
F	24. In building simulation model, we always have to start data collection after validation.

Question #2:

Given the following functions:

A	$np(1-p)$
B	$\frac{q}{p^2}$
C	p
D	$p(1-p)$
E	$\frac{e^{-\alpha}\alpha^{x}}{x!}$
F	$\frac{kq}{p^2}$

G	$\Gamma(\beta) = \int_0^{\infty} x^{\beta-1}e^{-x}\,dx$
H	$\frac{\lambda^k x^{k-1}e^{-\lambda x}}{(k-1)!}$ for $x, \lambda \geq 0,$
I	$\frac{1}{\sigma\sqrt{2\pi}}\exp\left[-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right]$
J	$\frac{\beta}{\alpha}\left(\frac{x-\nu}{\alpha}\right)^{\beta-1}\exp\left[-\left(\frac{x-\nu}{\alpha}\right)^{\beta}\right], \quad x \geq \nu$
K	$\frac{\beta\theta}{\Gamma(\beta)}(\beta\theta x)^{\beta-1}e^{-\beta\theta x}, \quad x > 0$ gamma
L	$\frac{1}{\sqrt{2\pi}\sigma x}\exp\left[-\frac{(\ln x-\mu)^2}{2\sigma^2}\right],$

M	$a+(b-a)\left(\frac{\beta_1}{\beta_1+\beta_2}\right)$
N	$e^{\mu+\sigma^2/2}$
O	$1-e^{-\lambda x},$
P	$\frac{1}{k\theta^2}$
Q	$\binom{n}{x}p^x q^{n-x},$

10
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Complete the following by choosing the correct function from above

1	The random variable X has ***Exponential*** distribution if if X has the CDF	O
2	The random variable X is has ***Erlang*** distribution if X has the pdf	H
3	The random variable X has ***Beta*** distribution if X has the pdf	J
4	The random variable X has ***Geometric*** distribution *(p)* then X has the a variance	B
5	The random variable X has ***Lognormal*** distribution if X has the pdf	L
6	A random variable X has ***Negative binomial*** distribution*(p)* then X has the a variance	F
7	The random variable X has ***Beta*** distribution if X has the pdf	J
8	The random variable X has ***Lognormal*** distribution, Then X has expected value	N
9	The random variable X has ***Beta*** distribution, then X has the expected value	M
10	The random variable X has ***Poisson*** distribution if X has the pdf	E
11	The random variable X has ***Bernoulli*** distribution *(p)*, then X has the expected value	C
12	The random variable X is has ***Erlang*** distribution *(k)*, then it has variance	P

Question #3:

A service station has three servers and a single waiting line. The servers serve customers in the order in which they arrive. Customers may leave the system without service due to long waiting time. The service time of the customers changes according to their gender and the type of service they request. The service facility provide four types of services.

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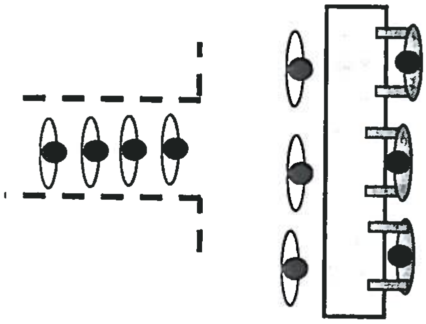

List al state variables and attributes and define the values of each one?

State Variables:

Customer arrival (cont)

Customer waiting (cont) values?

Customer leave without service (0)

Customer leave with Service (1)

att.

Server #1 busy(1)/ idle 0

Server #2 busy(1) idle 0

Server #3 busy(1) idle 0

– num of Customers (discrete)

– Customer departure (cont)

attributes:

gender (Female, male), Type of service (1,2,3,4), Service time for the customer (Continuous)

Arrival. ?

more?

Question #4:

High temperature (°F) in a city on July 21, denoted by the random variable X, has the following probability density function, where X is in degrees F.

