

College of Science.
Department of Statistics & Operations
Research

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قسم الإحصاء وبحوث العمليات

First Midterm Exam
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Exam Information معلومات الامتحان			
Course name	Modeling and Simulation		اسم المقرر
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Classroom No.	Lecturing Class Room		رقم قاعة الاختبار
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Serial No.		الرقم التسلسلي

Notes for the Exam:

1. Each question shouldn't take from you more than 20 minutes
2. Answers on the same papers
3. Use the back of the pages for more space make sure you write the question number.

Question #1: Answer the following with True or False:

T	1. Simulation model always gives different results in each simulation run.
F	2. In call center model, the number of callers in the system is an attribute
T	3. Simulation modeling is not good if there is less data or no estimates available.
T	4. "Verification" step is to make sure that the simulation cod is running correctly.
F	5. In the call center, the time that a caller takes on line is a state variable
T	6. The state variable is defined for all time of simulation
T	7. "Validation" step is to make sure that the logic of the simulation program is running correctly as in the system.

F	8. Simulating flight distance for an airplane is a discrete system simulation.
F	9. In the call center, the line is busy is an attribute
F	10. In Bank simulation, the variable (X = amount of money a customer has) is an a state variable for the system.
T	11. Results in simulation is always need statistical analysis and doesn't give exact solution
F	12. Every simulation run for the same model give the same results.
T	13. The sequence of random numbers generated from a given seed is called a random number a <i>Stream</i> .
F	14. LCG has full period if and only if we get exactly $(m-1)$ random numbers.
F	15. The second condition for LCG to have Full period is that all prim divisors of c divide $m-1$
F	16. Always, if the LCG repeat the starting value R_0 then the function has a full period.
F	17. Every LCG must satisfy all three conditions to have full period
F	18. Every time you run the simulation model you get the same output data.
F	19. The LCG is the function, it is possible always to find values for: R_0 , a , c and m to generate more than m different random numbers.
F	20. The LCG is the function $X_n = (aX_{n-1} + c) / (m)$.
T	21. The LCG is always used to generate pseudo-random numbers between 0 and 1.
F	22. The state variable $X(t)$ total fuel consumed in a car is used in a discrete simulation systems
T	23. Static systems means the parameter of the system are fixed over time.

Question #2:

For each of the experiments below, answer with () or () in the next column.

The Experiment	The Property
1. Customers arrive to a service either male or female. Percentage of male is p . The random variable X is number of male arrivals until the 1 st female.	<input checked="" type="checkbox"/> The experiment is Poisson <input checked="" type="checkbox"/> The experiment is Binomial <input checked="" type="checkbox"/> The experiment is Negative-Binomial <input checked="" type="checkbox"/> The vales of $X = 0,1,2, \dots n$ <input checked="" type="checkbox"/> The expected value is np <input checked="" type="checkbox"/> $P\{X=x\} = p(1-p)^{x-1}$
2. Customers arrive to a service either male or female. Average number of male arriving per hour to the service is p . The random variable X is number of male arrivals after in one hour.	<input checked="" type="checkbox"/> The experiment is Geometric <input checked="" type="checkbox"/> The experiment is Exponential <input checked="" type="checkbox"/> The vales of $X = 0,1,2, \dots n$ <input checked="" type="checkbox"/> The expected value of X is p <input checked="" type="checkbox"/> The variance of X is npq <input checked="" type="checkbox"/> $P\{X=x\} = \frac{p^x}{x!} e^{-p}$
3. Customers arrive to a service either male or female. Percentage of male is p . The random variable X is number of male arrivals after the 10 th customer.	<input checked="" type="checkbox"/> The experiment is Geometric <input checked="" type="checkbox"/> The experiment is Binomial <input checked="" type="checkbox"/> The experiment is Exponential <input checked="" type="checkbox"/> The vales of $X = 0,1,2, \dots 10$ <input checked="" type="checkbox"/> The expected value is npq <input checked="" type="checkbox"/> $P\{X=x\} = \binom{10}{x} p^x (1-p)^{x-1}$
4. Customers arrive to a service either male or female. Average number of female arriving per hour to the service is q . The random variable X is the time for next female arrival (in hour).	<input checked="" type="checkbox"/> The experiment is Geometric <input checked="" type="checkbox"/> The experiment is Exponential <input checked="" type="checkbox"/> The experiment is Poisson <input checked="" type="checkbox"/> The vales of $X = 0,1,2, \dots$ <input checked="" type="checkbox"/> The expected value of X is p <input checked="" type="checkbox"/> The expected value of X is q <input checked="" type="checkbox"/> The expected value of X is $1/q$ <input checked="" type="checkbox"/> The variance of X is npq <input checked="" type="checkbox"/> The variance of X is p <input checked="" type="checkbox"/> $P\{X=x\} = \frac{p^x}{x!} e^{-p}$ <input checked="" type="checkbox"/> $P\{X \leq x\} = 1 - e^{-qx}$

