Third Midterm Exam Year 1440-1441 H First Semester		
Course name & code	OPER 441 – Modeling and Simulation النمذجة والمحاكاة – 441 بحث	اسم ورمز المقرر
Date and Time	Wed. 4 – Dec.–2019 (12:00 pm 2 Hours)	الوقت والتاريخ
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Question #1:

A car repair workshop manager wants to develop a simulation model. For one particular repair, the times to completion can be represented by the following distribution (x in days):

$$f(x) = \begin{cases} \dfrac{x}{8} - \dfrac{1}{4} & ; \quad 2 \le x \le 4 \\[2ex] \dfrac{10}{24} - \dfrac{x}{24} & ; \quad 4 \le x \le 10 \end{cases}$$

(a) Compute the CDF of the function $f(x)$.

(b) Give the inverse transform to generate random numbers for repair time.

(c) Using U[0,1] random number in the following table, using the inverse transform in part (b) to determine the time of each car repair.

Q#1: $f(x) = \begin{cases} \frac{x}{8} - \frac{1}{4} & ; 2 \le x \le 4 \\ \frac{1}{24}(10 - x) & ; 4 \le x \le 10 \end{cases}$

(a) CDF: $2 \le x \le 4$

$F(x) = \int_2^x \frac{y}{8} - \frac{1}{4}\, dy$

$= \left[\frac{y^2}{16} - \frac{y}{4}\right]_2^x$

$= \frac{x^2}{16} - \frac{x}{4} + \frac{1}{4}$; $2 \le x \le 4$

CDF: $4 \le x \le 10$

$F(x) = \int_2^4 f(y)\, dy + \int_4^x \frac{1}{24}(10 - y)\, dy$

$= 1/4 + \frac{1}{24}\left[10y - \frac{1}{2}y^2\right]_4^x$

$= \frac{10}{24}x - \frac{1}{48}x^2 - \frac{52}{48}$

for $4 \le x \le 4$

	1	2	3	4	5	6	7	8	9	10
U[0,1]	0.138	0.776	0.911	0.259	0.458	0.343	0.105	0.940	0.188	0.343
Repair Time	3.486	6.721	7.933	4.036	4.899	4.384	3.296	8.303	3.734	4.384

(b). Inverse $F(x) = u$ $\quad u \sim U[0,1]$

$$\frac{x^2}{16} - \frac{x}{4} + \frac{1}{4} = u$$

$$\Rightarrow \frac{x^2}{16} - \frac{x}{4} + \left(\frac{1}{4} - u\right) = 0$$

$$x = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A} = \frac{-(-1) \pm \sqrt{1 - 4(1-4u)(1/4)}}{2(1/4)}$$

$$\Rightarrow x = 2 \pm 2\sqrt{4u}$$

take $\boxed{x = 2 + 4\sqrt{u}}$

$$2 \le x \le 4$$

$$2 \le 2 + 4\sqrt{u} \le 4$$

$$0 \le 4\sqrt{u} \le 2$$

$$\Rightarrow \boxed{0 \le u \le 1/4}$$

Inverse $F(x) = u$

$$\frac{10}{24}x - \frac{1}{48}x^2 - \frac{52}{48} = u$$

$$\Leftrightarrow \frac{1}{48}x^2 - \frac{10}{24}x + \left(\frac{52}{48} + u\right) = 0$$

$$\therefore x = \frac{\left(\frac{10}{24}\right) \pm \sqrt{\left(\frac{10}{24}\right)^2 - 4\left(\frac{1}{48}\right)\left(\frac{52}{48} + u\right)}}{(2/48)}$$

$$\therefore x = 10 \pm 24\sqrt{\left(\frac{5}{12}\right)^2 - \frac{4}{48}\left(\frac{13}{12} + u\right)}$$

since $4 \le x \le 10$

$\Rightarrow$ Take $\boxed{x = 10 - 24\sqrt{\left(\frac{5}{12}\right)^2 - \frac{4}{48}\left(\frac{13}{12} + \right.}}$

and $\boxed{1/4 \le u \le 1}$

(d) Let the time between car arrival is shifted binomial distribution with parameters n = 3 and p = 0.45 with shift value δ where the shift value is uniform [1,3]. Write the algorithm for generating the arrival time of job (n).

