استعن بالله أولا.. وكن على يقين بأن كل ما ورد في هذه الورقة
تعرفه جيدا وقد تدربت عليه بما فيه الكفاية، فكن مطمئناً

College of Science.
Department of Statistics & Operations Research

جامعة الملك سعود
King Saud University
كلية العلوم
قسم الإحصاء وبحوث العمليات

Second Midterm Exam
Academic Year 1443-1444 Hijri- First Semester

Exam Information معلومات الامتحان			
Course name	Modeling and Simulation	النمذجة والمحاكاة	اسم المقرر
Course Code	OPER 441	441 بحث	رمز المقرر
Exam Date	2021-11-24	1443-04-19	تاريخ الامتحان
Exam Time	12: 30 PM		وقت الامتحان
Exam Duration	2 hours	ساعتان	مدة الامتحان
Classroom No.			رقم قاعة الاختبار
Instructor Name	د. خالد النويبيت		اسم استاذ المقرر

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Serial Number	٢	الرقم التسلسلي

General Instructions:

- Your Exam consists of ____ PAGES (except this paper)
- Keep your mobile and smart watch out of the classroom.
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تعليمات عامة:

- عدد صفحات الامتحان ____ صفحة (يستثناء هذه الورقة)
- يجب إبقاء الهواتف والساعات الذكية خارج قاعة الامتحان.
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هذا الجزء خاص بأستاذ المادة
This section is ONLY for instructor

#	Course Learning Outcomes (CLOs)	Related Question (s)	Points	Final Score
1	To know the basics of pseudo random generation and apply different methods of random generation techniques			
2	Chose and fit theoretical distribution on collected data			
3	Define and compute performance measures from simulation models			
4	Recognize and analyze simple models and its main elements for simulation			
5	Understanding how to use computer software (ECXEL) for simulation models			
6	use appropriate statistical techniques to analyze and evaluate outputs of simulation models			
7	Generate random variates from different probability functions and directly from collected data			
8	build simple simulation models of real-life problems			

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Question #1:

Given the *LCG* with $a = 5$, $c = 1$ and $m = 19$. Answer the following:

1. Does this generator have a full cycle? Prove your answer?
2. Using this *LCG* and a seed value $R_0 = 13$, simulate tossing an unfair coin with $P\{H\} = 0.6$ until head appear twice.

① by testing 3 conditions:-

①
$m = 19 \rightarrow$ Factors $\{1, 19\}$
$c = 1$, factor $\{1\}$
No other common factor than 1 satisfied

②
$a - 1 = 4$
primery factor for $m = 1$
$\rightarrow \frac{4}{1} = 4$
satisfied

③
$\frac{m}{4} = \frac{19}{4} = 4.75$
$\frac{a-1}{4} = \frac{4}{4} = 1$
not satisfied

→ since the third conditions not satisfied then It's not a full cycle

2)
$R_0 = 13$

$R_{i+1} = (a R_i + c) \text{ Mod } m$

$R_1 = (5(13) + 1) \text{ Mod } 19 = 66 \text{ Mod } 19 = 9 \rightarrow U_1 = \frac{9}{19} = 0.47368$

$R_2 = (5(9) + 1) \text{ Mod } 19 = 8 \rightarrow U_2 = \frac{8}{19} = 0.42105$

$R_3 = (5(8) + 1) \text{ Mod } 19 = 3 \rightarrow U_3 = \frac{3}{19} = 0.15789$

$R_4 = (5(3) + 1) \text{ Mod } 19 = 16 \rightarrow U_4 = \frac{16}{19} = 0.842105$

$R_5 = (5(16) + 1) \text{ Mod } 19 = 5 \rightarrow U_5 = \frac{5}{19} = 0.26315$

$R_6 = (5(5) + 1) \text{ Mod } 19 = 7 \rightarrow U_6 = \frac{7}{19} = 0.3684$

tossing unfair coin until head appear twice:

$P\{H\} = 0.6$, $P\{T\} = 0.4$

→ RP ①: tossing a coin with $P\{H\} = 0.6$

→ RP ②: use LCG for this experiment

8
15

Question #2:

The period of time (in months) between rainfalls in Abha city is modeled using the following pdf:

$$f(x) = 1.06\, e^{\frac{-x}{2}} \quad ; \quad 1 \le x \le 4$$

Where random variable X is time between rainfalls in months.

a) Write the inverse transform for measuring the time between rainfalls.
b) Simulate the next 4 rainfalls (in months) in Abha city.
c) If the we want to simulate the rainfall that is at least 8 months from now. Write the simulated values.