Question #3:

Patients arrive to a hospital's emergency room in three different health conditions. Patients are categorized according to their condition as critical, serious, or stable. In the past year:

- i. 10% of the emergency room patients were critical;
- ii. 30% of the emergency room patients were serious;
- iii. the rest of the emergency room patients were stable;
- iv. 40% of the critical patients died;
- v. 10% of the serious patients died; and
- vi. 1% of the stable patients died.

Answer the following:

1. What is the probability that any patient enter the Emergency room will live?
2. Given that a patient survived, calculate the probability that the patient was categorized as serious upon arrival.
3. Given that a patient died, calculate the probability that the patient was categorized as stable condition upon arrival.
4. How do you evaluate the accuracy of the ER in checking the patient's conditions?

Solution

Let **Crt** : patient is in **critical** condition $\rightarrow P\{\text{Crt}\} = 0.10$

Ser: patient is in **serious** condition $\rightarrow P\{\text{Ser}\} = 0.30$

Stb: patient is in **stable** condition $\rightarrow P\{\text{Stb}\} = 0.60$

Also **Die**: patient leave the ER Died and **Sur**: patient leave the ER Survived.

$Pr\{\text{Die} | \text{Crt}\} = 0.40$ $Pr\{\text{Die} | \text{Ser}\} = 0.10$ $Pr\{\text{Die} | \text{Stb}\} = 0.01$

By complement:

$Pr\{\text{Sur} | \text{Crt}\} = 0.60$ $Pr\{\text{Sur} | \text{Ser}\} = 0.90$ $Pr\{\text{Sur} | \text{Stb}\} = 0.99$

1. $P\{\text{any patient enter the Emergency room will live}\} = P\{I\}$

$$= Pr\{\text{Sur} | \text{Crt}\}P\{\text{Crt}\} + Pr\{\text{Sur} | \text{Ser}\}P\{\text{Ser}\} + Pr\{\text{Sur} | \text{Stb}\}P\{\text{Stb}\}$$
$$= 0.60 (0.1) + 0.90 (0.3) + 0.99 (0.6) = \mathbf{0.924}$$

2. $Pr\{\text{patient was categorized as serious} | \text{patient Survived}\}$

$$= Pr\{\text{Ser} | \text{Sur}\} = Pr\{\text{Ser}, \text{Sur}\} / Pr\{\text{Sur}\} = 0.90 (0.3) / 0.924 = 0.2922$$

3. $Pr\{\text{patient was categorized as stable} | \text{patient Died}\}$

$$= Pr\{\text{Stb} | \text{Die}\} = Pr\{\text{Stb}, \text{Die}\} / Pr\{\text{Die}\}$$

$$Pr\{\text{Die}\} = P\{\text{Die} | \text{Crt}\}P\{\text{Crt}\} + P\{\text{Die} | \text{Ser}\}P\{\text{Ser}\} + P\{\text{Die} | \text{Stb}\}P\{\text{Stb}\}$$
$$= 0.40 (0.10) + 0.10 (0.30) + 0.01 (0.60) = 0.076$$

$$Pr\{\text{Stb} | \text{Die}\} = Pr\{\text{Stb}, \text{Die}\} / Pr\{\text{Die}\} = 0.01 (0.60) / 0.076 = 0.079$$

4. How do you evaluate the accuracy of the ER in checking the patient's conditions?

From part 3 the probability that the patient is not categorized stable given that the patient died after treatment is very minimal (0.079) which means that the error in categorizing the patient is very low. So the ER categorization process is very accurate.

