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King Saud University جـامـعـة الملك سعود

College of Science. كلية العلوم
Department of Statistics & Operations Research قسم الإحصاء وبحوث العمليات

Final Exam
Academic Year 1442-1443 Hijri- First Semester

Exam Information معلومات الامتحان			
Course name	Modeling and Simulation	النمذجة والمحاكاة	اسم المقرر
Course Code	OPER 441	441 بحث	رمز المقرر
Exam Date	2021-12-23	1443-05-19	تاريخ الامتحان
Exam Time	08: 00 AM		وقت الامتحان
Exam Duration	3 hours	ثلاث ساعات	مدة الامتحان
Classroom No.			رقم قاعة الاختبار
Instructor Name			اسم استاذ المقرر

Student Information معلومات الطالب		
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Section No.		رقم الشعبة
Serial Number		الرقم التسلسلي

General Instructions: **تعليمات عامة:**

- Your Exam consists of [ ] PAGES (except this paper)
- Keep your mobile and smart watch out of the classroom.
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- عدد صفحات الامتحان [ ] صفحة. (بإستثناء هذه الورقة)
- يجب إبقاء الهواتف والساعات الذكية خارج قاعة الامتحان.
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هذا الجزء خاص بأستاذ المادة
This section is ONLY for instructor

#	Course Learning Outcomes (CLOs)	Related Question (s)	Points	Final Score
1	To know the basics of pseudo random generation and apply different methods of random generation techniques			
2	Chose and fit theoretical distribution on collected data			
3	Generate random variates from different probability functions and directly from collected data			
4	build simple simulation models of real-life problems			
5	Define and compute performance measures from simulation models			
6	Recognize and analyze simple models and its main elements for simulation			
7	Understanding how to use computer software (ECXEL) for simulation models			
8	use appropriate statistical techniques to analyze and evaluate outputs of simulation models			

	Q. #1	Q. #2	Q. #3	Q. #4	Q. #5	Q. #6	Total
	23	20	13	15	24	20	115
Score	16	20	13	15	23	20	107

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Question # 1:

Consider a car insurance company collected the following report about their clients:

	Male				Female			
Age	**Total # of clients**	**Total #accidents per group**	**Min Claim (per client)**	**Max Claim (per client)**	**Total # of clients**	**Total #accidents per group**	**Min Claim (per client)**	**Max Claim (per client)**
18 – 28	113	284	350	9000	312	435	550	7500
28 – 38	258	314	950	13000	271	361	1050	11300
38 – 48	546	395	1300	6500	687	302	1450	9300
48 – 58	420	150	2000	11000	348	129	2800	11800
58 – 68	302	145	4000	20000	354	159	3700	32000
68 – 90	93	23	15000	80000	12	43	21000	65000
Total	**1732**	**1311**			**1984**	**1429**		

Answer the following questions:

1. How to simulate the gender type of the client from the table? Write the algorithm.

Total clients = 1732 + 1954 = 3716

male $0 \le u < 0.466$

female $0.466 \le u < 1$

2. Give a simulated gender type of the 4 clients using the following uniform values:

u~U[0,1]	0.1466	0.7061	0.8585	0.4944
	male	female	female	female

3. Given that you want to simulate the age of a male client between 28 and 38, what you will do assuming uniform distribution? let x is age of male [28,38]

$x = 28 + 10u$

4. How to simulate the exact age of the client from the table assuming uniform distribution? Write the algorithm. x is age [18,90]

$x = 18 + 72u$

?

 $x = 18 + 72u$

5. Give a simulated exact age (round to 1 decimal digit ##.#) for 4 clients using the following uniform values assuming uniform distribution in each category:

u~U[0,1]	0.5915	0.1998	0.9780	0.6281
	50.6	32.4	88.4	63.2

6. How to generate the amount of the claim for each client from the table assuming integer uniform? Write the general algorithm.

$x \in [a, b]$

$x = a + (b - a + 1)u$.