0.507	0.103	0.362	0.029	0.584
0.068	0.420	0.742	0.653	0.021
0.432	0.714	0.259	0.799	0.207
0.896	0.237	0.929	0.270	0.524
0.108	0.183	0.466	0.721	0.793
0.131	0.321	0.868	0.428	0.160

a) $F(x) \begin{cases} 0 & x < 1 \\ & 1 \le x \le 4 \\ 1 & x > 4 \end{cases}$ $\rightarrow \int_1^x f(t)\,dt = \int_1^x 1.06\, e^{-\frac{t}{2}} dt = -2.12\, e^{-\frac{x}{2}} - 12.12\, e^{-\frac{1}{2}}$

$= -12.12\,(e^{-\frac{x}{2}} + e^{-\frac{1}{2}})$

Let $u \in U(0,1)$

$u = -12.12\,(e^{-\frac{x}{2}} + e^{-\frac{1}{2}})$

$Ln(u) = -12.12\,(-\frac{x}{2} - \frac{1}{2})$

$Ln(u) = +6.06X + 6.06$

$\frac{Ln(u) - 6.06}{6.06} = x \leftarrow F^{-1}(u)$

b) → RP①: Let Rainfall RF_n

X_n: time between rainfalls in months

$RF_n = RF_{n-1} + X$

RP②: $RF_F = 4$

c) $X = \frac{\ln(u) - 6.66}{6.06}$ →

$X_1 = \frac{\ln(0.507) - 6.06}{6.06} = -1.11208$

$X_2 = \frac{\ln(0.103) - 6.06}{6.06} = -1.375$

$X_3 = -1.1676$

$X_4 = -1.58423$

$X_5 = -1.038$

$X_6 = -1.44$

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Question #3:

Students arrive at a self-service cafeteria at the rate of one every 30±20 seconds. Is estimated that 40% of students go to the sandwich bar, where every student prepares has own sandwich in 60±30 seconds. The rest go to the main counter, where one server spoons the prepared meal onto a plate in 45±30 seconds. All students take their seats in the cafeteria and spend 20 ± 10 minutes eating. After eating, 10% of the students go back for dessert, and return to their table to spend an additional 10±2 minutes in the cafeteria.

1. Simulate until 20 people have left the cafeteria using the following table of U[0,1] numbers.
2. At the final simulation time, estimate the following from the simulation data:
    a. How many students are still in the cafeteria 9
    b. What percentage of students at the sandwich bar. 40%
    c. What percentage of students at the main course counter. 60%
    d. What percentage of students on tables
    e. What percentage of students take dessert. 10%
3. From the simulation data, what is the average time that a student who gets a sandwich spends in the cafeteria until he leaves after finishing his entire meal.

	SI	Sgs	Sr	mP	eT	Sbd	TD	
Std 1	0.454	0.516	0.922	0.405	0.965	0.686	0.623	0.327
Std 2	0.046	0.239	0.356	0.686	0.577	0.234	0.439	0.588
Std 3	0.024	0.034	0.134	0.534	0.648	0.244	0.525	0.340
Std 4	0.162	0.032	0.224	0.209	0.441	0.493	0.850	0.607
Std 5	0.359	0.946	0.607	0.420	0.058	0.197	0.336	0.353
Std 6	0.908	0.385	0.181	0.683	0.067	0.856	0.736	0.328
Std 7	0.287	0.537	0.196	0.087	0.297	0.772	0.564	0.633
Std 8	0.980	0.383	0.485	0.909	0.061	0.201	0.356	0.361
Std 9	0.253	0.671	0.545	0.765	0.651	0.030	0.839	0.546
Std 10	0.160	0.498	0.090	0.432	0.187	0.588	0.248	0.954

$\frac{10}{15}$

Question #4:

A traffic department of a small city wants to simulate the occurrence time for the accident in an intersection using some different choices of distributions to estimate the number of accidents per month (30 days). Use the following U(0,1) numbers to compute the simulation under the following assumptions.

1. Estimate the number of accidents in the 1st month assuming the time between accident is an exponential distribution with mean 4 days.
2. Estimate the number of accidents in the 1st month assuming the time between accident is a discrete uniform distribution with mean 4 days and ±2 days.
3. Estimate the number of accidents in the 1st month assuming the time between accident is Binomial distribution with parameters $n = 6$ and $p = 0.75$.
4. Compare the result from simulation in (1), (2) and (3) with exact mean for each question.