Let T be the time between cars: T = δ+N

1. **Get u_1 ~U[0,1]**
2. **Generate δ ~ U[1,3] : δ = 1+2u_1**
3. **Get u_2 ~ U[0,1]**
4. **Generate N from inverse Binomal (3, p=0.45)**
5. **Compute T = δ + N**

	0	**1**	**2**	**3**
p{n}	0.166	0.408	0.334	0.091
CDF{N}	0.166	0.575	0.909	1.000

(e) Using U[0,1] random number in the following table and using the answer in part (d), determine the arrival time of each car for repair.

	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
U_1 [0,1]	0.301	0.120	0.491	0.145	0.448	0.048	0.049	0.846	0.590	0.509
U_2 [0,1]	0.138	0.776	0.911	0.259	0.458	0.343	0.105	0.940	0.188	0.343
δ	**1.60**	**1.24**	**1.98**	**1.29**	**1.90**	**1.10**	**1.10**	**2.69**	**2.18**	**2.02**
Bin	**0.00**	**2.00**	**3.00**	**1.00**	**1.00**	**1.00**	**0.00**	**3.00**	**1.00**	**1.00**
Time bet. cars	**1.60**	**3.24**	**4.98**	**2.29**	**2.90**	**2.10**	**1.10**	**5.69**	**3.18**	**3.02**
Arr. Time	**1.60**	**4.84**	**9.82**	**12.11**	**15.01**	**17.11**	**18.20**	**23.90**	**27.08**	**30.09**

(f) From you answers, compute the average rate of car arrival to the repair shop per week.

Total number of cars arrived in simulation = 10 cars

Total time for arrival = 30.09 days

Average number of cars per week = (10 cars)(7 days)/(30.09) = 2.33 cars/ week

(g) From you answers, compute the average repair time.

Average repair time = 5.12 days

Question #2:

Busses arrive to a bus station at random. It is assumed that time between bus arrival is Erlang with parameters $k=2$ and $\lambda = 4$ busses/hour. Each bus has a maximum of 5 seats. Any bus arrives to the station carries a random number of passengers. Past data shows that the distribution of number of passengers is binomial distribution with mean 3.5 passengers.

(a) Write the steps and required functions for simulation of bus arrival.
(b) Write the steps and functions for simulation of number of passengers in the bus.
(c) Using the following random U[0,1], do simulation for bus arrivals during the first 1:30 hours and number of passenger in each bus.

BUS #	$u_1 \sim$ U[0,1]	$u_2 \sim$ U[0,1]	$u_3 \sim$ U[0,1]	$u_4 \sim$ U[0,1]	u_1, u_2 $T(i)$	hrs $AT(i)$	u_3 $N(i)$	
1	0.150	0.130	0.176	0.614	0.075	0.075	3	
2	0.339	0.180	0.453	0.301	0.153	0.229	3	
3	0.220	0.306	0.484	0.139	0.153	0.382	4	
4	0.516	0.603	0.949	0.666	0.412	0.794	5	
5	0.188	0.213	0.504	0.324	0.112	0.906	4	
6	0.804	0.755	0.465	0.237	0.759	1.665	3	
7	0.795	0.347	0.548	0.072	0.503	2.168	4	
8	0.918	0.355	0.206	0.118	0.735	2.903	3	
9	0.742	0.050	0.873	0.463	0.352	3.254	5	
10	0.385	0.196	0.517	0.011	0.176	3.431	4	

1.30 hrs

NOTE: Use u_1, u_2, u_3, u_4, as needed for each bus.

(a) Process #1 : Bus Arrival Time:

Let $T(i)$ time between busses $\Rightarrow T(i) \sim Er[k=2, \lambda=4)$

Let $AT(i)$ the arrival time of Bus (i)

$\therefore \quad AT(i) = AT(i-1) + T(i)$

$$T(i) = -\frac{1}{4}\left(\ln(1-u_1) + \ln(1-u_2)\right)$$

Algorithm:

1. get $u_1, u_2 \sim U[0,1]$
2. get $T(i)$
   $T(i) = -\frac{1}{4}(\ln(1-u_1) + \ln(1-u_2))$
3. get $AT(i) = A(i-1) + T(i)$

(b) Process #2. # of passengers in Bus i.

