بسم الله الرحمن الرحيم

جامعة الملك سعود
King Saud University

College of Sciences
Department of Physics & Astronomy

كلية العلوم
قسم الإحصاء وبحوث العمليات

Final Exam
Academic Year 1441 Hijri- Second Semester

Exam Information معلومات الامتحان			
Course name	Modeling and Simulation	النمذجة والمحاكاة	اسم المقرر
Course Code	OPER 441	441 بحث	رمز المقرر
Exam Date	06 - May - 2020	13 رمضان 1441 هـ	تاريخ الامتحان
Exam Starting Time	9:00 am	الساعة 9:00 صباحا	وقت بدأ الامتحان
Exam Duration	4 hours	4 ساعات	مدة الامتحان
Classroom No.	Take Home Exam		رقم قاعة الاختبار
Instructor Name	Dr. Khalid Alnowibet	د. خالد النويبت	اسم استاذ المقرر

Student Information معلومات الطالب		
Student's Name		اسم الطالب
ID number		الرقم الجامعي
Section No.		رقم الشعبة
Serial Number		الرقم التسلسلي

General Instructions **تعليمات عامة:**

1. This is a TAKE-HOME EXAM that will be sent to you by email.
2. You **MUST** immediately reply the same email to confirm receiving the exam paper.
3. You will receive the exam paper as an electronic attachment via your KSU-email by 8:45 am on Wednesday May 6, 2020 sent to you by the instructor's email knowibet@ksu.edu.sa only.
4. The duration of the exam is **four hours**. Starting from 9:00 am. The last time for submitting and receiving your answers is **1:15 pm Wednesday May 6, 2020**. Any answers sent after this time will not be considered and the student will be marked as **ABSENT** in the Final Exam.
5. This a take-home exam, so you will have full access to the lecture notes, the text book, and Excel for your answer.
6. **DO NOT** copy/paste from the lecture notes. You **MUST** create your own answers based on what you learned and what you read from the resources of the course.
7. You are the guardian of your behavior in this Exam. This Final Exam is totally for your independent effort. **Do not attempt** to collaboration or communication with **anyone** about the questions of the exam, it is totally not allowed by any means. If similarities are spotted between two or more students, all of the students involved will get **ZERO** on the questions with similarity.
8. You **MUST** fill out the cover page with your information. Write your answers on a word document and email the document on PDF and WORD format. Write the subject of the email as:

OPER-441-Final-Exam <<Section Number>> , << your name>> , <<your KSU ID >>

9. Put all you of answers in one document and one Excel file with all worksheets. In Excel file, put each question in a separate worksheet with the name Q1, Q2, Q3, … If you have part of the answers on excel, capture the answer from the screen and insert it in the document.

وَاللهُ وَلِيُّ التَّوفِيق

Question #1:

A trader is investing his budget in stock market. He has an investment portfolio with **three** different companies: CO-A, CO-B and CO-C. The current situation of his portfolio is shown in the following table. By the end of every day, the change in closing price for each share varies between −10% to 8% as shown in the table. The investor wants to start with all company shares have about equal total value in the portfolio. This means that

	Current Portfolio		Percentage of Daily Price Change					
	Total value	**Price Per Share**	**−10%**	**−5%**	**0%**	**3%**	**5%**	**8%**
CO-A	200,000	32 SR	0.1	0.2	0.3	0.2	0.15	0.05
CO-B	200,010	15 SR	0.2	0.15	0.05	0.1	0.2	0.3
CO-C	200,070	95 SR	0.05	0.15	0.2	0.3	0.2	0.1

(a) Let PV(n) be the portfolio value on day (n). Write a detailed algorithm to simulate PV(n).

(b) Using the following U(0,1) numbers, simulation for 10 days to evaluate the average value of the portfolio in 10 days. Use the following table to evaluate the average value of the portfolio after 10 days.

Day	1	2	3	4	5	6	7	8	9	10
U(0,1)	0.266	0.547	0.325	0.514	0.951	0.721	0.33	0.481	0.961	0.358
U(0,1)	0.342	0.657	0.785	0.603	0.793	0.677	0.87	0.362	0.08	0.231
U(0,1)	0.996	0.959	0.237	0.669	0.382	0.594	0.845	0.583	0.629	0.806

(c) Form a data table in Excel to simulate the daily value of the portfolio PV(n) for one year. Then compute the Average monthly APV(n) with 95% confidence interval and draw the empirical distribution of the PV(n) from the data table (fix your results in the data table). Put your results in an Excel sheet and <Past> the screen shot of Excel Data Table for first 20 days with numbers and with cell functions (Ctrl + ~).