Use uniform numbers (as needed) by columns until you finish all numbers in the column then move to the next.

↓ Part 1	↓ Part 2	↓ Part3
0.737	0.454	0.516
0.293	0.046	0.239
0.136	0.024	0.034
0.848	0.162	0.032
0.692	0.359	0.946
0.727	0.908	0.385
0.116	0.287	0.537
0.074	0.980	0.383
0.262	0.253	0.671
0.385	0.160	0.498
0.317	0.815	0.328
0.923	0.500	0.336
0.057	0.872	0.600
0.441	0.993	0.965

① time between accident $TB \sim \exp(4)$

$F^{-1}(u) = -E(X)\ln(1-u)$

$TB = -4\ln(1-u)$

$Acc_1 = Acc_0 + TB$

If $TB \geq 30 \to$ stop

$Acc_1 = 0 - 4\ln(1-0.737) = 5$ accident in day 5

$Acc_2 = 5 - 4\ln(1-0.293) = 4$ acc in day 4

$Acc_3 = 4 - 4\ln(1-0.136) = 4$ a in day 5

$Acc_4 = 4 - 4\ln(1-0.848) = 3$

$Acc_5 = 3 - 4\ln(1-0.692) = 1$

$Acc_6 = 1 - 4\ln(1-0.727) = 4$

$Acc_7 = 1 - 4\ln(1-0.116) = 1$

$Acc_8 = 1 - 4\ln(1-0.74) = 1$

$Acc_9 = 1 - 4\ln(1-.762) = 0$

$Acc_{10} = 0 - 4\ln(1-.385) = 1$

$Acc_{11} = 1 - 4\ln(1-.317) = 0$

$Acc_{12} = 0 - 4\ln(1-.923) = 10$

$Acc_{13} = 10 - 4\ln(1-0.57) = 10$

$Acc_{14} = 10 - 4\ln(1-0.441) = 8$

② $TB \sim \text{Uniform}(2,6)$

$F(x) = \frac{x-a}{b-a} = \frac{x-2}{4}$

$u = \frac{x-2}{4} \to 4u = x-2 \to 4u+2 = x$

③ $TB \sim \text{Bin}(6, 0.75)$

$\frac{9}{15}$

Question #5:

Consider a continuous random variable with the following pdf:

$$f(x) = \begin{cases} \frac{1}{2} & 0 \leq x \leq 1 \\ \frac{3}{4} - \frac{x}{4} & 1 \leq x \leq 3 \end{cases}$$

(a) Construct the algorithm for obtaining random numbers from $f(x)$ using the inverse transform method.

(b) Using your answer in (a), give 10 random numbers from $f(x)$ using the following uniform numbers.

1	2	3	4	5	6	7	8	9	10
0.387	0.336	0.466	0.074	0.184	0.336	0.900	0.875	0.475	0.636
0.774	0.672	0.892	0.148	0.368	0.672	1.8	1.75	0.95	1.272

(c) Assume that the variable X represents the time between arrival to a service station (in hours). The station work hours are from 8 am to 3 pm. The owner wants to determine the expected number arrivals per day using simulation output for 3 days. Using the following uniform numbers for each day, estimate the average number of arrivals per day.

	Day-1	Day-2	Day-3
1	0.194	0.489	0.029
2	0.790	0.300	0.800
3	0.084	0.791	0.764
4	0.111	0.918	0.553
5	0.954	0.638	0.452
6	0.723	0.890	0.744
7	0.551	0.079	0.139
8	0.919	0.926	0.281
9	0.337	0.310	0.384
10	0.831	0.086	0.100

(a) $F(x) = \begin{cases} 0 & 0 > x \\ \frac{1}{2}x & 0 \leq x \leq 1 \\ \frac{3 - x^2}{8} - \frac{1}{2} & 1 \leq x \leq 3 \\ 1 & x > 3 \end{cases}$

$0 \leq x \leq 1 \rightarrow u = \frac{1}{2}x \rightarrow 2u = x$

$1 \leq x \leq 3 \rightarrow u = \frac{3 - x^2}{8} - \frac{1}{2} \rightarrow u + \frac{1}{2} = \frac{3}{8} - \frac{x^2}{8}$

$u + \frac{1}{2} - \frac{3}{8} = -\frac{x^2}{8}$

$8(u + \frac{1}{2} - \frac{3}{8}) = -x^2$

$x = (-8(u + \frac{1}{2} - \frac{3}{8}))^{\frac{1}{2}}$