Let $N(i) \sim$ Binomial (mean $= 3.5$) max. seats $= 5$

$\therefore \quad n=5$ and $np = 3.5 \Rightarrow p = \frac{3.5}{5} = 0.7$

$\therefore \quad Pr\{N=n\} = \binom{5}{n}(0.7)^n(0.3)^{5-n}$

n	0	1	2	3	4	5
$Pr\{n\}$	0.00243	0.0284	0.1323	0.3087	0.3602	0.1681
$Pr\{N \le n\}$	0.00243	0.0308	0.163	0.4718	0.8319	1.0

Inverse

u	0 – 0.0024	0.0024 – 0.0308	0.0308 – 0.163	0.163 – 0.471	0.472 – 0.8319
N	0	1	2	3	4

else 5

Question #3:

Consider the following probability density function:

$$f(x) = \frac{4}{80}x^3 ; \quad 1 \le x \le 3$$

Assume that there are two types of breakdowns happen on a machine: BKD-1 and BKD-2. BKD-1 needs a random amount to repair and it follows the pdf in (a). BKD-2 needs a random amount to repair and it follows the exponential distribution with mean 2 hours. From past data 40% of the time BKD-1 happens.

(a) and the time Let Y be the repair time. Write the CDF of Y($F(y)$)

(b) Write the steps to generate observations for the repair time.

(c) Use the following table for simulation of 5 breakdowns in the machine.

	1	2	3	4	5
$U_1[0,1]$	0.0129	0.1164	0.6804	0.9513	0.2017
$U_2[0,1]$	0.804	0.755	0.465	0.237	0.1105
Type of breakdown	BKD1	BKD1	BKD2	BKD2	BKD1
Repair Time of BKD	2.8429	2.799	0.31274	0.1353	1.771

(a) Y : repair time => CDF Y

$F_Y(y) = 0.4\, F_{X_1}(y) + 0.6\, F_{X_2}(y)$

X_1 : repair time for BKD-1 → $F_{X_1}(x_1)$

X_2 : ~ ~ ~ BKD-2 → $F_{X_2}(x_2)$

(b)

Y repair time

BKD1 (0.4) → $f(x)$ => use inverse

$F_X(x) = u = \frac{1}{80}(x^4 - 1)$

$\boxed{X = \sqrt[4]{80u+1}}$

BKD2 (0.6) → Exp. (mean = 2)

$Y = -\frac{1}{2}\ln(1-u)$

$F_X(x) = \frac{4}{80}\int_1^x y^3\,dy = \frac{4}{80}\left[\frac{1}{4}y^4\right]_1^x = \frac{1}{80}(x^4 - 1)$

Steps

1. get $u_1 \sim U$
2. Test u_1
   if $u_1 \le 0.4$ => BKD 1
   if $u_1 > 0.4$ => BKD 2
3. get $u_2 \sim U$
   if BKD1 → $Y = \sqrt[4]{80u+1}$
   if BKD2 → $Y = \frac{-1}{2}\ln(1-u_2)$

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Question #4:

1. If time between event is integer random uniform between 5 min and 10 min. Write the Excel function in the screen shot for generating the time for 1st and 2nd events.

O32

	A	B	C	D
1				
2		u~U(0,1)	Time Between	Event Time
3	Event 1	RAND()	= 5 + INT(6 * B3)	C3
4	Evant 2	RAND()	= 5 + INT(6 * B4)	C4 + D3
5	Evant 3	RAND()	= 5 + INT(6 * B5)	C5 + D4
6				
7				
8				

2. If time between events is continuous random uniform between 5 min and 10 min. Write the Excel function in the screen shot for generating the time for 1st , 2nd events and 3rd event.

O32

	A	B	C	D
1				
2		u~U(0,1)	Time Between	Event Time
3	Event 1	RAND()	= 5 + (5 * B3)	C3
4	Evant 2	RAND()	= 5 + (5 * B4)	C4 + D3
5	Evant 3	RAND()	= 5 + (5 * B5)	C5 + D4
6				
7				
8				

3. If time between events is random with integer values from Normal distribution with positive values only and with parameters $\mu = 3$, $\sigma = 9$. Write the Excel function in the screen shot for generating the time for 1st , 2nd events and 3rd event.

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	A	B	C	D	E	F	G	H	I	J
1										
2										
3	Event 1	RAND()	= NORM.INV(B3,3,9)		ABS(INT(C3))	E3				
4	Evant 2	RAND()	= NORM.INV(B4, 3,9)		ABS(INT(C4))	E4 + F3				
5	Evant 3	RAND()	= NORM.INV(B5,3,9)		ABS(INT(C5))	E5 + F4				
6										