(d) Now consider the case where the distribution for the daily change of each share is given as a continuous variable as in the table below. Also, the investor will maintain the equal value for each share as in the table. Using Excel simulation, build a data table for one month (30 day) what is the simulated value of the portfolio by the end of the month. Put your results in an Excel sheet and <Past> the screen shot of model simulation table for first 20 days with numbers and with cell functions (Ctrl + ~). Send the excel file with your answer sheet.

	Current Portfolio		Percentage of Daily Price Change is Normal distribution	
	Total value	**Price Per Share**	**Mean μ**	**Standard deviation σ**
CO-A	200,000	32 SR	3%	5%
CO-B	200,010	15 SR	6%	10%
CO-C	200,070	95 SR	8%	15%

(e) Find the expected **monthly** value of the portfolio in part (d) for 100 months. Give the average monthly value and the 95% confidence interval. Put your results in an Excel sheet and <Past> the screen shot of model simulation table for first 20 days with numbers and with cell functions (Ctrl + ~). Send the excel file with your answer sheet.

Question #2:

Consider the **CDF** function for the random variable X:

$$F(x) = \begin{cases} 0 & ; \quad x < 1 \\ \dfrac{(x-1)^2}{15} & ; \ 1 \le x \le 4 \\ 1 - \dfrac{(6-x)^2}{10} & ; \ 4 < x \le 6 \\ 1 & ; \quad x > 6 \end{cases}$$

Answer the following:

(a) Derive an inverse transform algorithm for this distribution, and write the detailed algorithm for simulating from F(x).

(b) Let X be the time between arrival of buses in a given station (in minutes). Using the U[0,1] random number in the following table, using the inverse transform in part **(b)** to determine the arrival time of the 1st ten buses. (Use uniforms as needed)

	1	2	3	4	5	6	7	8	9	10
$U_1[0,1]$	0.485	0.389	0.601	0.374	0.808	0.527	0.533	0.532	0.122	0.854
$U_2[0,1]$	0.328	0.708	0.027	0.653	0.283	0.113	0.662	0.701	0.169	0.283
X										
Arrival Time of										

(c) Assume that all buses are of the same size with maximum of 20 seats. Let N be the number of passengers on board in each bus arrived. Where N is random variable Geometric distribution with minimum 5 and max 20. Write the inverse function of the N, and write the detailed algorithm for simulating from *f*(N):

$$f(N = n) = (1 - 0.9)(0.9)^n \qquad ; \qquad n = 0, 1, 2, \ldots$$

(d) Using Excel, data table in Excel and your answers, **run a simulation model for 1000 bus** to estimate average number of passengers arrived to the bus station per hour with the 95% confidence interval (fix the results in Excel for the data table). Put your results in an Excel sheet and <Past> the screen shot of the model simulation table for first 20 days with numbers and with cell functions (Ctrl + ~). Send the excel file with the answer sheet.

Question #3:

Consider the following probability density function:

$$f(x) = \begin{cases} 0.5x - 1 & 2 \le x \le 4 \\ 0 & \text{otherwise} \end{cases}$$

(a) Compute the CDF of the function *f*(x).

(b) Using your answer in part **(a)**, Derive an inverse transform algorithm for this distribution.

(c) Let X be the time between calls to a call center. Using U[0,1] random number in the following table, use the inverse transform in part **(b)** to simulate the call arrival time of each customer.

Call #	1	2	3	4	5	6	7	8	9	10
U[0,1]	0.013	0.116	0.681	0.951	0.202	0.328	0.708	0.027	0.653	0.283
X										
Arrival Time										

(d) Assume that the call center has one line. Simulate the call time spend on the line assuming that the call takes a random amount of time that follows the shifted *pdf*:

$$f(x) = \frac{2}{x^3} \quad ; \quad x > 1 \quad \text{with shift parameter = 1 min.}$$

Call #	1	2	3	4	5	6	7	8	9	10
Arrival Time										
U[0,1]	0.370	0.143	0.978	0.067	0.232	0.052	0.691	0.173	0.276	0.137
Call Time										

(e) Estimate the percentage of calls answered from simulation output. Show all your answers on paper.

