استعن بالله أولا.. وكن على يقين بأن كل ما ورد في هذه الورقة
تعرفه جيدا وقد تدربت عليه بما فيه الكفاية، فكن مطمئناً

College of Science.
Department of Statistics & Operations Research

كلية العلوم
قسم الإحصاء وبحوث العمليات

Second Midterm Exam
Academic Year 1443-1444 Hijri- First Semester

Exam Information معلومات الامتحان			
Course name	Modeling and Simulation	النمذجة والمحاكاة	اسم المقرر
Course Code	OPER 441	441 بحث	رمز المقرر
Exam Date	2021-11-24	1443-04-19	تاريخ الامتحان
Exam Time	12: 30 PM		وقت الامتحان
Exam Duration	2 hours	ساعتان	مدة الامتحان
Classroom No.			رقم قاعة الاختبار
Instructor Name			اسم استاذ المقرر

Student Information معلومات الطالب		
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Section No.		رقم الشعبة
Serial Number	1	الرقم التسلسلي

General Instructions:

- Your Exam consists of PAGES (except this paper)
- Keep your mobile and smart watch out of the classroom.
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تعليمات عامة:

- عدد صفحات الامتحان صفحة (باستثناء هذه الورقة)
- يجب إبقاء الهواتف والساعات الذكية خارج قاعة الامتحان.
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هذا الجزء خاص بأستاذ المادة

This section is ONLY for instructor

#	Course Learning Outcomes (CLOs)	Related Question (s)	Points	Final Score
1	To know the basics of pseudo random generation and apply different methods of random generation techniques			
2	Chose and fit theoretical distribution on collected data			
3	Define and compute performance measures from simulation models			
4	Recognize and analyze simple models and its main elements for simulation			
5	Understanding how to use computer software (ECXEL) for simulation models			
6	use appropriate statistical techniques to analyze and evaluate outputs of simulation models			
7	Generate random variates from different probability functions and directly from collected data			
8	build simple simulation models of real-life problems			

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Question #1:

Given the ***LCG*** with $a = 5$, $c = 1$ and $m = 19$. Answer the following:

1. Does this generator have a full cycle? Prove your answer?
2. Using this ***LCG*** and a seed value $R_0 = 13$, simulate tossing an unfair coin with P{H} = 0.6 until head appear twice.

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1. Cond 1 m and c most have no commun deviser other than 1

$c = \{\boxed{1}\}$ $m = \{\boxed{1}, 19\}$ ∴ then cond 1 holds

2. if q is a prime number that devides m then it should devide (a-1)

pime numbers in $m = \{1, 19\}$ then $a - 1 = \frac{4}{1} = 4$

$= \frac{4}{19} = \times$ since 19 is not a devider of (a-1) then Cond #2 does not hold which mean not all three conditions were satisfied So this LCG does not have a full Period.

2 $R_0 = 13$, $a = 5$, $C = 1$, $m = 19$

$R_1 = (aR_0 + c) \bmod m = (5(13)+1) \bmod 19 = 66 - 19(3) = \boxed{9}$

$U_1 = \frac{R_1}{m} = \frac{9}{19} = 0.473$

$R_2 = (5(9)+1) \bmod 19 = 46 - 19(2) = \boxed{8}$, $U_2 = \frac{8}{19} = 0.421$

$R_3 = (5(8)+1) \bmod 19 = 41 - 19(2) = \boxed{3}$, $U_3 = \frac{3}{19} = 0.157$

$R_4 = (5(3)+1) \bmod 19 = 16 - 19(0) = \boxed{16}$, $U_4 = \frac{16}{19} = 0.842$

$R_5 (5(16)+1) \bmod 19 = 81 - 19(4) = \boxed{5}$, $U_5 = \frac{5}{19} = 0.263$

$R_6 (5(5)+1) \bmod 19 = 26 - 19(1) = \boxed{7}$, $U_6 = \frac{7}{19} = 0.368$

$R_7 (5(7)+1) \bmod 19 = 36 - 19(1) = \boxed{17}$, $U_7 = \frac{17}{19} = 0.894$

$R_8 (5(17)+1) \bmod 19 = 86 - 19(4) = \boxed{10}$, $U_8 = \frac{10}{19} = 0.526$

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Question #2:

The period of time (in months) between rainfalls in Abha city is modeled using the following pdf:

$$f(x) = 1.06\, e^{\frac{-x}{2}} \quad ; \quad 1 \le x \le 4$$

Where random variable X is time between rainfalls in months.

a) Write the inverse transform for measuring the time between rainfalls.
b) Simulate the next 4 rainfalls (in months) in Abha city.
c) If the we want to simulate the rainfall that is at least 8 months from now. Write the simulated values.