$x \in [350, 800000]$

$x = 350 + 799651u$

7. Give a simulated exact amount of the claim for 4 clients using the following uniform values assuming discrete uniform distribution in each category:

u~U[0,1]	0.3887	0.0739	0.9482	0.7715
	309955	59212.4	755604.6	614860.6

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Question # 2:

Consider an investment with a monthly return on investment. The monthly percentage of return on investment is a random variable $X\%$ given by the following probability function:

$$f(x)=\begin{cases}\frac{|x|}{10}, & -2\le x\le 4\\ 0, & \text{otherwise.}\end{cases} \quad \textit{where} \quad f(x)=\begin{cases}\frac{-x}{10} & ; -2\le x\le 0\\ \frac{x}{10} & ; 0\le x\le 4\end{cases}$$

Answer the following:

1) Write the inverse algorithm for generating the monthly percentage of return on investment.

$F(x)$ for $-2\le x\le 0 \Rightarrow -\frac{1}{10}\int_{-2}^{x} y\,dy = -\frac{1}{20}\left.y^2\right]_{-2}^{x} \Rightarrow -\frac{x^2}{20}+\frac{4}{20}$

$F(x)$ for $0\le x\le 4 \Rightarrow -\frac{0^2}{20}+\frac{1}{5}+\int_0^x \frac{y}{10}\,dy = \frac{1}{5}+\frac{x^2}{20}$

$F(x) = \begin{cases} -\frac{x^2}{20}+\frac{1}{5}, & -2\le x\le 0 \\ \frac{1}{5}+\frac{x^2}{20}, & 0\le x\le 4 \end{cases}$

① $-\frac{x^2}{20}+\frac{1}{5}=u \Rightarrow 20u-4=-x^2 \Rightarrow -\sqrt{4-20u}=x$ for $0\le u<0.2$

② $\frac{x^2}{20}+\frac{1}{5}=u \Rightarrow 20u-4=x^2 \Rightarrow \sqrt{20u-4}=x$, $0.2\le u\le 1$

F

2) Using simulation and the following U[0,1] numbers, evaluate the results of the investment for 6 months with initial budget of 100,000 SR.

Mon.	U	X %	$100,000(1+X)^6$
1	0.032	−1.53 %	89510
2	0.138	−1.11 %	93522
3	0.629	2.91 %	118781
4	0.691	3.13 %	100312
5	0.006	−1.97 %	88747
6	0.413	2.06 %	113014

3) From simulation, compute the average and standard deviation of the monthly percentage of return on investment. $\text{average} = \frac{\sum \text{rates}}{\text{num}} = 0.53\%$

$E(X^2) = \frac{1}{6}(0.0153^2 + 0.0111^2 + \dots + 0.0206^2) = 0.0005$

$Var(X) = 0.0005 - 0.0053^2 = 0.00047 \Rightarrow SD = \sqrt{Var} = 0.02 = 2\%$

4) From simulation compute the probability that the company will have profit more than 10000 SR per month. $P(\text{Profit} > 10000) = P(\text{inv} > 110000) = 0$

other way $Pr(\text{rate} > 10\%) = 0$ impossible because $\max(X) = 4$.

5) From simulation output, what is the probability of losing.

$\frac{\text{num inv} < 100000}{\text{num of inv}} = \frac{3}{6} = \frac{1}{2}$

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Question #3:

Patients arrive to a hospital's emergency room according to a Poisson process with rate 8 patients per hour. Patients come in three different health conditions. The patients are categorized according to their condition as critical, serious, or stable. In the past year, statistics show that:

i. 10% of the emergency room patients were ***critical***; and take random amount of treatment of Erlang with parameters $\alpha = 2$ and $\lambda = 0.5$ per hour

ii. 30% of the emergency room patients were ***serious***; and take random amount of treatment of Exponential with average time of 2 hours.

iii. the rest of the emergency room ***patients*** were stable; and take random amount of treatment of integer uniform between 15 min and 30 min.

After treatment at the ER, statistics show that: 40% of the critical patients died; 10% of the serious patients died; and 1% of the stable patients died.