Question #4

An insurance company insures a large number of homes. The insured value (in 100,000 SR), X , of a randomly selected home is assumed to follow a distribution with density function

$$f(x) = \begin{cases} 3x^{-4}, & x > 1 \\ 0, & \text{otherwise.} \end{cases}$$

Answer the following:

1. What is the expected insurance value?
2. What is the CDF function of X ?
3. What is the variance of the insurance value?
4. Given that a randomly selected home is insured for at least 1.5, calculate the probability that it is insured for less than 2.

Solution

1. $E[\text{insurance value}] =$

$$\int_1^{\infty} x(3x^{-4})dx = \left[\frac{3x^{-2}}{-2} \right]_1^{\infty} = 0 + \frac{3}{2} = 1.5 \text{ (100,000) SR}$$

2. $CDF = Pr\{X \leq x\} =$

$$= \int_1^x (3t^{-4})dt = \left[\frac{3t^{-3}}{-3} \right]_1^x = 1 - x^{-3} \quad ; x > 1$$

3. $Var[X] = E[X^2] - (E[X])^2$

$$E[X^2] = \int_1^{\infty} x^2(3x^{-4})dx = \left[\frac{3x^{-1}}{-1} \right]_1^{\infty} = 0 - \frac{3}{-1} = 3$$
$$Var[X] = E[X^2] - (E[X])^2 = 3 - 1.5^2 = 0.75$$

4. $Pr\{X \leq 2 \mid X \geq 1.5\} ??$

$$Pr\{X \geq 2 \mid X \geq 1.5\} = \frac{Pr\{X \leq 2 \text{ and } X \geq 1.5\}}{Pr\{X \geq 1.5\}}$$
$$Pr\{X \geq 2 \mid X \geq 1.5\} = \frac{F(2) - F(1.5)}{1 - F(1.5)} = \frac{(1 - 2^{-3}) - (1 - (1.5)^{-3})}{(1.5)^{-3}}$$
$$= \frac{(-0.125) + (0.2963)}{0.2963} = 0.578$$

Question #5:

Conceder an LCG generator with the following sequence of random numbers:

R(n)	U(n)
0	0.000
1	0.053
6	0.316
12	0.632
4	0.211
2	0.105
11	0.579
18	0.947
15	0.789
16	0.842
5	0.263
7	0.368
17	0.895
10	0.526
13	0.684
9	0.474
8	0.421
3	0.158
14	0.737

Solution

1. what is the value of m for the LCG? why?

We know that $U_i = R_i/m$

Then from $R(n) = 1 \rightarrow 0.053 = 1/m \rightarrow m = 1/0.053 = 18.86$

Then m must equal to 19 because total number of of $R(n) = 19$

2. Using $R_0 = 0$ and $R_1 = 1$, find the value of c for the LCG?

$R_0 = 0$ and $R_1 = 1$

$R_1 = (a R_0 + c) \text{ mod } 19$

$1 = (a \cdot 0 + c) \text{ mod } 19 \rightarrow (c) \text{ mod } 19 = 1$

Since always $c < 19 \rightarrow c = 1$

3. Using $R_0 = 0$, $R_1 = 1$ and $R_2 = 6$, find the value of a for the LCG?

$R_0 = 0$ and $R_1 = 1$ and $R_2 = 6$

$R_1 = (a R_0 + 1) \text{ mod } 19$

$R_2 = (a R_1 + 1) \text{ mod } 19$

$1 = (1) \text{ mod } 19 \rightarrow (c) \text{ mod } 19 = 1 \rightarrow c = 1$

$6 = (a \cdot 1 + 1) \text{ mod } 19 \rightarrow (a + 1) \text{ mod } 19 = 6 \rightarrow \text{since } a < 19 \rightarrow a = 5$

Answer the following:

- From the sequence above, what is the value of m for the LCG? why?
- Using your answer in (1) and the first two values of the sequence $R_0 = 0$ and $R_1 = 1$, find the value of c for the LCG?
- Using your answer in (1) and (2) and the first three values in the sequence $R_0 = 0$, $R_1 = 1$ and $R_2 = 6$, find the value of a for the LCG?
- After finding a , c , m of the LCG, Using the **three conditions**, show that the above LCG has full period or not.