$\frac{10}{12}$

$$f(x) = \begin{cases} \frac{2(x-85)}{119}, & 85 \le x \le 92 \\ \frac{2(102-x)}{170}, & 92 < x \le 102 \\ 0, & \text{otherwise} \end{cases}$$

5349.4

(a) What is the mean of the temperature E(X)?
(b) What is the variance of the temperature V(X)?
(c) What is the median temperature?

a) $E(x) = \int_{85}^{92} x \frac{2(x-85)}{119} dx + \int_{92}^{102} x \frac{2(102-x)}{170} dx$

$= \frac{2}{119} \int_{85}^{92} x^2 - 85x \, dx + \frac{2}{170} \int_{92}^{102} 102x - x^2 \, dx$

$= \frac{2}{119} \left(\frac{x^3}{3} - \frac{85x^2}{2} \right) \Big]_{85}^{92} + \frac{2}{170} \left(\frac{102x^2}{2} - \frac{x^3}{3} \right) \Big]_{92}^{102}$ ✓

$36.92 + 56.078 \Longrightarrow \underline{E(x) \simeq 93}$ ✓

b) $V(x) = E(x^2) - (E(x))^2$

$E(x^2) = \int_{85}^{92} x^2 \frac{2(x-85)}{119} dx + \int_{92}^{102} x^2 \frac{2(102-x)}{170} dx$

$= \frac{2}{119} \left(\frac{x^4}{4} - \frac{85x^3}{3} \right) \Big]_{85}^{92} + \frac{2}{170} \left(\frac{102x^3}{3} - \frac{x^4}{4} \right) \Big]_{92}^{102}$

3311.75 ✓ $+ 5349.4 \Longrightarrow E(x^2) = 8661.15$

$\therefore V(x) = 8661.15 - 93^2 \Longrightarrow \underline{V(x) = 12.15}$ ✓

c) median X is $\frac{102 + 85}{2} = 93.5$

$f(93.5) = \frac{2(102 - 93.5)}{170} = 0.1$ ✗

$\therefore$ median Temp is 93.5 F° with Probability 0.1

B

17
20

Question #5:

$\lambda = \frac{1}{15}$

Busses arrive to a station at random. It is estimated that the time between busses is an exponential distribution with mean 15 minutes. The number of passengers on the bus is also random follows a binomial distribution with parameter 10 and probability 0.75.

n p

(1) If you arrive at 10:00 am, What is the probability you will wait more than 30 min for your bus?
(2) What is the expected number of busses that will arrive to the station between 10:00 to 11:00 am?
(3) What is the expected number of passengers that will drop off to the station between 10:00 to 11:00 am?
(4) Given that 15 passengers arrived between 10:00 to 11:00 am what is the probability that the next bus will have 3 passengers on board?
(5) Draw the flowchart that will simulate the bus arrival and passenger's drop-off to this station?
(Use the command *Generate RV from Dist.*--- to complete your flowchart)

let x Time between busses : $x \sim \exp(\frac{1}{15})$

1) $P(x > 30) = 1 - p(x < 30) = 1 - (1 - e^{-\frac{1}{15}(30)}) = \underline{0.135}$ ✓

2) $\frac{60}{E(x)} = \frac{60}{15} = \underline{4}$ busses ✓

3) let y num of passengers ..., $y \sim Bin(p = 0.75, n = 10)$

$E(y) = np = 7.5$ ✗

$Var(y) = npq = 1.875$

drop → q ∴ every 15 min = $nq = 2.5$ in 1 hour = $\underline{10 \text{ drop}}$ ✗

4) Pr(3 Passengers | 15 arrived 10 to 11) = Pr(3 Passengers on board)

from memory lack of exponential

$pr(y = 3) = \binom{10}{3}(0.75)^{3}(0.25)^{7} = 0.003$ ✓

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let k=1, j=1
① y(0) [illegible]
② enter B(j)

Question #6:

العينة السابقة ١٥

A supermarket sells fresh milk daily. Customers arrive to supermarket buy milk. Customers may buy 1 bottle, 2 bottle or 3 bottles at random (let B(*j*): number of bottles bought by customer *j*). The number of customers demanding the milk varies between 5 to 15 customers daily (let N(*k*): number customers arrived in day *k* asking for milk). The supermarket stores the milk in a refrigerator that can hold up to 20 bottles of milk. By the end of each day, the owner will decide wither to order more milk for next day or not. If he finds 10 or less in that the refrigerator then he will refill the refrigerator for next day.