Answer the following with ***True*** or ***False***:

F	**1.**	To simulate the ER system, we have to simulate 3 independent random processes only: Patients Arrival Time PAT(n), Patient Treatment Time PTT(n) and Patient Exit Conditions PEC(n). + CP
T	**2.**	To simulate the patient's arrival to ER system, we need to use the inverse transform for exponential distribution with mean value = 1/8.
F	**3.**	The time between arrivals T(n) is simulated by the function: $-8\, ln(1-u)$ where $u \sim U[0,1]$
F	**4.**	The arrival time of the patient (n) = PAT(n) is equal to PAT(n−1) − T(n). Where T(n) time between arrivals from simulation.
F	**5.**	The condition of the patient CP(n) can be generated as a random process with discrete uniform with values 1, 2, 3.
F	**6.**	To simulate the condition of the patient CP(n), get $u_1 \sim U[0,1]$: if $0 \le u_1 \le 0.1$ then CP(n)=1 (***Critical***). If $0.1 < u \le 0.3$ then CP(n)=2 (***serious***). If $0.3 < u \le 1$ then CP(n)=3 (***stable***) 0.4 0.4
T	**7.**	Patient Treatment Time PTT(n) is a random variable depends on the condition of the patient CP(n).
F	**8.**	If the condition of the patient CP(n)=2 (***serious***) then PTT(n)~DU (15, 30 min); using discrete inverse method: take $u_2 \sim U[0,1]$ and PTT(n) = 15 + int[(1+30 − 15) u_2]
F	**9.**	If the condition of the patient CP(n) = 1 (***critical***) then we generate the Patient Treatment Time PTT(n) ~ Er($\alpha = 2$ and $\lambda = 0.5$ per hour) by using inverse transform method directly.
T	**10.**	It is always possible to use acceptance/rejection method to generate the Patient Treatment Time PTT(n) ~ Er($\alpha = 2$ and $\lambda = 0.5$ per hour).
T	**11.**	Patient Exit Conditions PEC(n) is also a random variable depends on the condition of the patient CP(n).
F	**12.**	If condition of the patient CP(n) = 2 (serious), then Patient Exit Conditions PEC(n) ~ Bernoulli (p = 0.1) with $w \sim U[0,1]$ If $w < 0.1$ then PEC(n) = ***Lived***, Else PEC(n) = ***Died***
T	**13.**	If condition of the patient CP(n) = 3 (stable), then Patient Exit Conditions PEC(n) ~ Bernoulli (p = 0.01) with $w \sim U[0,1]$ If $w < 0.99$ then PEC(n) = ***Lived***, Else PEC(n) = ***Died***
T	**14.**	The patient Departure time DT(n) is equal to patient arrival time PAT(n) + Patient Treatment Time PTT(n).

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Question #4:

An insurance policy pays for the insured 1000 SR per day spent admitted in a hospital for up to three days. If the insured spend more than three days, the insurance company pays 500 SR per day for each extra day of hospitalization thereafter. The number of days that an insured customer spends in the hospital, X, is a discrete random variable with probability function:

$$P[X=k]=\begin{cases}\frac{6-k}{15}, & k=1,2,3,4,5\\ 0, & \text{otherwise.}\end{cases}$$

Answer the following:

1) Write the inverse algorithm for generating number of days for the insured clients
2) Write the simulation algorithm and use the following U[0,1] numbers, give the results for the 9 insured clients.

u	Days	Cost (n)	u	Days	Cost (n)	u	Days	Cost (n)
0.032	1	1000	0.691	3	3000	0.823	4	3500
0.138	1	1000	0.006	1	1000	0.776	3	3000
0.623	3	3000	0.413	2	2000	0.306	1	1000

3) Compute the average and standard deviation of amount that the insurance company will pay per claim.
4) Compute the average and standard deviation of number of days that a client spends in the hospital.
5) From simulation compute the probability that the company will pay more than 2000 SR.