(f) Estimate average number of answered calls per hour from simulation output. Show all your answers on paper.

(g) The management decided to buy a new line. Show that the new line will affect the Expected average number of answered calls using simulation in (c) and (d). Show all your answers.

(h) Assume that the call center has one line and takes <u>only 20 calls per day</u>. Build a **<u>five</u>** different date table using Excel with 100 days for each table and compute:

a. percentage of calls answered with confidence 95% interval.

b. number of answered calls per hour with confidence 95% interval.

measure	**Ave.**	**STD.**	**LL-95%**	**UL-95%**
Ave. percentage calls answered				
Ave. number calls answered per hour				

Your results will be something like:

	measure	Ave.	STD.	LL-95%	UL-95%
Data Table #1 outputs	Ave. percentage calls answered				
	Ave. number calls answered per hour				
Data Table #2 outputs	Ave. percentage calls answered				
	Ave. number calls answered per hour				
Data Table #3 outputs	Ave. percentage calls answered				
	Ave. number calls answered per hour				
Data Table #4 outputs	Ave. percentage calls answered				
	Ave. number calls answered per hour				
Data Table #5 outputs	Ave. percentage calls answered				
	Ave. number calls answered per hour				

Finally write the summary for all 5 data tables:

	Ave.	STD.	LL-95%	UL-95%
Ave. percentage calls answered from all data tables				
Ave. number calls answered per hour from all data tables				

Send the excel file with your answers sheet

Question #4:

SimAir is a local airline company that runs small jet airplanes between some local cities. Each airplane can take a maximum of 100 passengers. Past data shows that, for any flight, the number of confirmed booking seats follows a shifted binomial distribution with shift 60 and mean 30. In addition, the number of confirmed booking passengers may miss the flight for some reason and do not show at the time of departure. The number of "*no-show*" passengers is a random integer that range between zero and 20% of the confirmed bookings. For every confirmed booking SimAir charge 750 SR. If the passenger did not show up he for his flight, he gets 50% refund. Also, any *unbooked* empty seat cost SimAir 250 SR.

(a) Write the total revenue for SimAir from flight (n) given the variables defined as follows:

- X(n): be the number of confirmed booking seats
- Y(n): be the number of no-show passengers.

(b) Define all parameters needed for simulation in Excel in the following cells location:

	A	B	C	D
1				
2				
3				
4				
5				
6				

You can use screen shot from Excel for the same cells.

(c) Using the parameters you write in part **(a),** write in the following cell the Excel function that you will use to simulate number of <u>confirmed booking seats</u>. *Your answered will be corrected based the cell location you used in (a).*

	A
10	

(d) Using the parameters you write in part **(a),** write in the following cell the Excel function that you will use to simulate number of <u>no-show passengers</u>. *Your answered will be corrected based the cell location you used in (a).*

	B
10	

(e) Using data table in Excel and your answers, **run a simulation model for 1000 flight** to provide the following outputs (*fix the result of data table in Excel*):

- The average revenue per flight and the 95% confidence interval
- The average number of actual passengers boarded the airplane with the 95% confidence interval
- The empirical CDF of the revenue
- The empirical CDF for the actual passengers traveled on the airplane.

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Academic Year 1441 Hijri- Second Semester

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والله ولي التوفيق

Question #1:

Rubric: 28 Points	
+2 Points-- (2)	Attempt to write something on PV(n)-- part (a)
+3 Points—(5)	writing correct PV with detailed algorithm to simulate PV(n)-- part (a)
+5 Points—(10)	Attempt to do short simulation by hand-- part (b)
+2 Points—(12)	Writing the correct calculations for Simulation by hand-- part (b)
+4 Points—(16)	Attempt to do simulation by Excel-- part (c)
+2 Points—(18)	All output and results are correct in Excel simulation-- part (c)
+3 Points – (23)	Attempt to solve the Extension -- part (d)
+2 Points—(22)	Correct solution of Part (d)
+3 Points—(25)	Attempt to solve part (e)
+3 Points—(28)	part (e)-- All output in excel is done and correct
Total	

A trader is investing his budget in stock market. He has an investment portfolio with **three** different companies: CO-A, CO-B and CO-C. The current situation of his portfolio is shown in the following table. By the end of every day, the change in closing price for each share varies between –10% to 8% as shown in the table. The investor wants to start with all company shares have about equal total value in the portfolio. This means that

	Current Portfolio		Percentage of Daily Price Change					
	Total value	**Price Per Share**	**-10%**	**-5%**	**0%**	**3%**	**5%**	**8%**
CO-A	200,000	32 SR	0.1	0.2	0.3	0.2	0.15	0.05
CO-B	200,010	15 SR	0.2	0.15	0.05	0.1	0.2	0.3
CO-C	200,070	95 SR	0.05	0.15	0.2	0.3	0.2	0.1

(a) Let PV(n) be the portfolio value on day (n). Write a detailed algorithm to simulate PV(n).