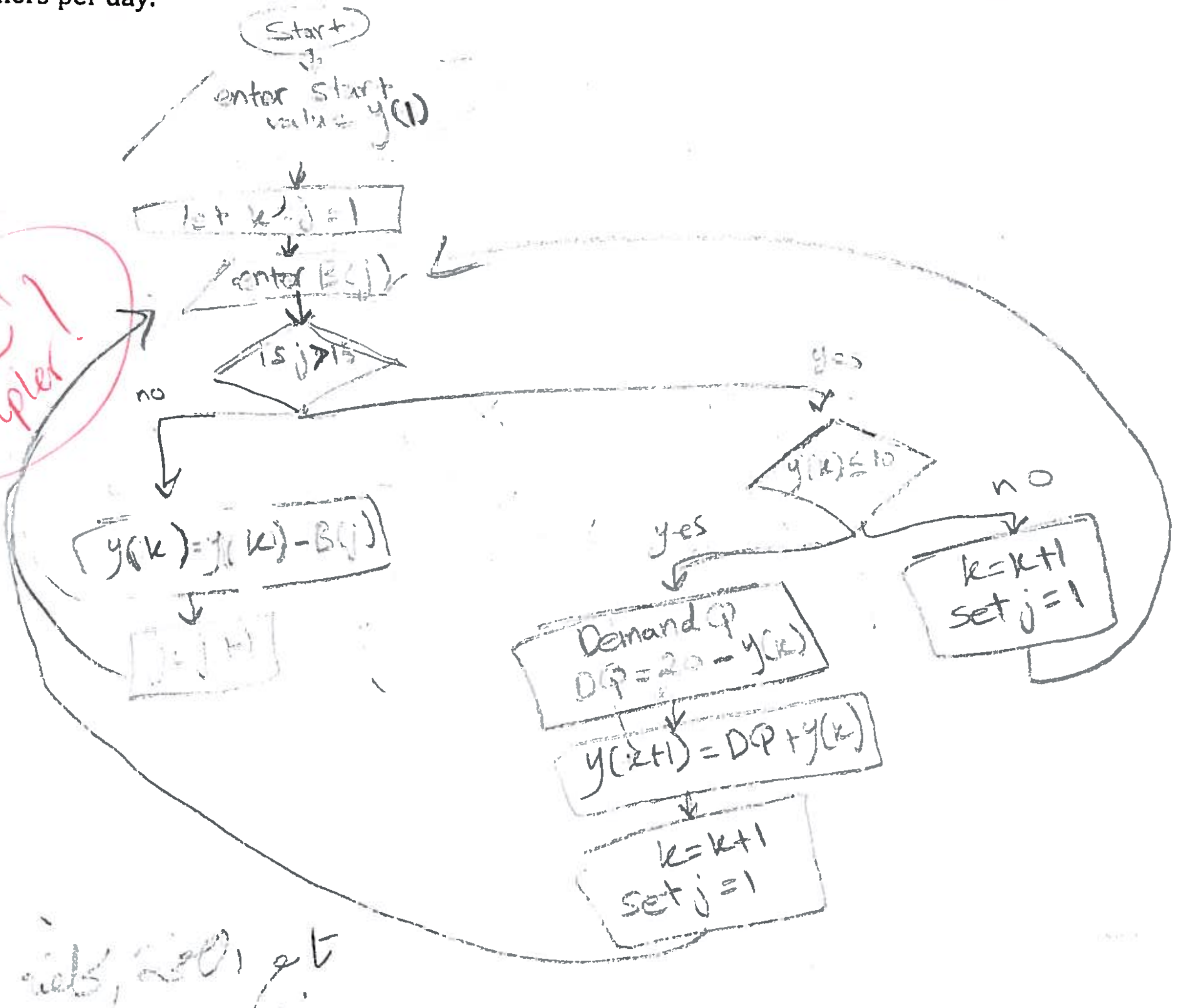
Day	Cust-1	Cust-2	Cust-3	Cust-4	Cust-5	Cust-6	Cust-7	Cust-8	Cust-9	Cust-10	Cust-11	Cust-12	Cust-13	Cust-14	Cust-15
1	1	1	2	1	3	1	1	3	3	1	2				
2	1	3	1	3	3	1	3	2	1	3	1	2	1	3	3
3	3	1	3	2	1	2	1	1	1						
4	2	3	1	2	3	3									
5	1	2	3	1	3	1	2	3							

(1) Draw the flowchart that will simulate number of sold bottles.

(2) Do the manual simulation to determine the status of customer (accepted, rejected) and amount of milk in the frig.

(3) From the manual simulation, estimate the number of sold units and the number of lost customers per day.

1)



should be simpler!

تابع الحل خلف