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0.507	0.103	0.362	0.029	0.584
0.068	0.420	0.742	0.653	0.021
0.432	0.714	0.259	0.799	0.207
0.896	0.237	0.929	0.270	0.524
0.108	0.183	0.466	0.721	0.793
0.131	0.321	0.868	0.428	0.160

a]

1. $cdf = \int_1^x 1.06 e^{-\frac{x}{2}} dx = \left. \frac{1.06 e^{-\frac{x}{2}}}{-\frac{1}{2}} \right|_1^x = -\frac{1.06 e^{-\frac{x}{2}}}{\frac{1}{2}} + \frac{1.06 e^{-\frac{1}{2}}}{\frac{1}{2}}$

$= -\frac{1.06 e^{-\frac{x}{2}}}{\frac{1}{2}} + 1.285 = u = F(x)$

$u = \frac{-1.06 e^{-\frac{x}{2}}}{\frac{1}{2}} + 1.285 \Rightarrow u - 1.285 = \frac{-1.06 e^{-\frac{x}{2}}}{\frac{1}{2}}$

$\Rightarrow \frac{1}{2}(u - 1.285) = -1.06 e^{-\frac{x}{2}} \Rightarrow -\frac{(u - 1.285)}{2(1.06)} = e^{-\frac{x}{2}} = \ln\left(\frac{-(u-1.285)}{2(1.06)}\right) = -\frac{x}{2}$

$-2\ln\left(\frac{-(u-1.285)}{2(1.06)}\right) = x$

b] $U = 0.507$, $x = -2\ln\left(\frac{-(0.507 - 1.285)}{2(1.06)}\right) = 2.004$

$U = 0.103$, $x = -2\ln\left(\frac{-(0.103 - 1.285)}{2(1.06)}\right) = 1.168$

$U = 0.362$, $x = -2\ln\left(\frac{-(0.362 - 1.285)}{2(1.06)}\right) = 1.663$

$U = 0.029$, $x = -2\ln\left(\frac{-(0.029 - 1.285)}{2(1.06)}\right) = 1.046$

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Question #3:

Students arrive at a self-service cafeteria at the rate of one every 30±20 seconds. Is estimated that 40% of students go to the sandwich bar, where every student prepares has own sandwich in 60±30 seconds. The rest go to the main counter, where one server spoons the prepared meal onto a plate in 45±30 seconds. All students take their seats in the cafeteria and spend 20 ± 10 minutes eating. After eating, 10% of the students go back for dessert, and return to their table to spend an additional 10±2 minutes in the cafeteria.

1. Simulate until ~~20~~ 10 people have left the cafeteria using the following table of U[0,1] numbers.
2. At the final simulation time, estimate the following from the simulation data:
   a. How many students are still in the cafeteria
   b. What percentage of students at the sandwich bar.
   c. What percentage of students at the main course counter.
   d. What percentage of students on tables
   e. What percentage of students take dessert.
3. From the simulation data, what is the average time that a student who gets a sandwich spends in the cafeteria until he leaves after finishing his entire meal.

