



الرقم:

الاسم:

Problem 1 : Let T be the random variable that give the period of time (in months) between two rainfalls in Abha city and its CDF given by المتغير العشوائي الذي يقيس الزمن بين هطولين للمطر بالأشهر في مدينة أبها. T ليكن

$$F(t) = 2.12(0.6 - e^{-\frac{t}{2}})$$

a) Find the inverse transform function $t = F^{-1}(u)$.

b) Use the random numbers in the tableau to determine the time of the 8th rainfall.

حدد زمن هطول المطر للمرة الثامنة مستعينا بتكملة الجدول التالي .

k	u_k	$t_k = F^{-1}(u_k)$	$x_i = \sum_{k=1}^i t_k$
1	0.967		
2	0.663		
3	0.381		
4	0.752		

k	u_k	$t_k = F^{-1}(u_k)$	$x_i = \sum_{k=1}^i t_k$
5	0.835		
6	0.453		
7	0.916		
8	0.484		

Problem 2 : Consider the following probability density function :

$$f(x) = \begin{cases} \frac{1}{2}, & 0 \leq x \leq 1 \\ \frac{3}{4} - \frac{x}{4}, & 1 \leq x \leq 3 \end{cases}$$

Determine the CDF $F(x)$ of this distribution.

Problem 3 : Consider the following cumulative density function (CDF):

$$G(x) = \begin{cases} 0, & x < 0 \\ \frac{1}{2}x, & 0 \leq x \leq 1 \\ 1 - \frac{(3-x)^2}{8}, & 1 \leq x \leq 3 \\ 1, & x \geq 3 \end{cases}$$

1. Determine the inverse transform $x = G^{-1}(u)$ of $G(x)$

2. Generate the values of X using the following sequence of (0,1) random numbers.

u	0.387	0.336	0.466	0.074	0.184	0.34
$x = G^{-1}(u)$						
u	0.9	0.875	0.475	0.64		
$x = G^{-1}(u)$						

Problem 4 : Consider the discrete random variable X with the following distribution

$$P(X = x) = \frac{7 - k}{21}, \quad k = 1, 2, 3, 4, 5, 6$$

1. Fill the table (إملا الجدول)

x	1	2	3	4	5	6
$P(X = x)$						
$F(x)$						

2. Find the inverse transform function $F^{-1}(u)$ to generate observations for the random variable X .

Problem 5 : Consider the following function

$$f(x) = \frac{x^3}{20}, \quad 1 \leq x \leq 3,$$

1. Find the greatest value f_{max} of the function $f(x)$ حدد أكبر قيمة للدالة $f(x)$

2. Use Acceptance/Rejection method to generate 10 observations (x_i, y_i) using the following formulas

استعمل طريقة " قبول / رفض " لتوليد 10 مشاهدات بتطبيق القوانين التالية

$$x_i = a + u_i(b - a), \quad y_i = r_i f_{max}$$

i	u_i	x_i	$f(x_i)$	r_i	$y_i = r_i f_{max}$	$acc./rej.$
1	0.622			0.311		
2	0.943			0.964		
3	0.851			0.827		
4	0.592			0.186		
5	0.084			0.165		
6	0.936			0.684		
7	0.016			0.768		
8	0.219			0.667		
9	0.091			0.257		
10	0.238			0.280		

Problem 6 : Consider the following function

$$f(x) = \begin{cases} \frac{x}{8} - \frac{1}{4}, & 2 \leq x \leq 4 \\ \frac{3}{4} - \frac{x}{4}, & 4 \leq x \leq 10 \end{cases}$$

1. Find the greatest value f_{max} of the function $f(x)$ حدد أكبر قيمة للدالة $f(x)$

2. Use Acceptance/Rejection method to generate 5 observations (x_i, y_i) using the following formulas

استعمل طريقة " قبول / رفض " لتوليد 10 مشاهدات بتطبيق القوانين التالية

$$x_i = a + u_i(b - a) , y_i = r_i f_{max}$$

i	u_i	x_i	$f(x_i)$	r_i	$y_i = r_i f_{max}$	$acc./rej.$
1	0.622			0.311		
2	0.943			0.964		
3	0.851			0.827		
4	0.592			0.186		
5	0.084			0.165		

Problem 7 : Consider the following cumulative density function (CDF):

$$G(x) = \begin{cases} 0, & x \leq 1 \\ \frac{1}{80}(x^4 - 1), & 1 \leq x \leq 3 \\ 1, & x \geq 3 \end{cases}$$

and its inverse transform $x = G^{-1}(u) = \sqrt[4]{80u + 1}$, $1 \leq x \leq 3$.

In a mechanic shop (ميكانيكي), suppose that there is 35 % chance that the repair time for a car is a random variable, X_1 , with CDF $G(x)$, and 65 % chance that the repair time is a random variable, X_2 , distributed according to an exponential distribution with mean 2 hours.

Complete the table to generate observations from the mixed distribution.

املاً الجدول لتوليد المشاهدات من التوزيع المختلط.

n	الأرقام عشوائية $U_1(0, 1)$	تحديد $F_{X_1}(x)$ أو $F_{X_2}(x)$	الأرقام عشوائية $U_2(0, 1)$	المشاهدات x
1	0.902		0.641	
2	0.818		0.984	
3	0.375		0.495	
4	0.341		0.592	
5	0.812		0.815	
6	0.148		0.111	
7	0.509		0.665	
8	0.653		0.379	