4. Three conditions:

Cond#1: divisors of $m=19 \in \{1, 19\}$

and divisors of $c=1 \in \{1\}$

the only common divisor is 1 \rightarrow True

Cond#2: prime divisors of $m=19 \in \{1, 19\}$ all must divide $a-1 = 4$

$4/1 = 4$ and $4/19$ is nor \rightarrow False

The LCG $a = 5$, $c = 1$, $m = 19$ does not have a full period .

Question #6:

An insurance company provide car insurance. The company has a call center with one line only for handling claims. Calls for calms arrive to the company at random if the call finds line busy then it is lost. The call takes a random amount of time (in minutes)to be processed. The company wants to evaluate the process using simulation.

1. Consider the LCG with $a=7$, $c=1$, $m=15$. Using the theorem, dose the LCG has Full period?
2. Starting with seed $R_0 = 0$ and using the LCG in (1), generate the stream of 7 uniform numbers using R_0 .
3. Let the call time be random computed as integer $[10*U+1]$. Using the uniform numbers in (2) compute the call time of the calls from 1 to 7 in the table below.

i	1	2	3	4	5	6	7
U_i	$\frac{1}{15} = 0.067$	$\frac{8}{15} = 0.533$	$\frac{12}{15} = 0.800$	$\frac{10}{15} = 0.667$	$\frac{11}{15} = 0.733$	$\frac{3}{15} = 0.200$	$\frac{7}{15} = 0.467$
Call Time	Int.[1.67] 1 min	Int.[6.33] 6 min	Int.[9.00] 9 min	Int.[7.7] 7 min	Int.[8.33] 8 min	Int.[3.00] 3 min	Int.[5.67] 5 min

4. Let the arrival time of calls is given in the following table

Request	1	2	3	4	5	6	7
Arrival	8:05	8:12	8:15	8:27	8:41	8:45	9:02

Define the events of the system as: Call Arrive (CA), Call End (CE), Call Lost (CL) and Idle Line (IL). Do the discrete event simulation for the model with given events and the information in (3) .

5. From the simulation results, what is the average call arrival per hour?
6. From the simulation, what is the average service time per call ?
7. From the simulation, what is the percentage of time that the line is busy?

1. Three conditions: LCG with $a=7$, $c=1$, $m=15$

Cond#1: divisors of $m=15 \in \{1, 3, 5, 15\}$

and divisors of $c=1 \in \{1\}$

the only common divisor is 1 \rightarrow **True**

Cond#2: prime divisors of $m=15 \in \{1, 3, 5\}$ all must divide $a-1 = 6$

$6/1 = 4$ (yes) and $6/3 = 2$ (yes) and $6/5 = 1.2$ (no) \rightarrow **False**

The LCG $a = 7$, $c = 1$, $m = 15$ does not have a full period .

4. Let the arrival time of calls is given in the following table

Request	1	2	3	4	5	6	7
Arrival	8:05	8:12	8:15	8:27	8:41	8:45	9:02
Call Time	1 min	6 min	9 min	7 min	8 min	3 min	5 min
	8:06	8:18	Lost	8:34	8:49	Lost	9:07

Discrete event simulation

Clock Time	Events	Clock Time	Events
8:00	IL	8:27	CA
8:05	CA	8:34	CE + IL (7min)
8:06	CE + IL (6min)	8:41	CA
8:12	CA	8:45	CL
8:15	CL	8:49	CE + IL (4min)
8:18	CE + IL (9min)	9:02	CA
		9:07	CE

5. Average call arrival per hour =

$$\frac{\text{total number of calls arrived}}{\text{total arrival time}} = \frac{7 \text{ calls}}{\left(\frac{62}{60}\right)} = 6.77 \text{ call/hr}$$

6. Average service time per call

$$\frac{\text{total call times for received calls only}}{\text{number of received calls}} = \frac{1 + 6 + 7 + 8 + 5}{5} = \frac{27}{5} = 5.4 \text{ minutes}$$

7. Percentage of time that the line is busy

$$\frac{\text{total idle times for the line}}{\text{total simulation time}} = \frac{6 + 9 + 7 + 4}{62 \text{ min}} = \frac{26}{62} = 0.4194$$

Percentage of time that the line is busy = 41.94 %