1.

k	1	2	3	4	5
$P(X=k)$	$\frac{1}{3}$	$\frac{4}{15}$	$\frac{1}{5}$	$\frac{2}{15}$	$\frac{1}{15}$

$\Rightarrow$

k	1	2	3	4	5
$F(k)$	$\frac{1}{3}$	$\frac{3}{5}$	$\frac{4}{5}$	$\frac{14}{15}$	1

$\therefore$ inverse fun $\Rightarrow$

$$\begin{cases}1, & 0\le u<\frac{1}{3}\\ 2, & \frac{1}{3}\le u<\frac{3}{5}\\ 3, & \frac{3}{5}\le u<\frac{4}{5}\\ 4, & \frac{4}{5}\le u<\frac{14}{15}\\ 5, & \frac{14}{15}\le u\le 1\end{cases}$$

3. $P(x)=\begin{cases}\frac{4}{9}, & x=1000\\ \frac{1}{9}, & x=2000\\ \frac{3}{9}, & x=3000\\ \frac{1}{9}, & x=3500\end{cases}$

$\therefore$ average $=\frac{\text{sum of cost}}{\text{num of claims}}=2055.5$ \ claim

$\therefore E(x^2)=5250000$

$var(x)=5250000-(2055.5)^2=1024919.75$

$SD(x)=1012.4$

4. $P(y)=\begin{cases}\frac{4}{9}, & y=1\\ \frac{1}{9}, & y=2\\ \frac{3}{9}, & y=3\\ \frac{1}{9}, & y=4\end{cases}$

$\therefore$ average $=\frac{\text{sum of Days}}{\text{num of claims}}=2.1$

$E(y^2)=5.6$

$var(y)=5.6-(2.1)^2=1.256$

$\therefore SD(y)=1.12$

5. $P(\text{company pay more than }2000)=P(x=3)+P(x=4)$

$=\frac{3}{9}+\frac{1}{9}$

$=\frac{4}{9}$

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2B

Question #5: Consider the following are functions in EXCEL.

A	RANDBETWEEN(*a*,*b*)	E	GAMMA.DIST(*a*,*b*,*c*,*d*)	I	RAND()
B	NORM.DIST(*a*,*b*,*c*,*d*)	F	DATA TABLE	J	BINOM.INV(*a*,*b*,*c*)
C	GAMMA.INV(*a*,*b*,*c*,*d*)	G	BINOM.DIST(*a*,*b*,*c*)	K	CONFIDENCE.T(*a*,*b*,*c*)
D	VLOOKUP(a,b,c)	H	NORM.INV(*a*,*b*,*c*)		

Read each statement and assign the letter of EXCEL function from the table to the statement.

The Statement	Excel Function
1. Used to compute the PDF function of the Normal distribution with $\mu=a$ and $\sigma=b$ at the value *a*	B
2. Used to generates random values from relative frequency table.	D
3. Used generate integer random values with equal distributions between (*a*) and (*b*) including the (*a*) and (*b*)	A
4. Used to generate random values from Exponential distribution	C
5. Used to compute the upper limit of the confidence interval from sample	K
6. Used generate integer random values with equal distributions between (*a*) and (*b*) without the (*a*) and (*b*)	A
7. Used to generate random values from Erlang distribution	C
8. Used to compute the PDF function of the Gamma distribution at the value *a*	E
9. Used to compute the CDF function of the Normal distribution with $\mu=a$ and $\sigma=b$ at the value *a*	B
10. Used to Generates random values from Gamma distribution	C
11. Used to generate random values from of integer values (*a*) to (*b*) with different probabilities	J
12. Used to compute the CDF function of the Gamma distribution at the value *a*	E
13. Used to generate continuous random values between *a* and *b* with uniform distribution	A
14. Used to compute the PMF function of the Binomial distribution with value *a*	G
15. Used to run the simulation model for many times and record the measures from every run	F
16. Used to compute the CDF function of the Binomial distribution with at the value *a*	G
17. Used to compute the CDF function of the Exponential distribution at the value *a*	E
18. Used to generate random values from standard deviation	H
19. Used to compute the CDF function of the Erlang distribution at the value *a*	E
20. Used to Generates random values from normal distribution with mean *b* and standard deviation *c*	H
21. Used to generate continuous random values between 0 and 1 with uniform distribution	I
22. Used to compute $Z_{\alpha/2}$	H
23. Used to compute the lower limit of the confidence interval from sample	K
24. Used to generate random values from binomial distribution	J
25. Used to compute the half width of the confidence interval from sample	K