Std 1	0.454	0.516	0.922	0.405	0.965	0.686	0.623	0.327
Std 2	0.046	0.239	0.356	0.686	0.577	0.234	0.439	0.588
Std 3	0.024	0.034	0.134	0.534	0.648	0.244	0.525	0.340
Std 4	0.162	0.032	0.224	0.209	0.441	0.493	0.850	0.607
Std 5	0.359	0.946	0.607	0.420	0.058	0.197	0.336	0.353
Std 6	0.908	0.385	0.181	0.683	0.067	0.856	0.736	0.328
Std 7	0.287	0.537	0.196	0.087	0.297	0.772	0.564	0.633
Std 8	0.980	0.383	0.485	0.909	0.061	0.201	0.356	0.361
Std 9	0.253	0.671	0.545	0.765	0.651	0.030	0.839	0.546
Std 10	0.160	0.498	0.090	0.432	0.187	0.588	0.248	0.954

$\frac{10}{15}$

Question #4:

A traffic department of a small city wants to simulate the occurrence time for the accident in an intersection using some different choices of distributions to estimate the number of accidents per month (30 days). Use the following U(0,1) numbers to compute the simulation under the following assumptions.

1. Estimate the number of accidents in the 1st month assuming the time between accident is an exponential distribution with mean 4 days.
2. Estimate the number of accidents in the 1st month assuming the time between accident is a discrete uniform distribution with mean 4 days and ±2 days.
3. Estimate the number of accidents in the 1st month assuming the time between accident is Binomial distribution with parameters $n = 6$ and $p = 0.75$.
4. Compare the result from simulation in (1), (2) and (3) with exact mean for each question.

Use uniform numbers (as needed) by columns until you finish all numbers in the column then move to the next.

↓ Part 1	↓ Part 2	↓ Part3
0.737	0.454	0.516
0.293	0.046	0.239
0.136	0.024	0.034
0.848	0.162	0.032
0.692	0.359	0.946
0.727	0.908	0.385
0.116	0.287	0.537
0.074	0.980	0.383
0.262	0.253	0.671
0.385	0.160	0.498
0.317	0.815	0.328
0.923	0.500	0.336
0.057	0.872	0.600
0.441	0.993	0.965

1| exp $cdf = 1 - e^{-\theta x}$ $E(x) = \frac{1}{\theta} = 4$

$= 1 - e^{-\frac{1}{4}(30)} = 0.99$

2| uniform $Cdf = \frac{(x-a)}{(b-a)}$ $-2 \le x \le 2$

$= \frac{(30+2)}{(2+2)} = 8$

3| binomial $\binom{n}{x} p^x (q)^{n-x}$

$= \binom{6}{4} (0.75)^4 (0.25)^2 =$

$\frac{4}{15}$

Question #5:

Consider a continuous random variable with the following pdf:

$$f(x) = \begin{cases} \frac{1}{2} & 0 \le x \le 1 \\ \frac{3}{4} - \frac{x}{4} & 1 \le x \le 3 \end{cases}$$

(a) Construct the algorithm for obtaining random numbers from $f(x)$ using the inverse transform method.

(b) Using your answer in (a), give 10 random numbers from $f(x)$ using the following uniform numbers.

1	2	3	4	5	6	7	8	9	10
0.387	0.336	0.466	0.074	0.184	0.336	0.900	0.875	0.475	0.636

?

(c) Assume that the variable X represents the time between arrival to a service station (in hours). The station work hours are from 8 am to 3 pm. The owner wants to determine the expected number arrivals per day using simulation output for 3 days. Using the following uniform numbers for each day, estimate the average number of arrivals per day.

	Day-1	Day-2	Day-3
1	0.194	0.489	0.029
2	0.790	0.300	0.800
3	0.084	0.791	0.764
4	0.111	0.918	0.553
5	0.954	0.638	0.452
6	0.723	0.890	0.744
7	0.551	0.079	0.139
8	0.919	0.926	0.281
9	0.337	0.310	0.384
10	0.831	0.086	0.100

a) cdf $\int_0^x \frac{1}{2}\,dx + \int_1^x \frac{3}{4} - \frac{x}{4}\,dx = \frac{1}{2}\left[x\Big|_0^x\right] + \frac{3x}{4} - \frac{x^2}{8}\Big|_1^x$

$= \frac{x}{2} + \frac{3x}{4} - \frac{x^2}{8} = \frac{10x}{8} - \frac{x^2}{8} = \frac{80x - 8x^2}{64} = \frac{8(10x - x^2)}{64}$

$= \frac{10x - x^2}{8}$