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Question #6:

Customer arrive to a car repair station for maintenance according to a random process with time between arrivals (in hours) follows Erlang distribution with parameters r=2 and $\lambda = 0.5$. The arriving cars come to a single repairman to perform the maintenance. It is assumed that the repair time (in hours) follows a random time that follows a Weibull distribution with parameters $\alpha = 2$ and $\beta=3$. The cars are repaired according to the order of their arrival. The repair station has a space that can hold any number of cars. The following data was collected.

1. Define all the variables and random process and the logic of the system
2. Draw the flow chart for the system for simulation of 10 cars.
3. Complete the results of simulation in the table below? *Use uniform values as needed starting from u_1*

car#	time btwn cars	Arrival Time	Service time	start time	Wait? (0/1)	Waiting Time	Exit time	Idle Time
u_1	0.164	0.092	0.588	0.260	0.608	0.094	0.722	0.260
u_2	0.938	0.412	0.879	0.138	0.018	0.371	0.247	0.138
1	5.9	5.9	2.8	5.9	0	0	8.7	5.9
u_1	0.129	0.874	0.841	0.308	0.165	0.136	0.368	0.308
u_2	0.355	0.992	0.690	0.040	0.661	0.904	0.308	0.040
2	1.15	7.05	4.1	8.7	1	1.65	12.8	0
u_1	0.130	0.371	0.220	0.090	0.927	0.680	0.032	0.090
u_2	0.012	0.563	0.668	0.262	0.416	0.607	0.737	0.262
3	0.3	7.35	1.5	12.8	1	5.45	14.3	0
u_1	0.999	0.176	0.429	0.131	0.830	0.371	0.362	0.131
u_2	0.020	0.071	0.079	0.221	0.891	0.190	0.067	0.221
4	13.9	21.25	2.2	21.25	0	0	23.45	6.95
u_1	0.840	0.021	0.492	0.843	0.428	0.062	0.112	0.843
u_2	0.128	0.137	0.340	0.038	0.446	0.848	0.944	0.038
5	3.9	25.15	2.5	25.15	0	0	27.65	1.7
u_1	0.584	0.211	0.855	0.344	0.844	0.296	0.311	0.344
u_2	0.577	0.647	0.659	0.850	0.977	0.558	0.646	0.850
6	3.5	28.65	4.2	28.65	0	0	32.85	1
u_1	0.684	0.815	0.407	0.053	0.072	0.969	0.442	0.053
u_2	0.305	0.682	0.212	0.045	0.783	0.014	0.076	0.045
7	3	31.65	2.2	32.85	1	1.2	35.05	0
u_1	0.318	0.117	0.843	0.614	0.217	0.009	0.532	0.614
u_2	0.716	0.408	0.439	0.874	0.429	0.332	0.287	0.874
8	3.3	34.95	4.1	35.05	1	0.1	39.15	0
u_1	0.108	0.034	0.028	0.328	0.325	0.243	0.825	0.328
u_2	0.532	0.833	0.628	0.280	0.144	0.157	0.131	0.280
9	1.7	36.65	0.5	39.15	1	2.5	39.65	0
u_1	0.918	0.115	0.798	0.260	0.960	0.750	0.268	0.260
u_2	0.835	0.991	0.569	0.823	0.056	0.603	0.848	0.823
10	8.6	45.25	3.8	45.25	0	0	49.05	5.6

4. From the simulation run, what is the percentage of customers who wait in line? 50%
5. What is the probability that the repairman is IDLE during the *simulation Time*? $\frac{\text{Idle time}}{\text{sim time}} = \frac{21.15}{49.05} = 0.43$
6. What is the average waiting time in line? $\frac{\text{wait}}{10} = 1.09$