CO-A ➔ $price\ change\ CO-A = PCA = F^{-1}(u) = \begin{cases} -0.1 & ; 0 \le u \le 0.1 \\ -0.05 & ; 0.1 < u \le 0.3 \\ 0 & ; 0.3 < u \le 0.6 \\ 0.03 & ;\ 0.6 < u \le 0.8 \\ 0.05 & ;\ 0.8 < u \le 0.95 \\ 0.08 & ;\ 0.95 < u \le 1 \end{cases}$

Closing value CO-A day (n) = CVA(n) = CVA(n-1)*(1+PCA(n))

CO-B ➔ $price\ change\ CO-B = PCB = F^{-1}(u) = \begin{cases} -0.1 & ; 0 \le u \le 0.2 \\ -0.05 & ; 0.2 < u \le 0.35 \\ 0 & ; 0.35 < u \le 0.4 \\ 0.03 & ;\ 0.4 < u \le 0.5 \\ 0.05 & ;\ 0.5 < u \le 0.7 \\ 0.08 & ;\ 0.7 < u \le 1 \end{cases}$

Closing value CO-B day (n) = CVB(n) = CVB(n-1)*(1+PCB(n))

CO-C ➔ $price\ change\ CO-B = PCC = F^{-1}(u) = \begin{cases} -0.1\ ; 0 \le u \le 0.05 \\ -0.05\ ; 0.05 < u \le 0.2 \\ 0\ ; 0.2 < u \le 0.4 \\ 0.03\ ;\ 0.4 < u \le 0.7 \\ 0.05\ ;\ 0.7 < u \le 0.9 \\ 0.08\ ;\ 0.9 < u \le 1 \end{cases}$

Closing value CO-C day (n) = CVC(n) = CVC(n-1)*(1+PCC(n))

PV(n) = CVA +CVB + CVC

(b) Using the following U(0,1) numbers, simulation for 10 days to evaluate the average value of the portfolio in 10 days. Use the following table to evaluate the average value of the portfolio after 10 days.

Day	1	2	3	4	5	6	7	8	9	10
U(0,1)	0.266	0.547	0.325	0.514	0.951	0.721	0.33	0.481	0.961	0.358
U(0,1)	0.342	0.657	0.785	0.603	0.793	0.677	0.87	0.362	0.08	0.231
U(0,1)	0.996	0.959	0.237	0.669	0.382	0.594	0.845	0.583	0.629	0.806

CO-A	200000									
CO-B	200010									
CO-C	200070									
Day(n)	1	2	3	4	5	6	7	8	9	10
U1	0.266	0.547	0.325	0.514	0.951	0.721	0.33	0.481	0.961	0.358
Percentage	-0.05	0	0	0	0.08	0.03	0	0	0.08	0
PV(n)	190000	190000	190000	190000	205200	211356	211356	211356	228264.5	228264.5
U2	0.342	0.657	0.785	0.603	0.793	0.677	0.87	0.362	0.08	0.231
Percentage	-0.05	0.05	0.08	0.05	0.08	0.05	0.08	0	-0.1	-0.05
PV(n)	190009.5	199509.975	215470.8	226244.3	244343.9	256561	277085.9	277085.9	249377.3	236908.5
U3	0.996	0.959	0.237	0.669	0.382	0.594	0.845	0.583	0.629	0.806
Percentage	0.08	0.08	0	0.03	0	0.03	0.05	0.03	0.03	0.05
PV(n)	216075.6	233361.648	233361.6	240362.5	240362.5	247573.4	259952	267750.6	275783.1	289572.3
Portfolio	596085.1	622871.623	638832.4	656606.8	689906.4	715490.4	748394	756192.5	753424.9	754745.2
Average		**693254.941**								

Note that the first line of PV(n) is for CO-A, the second PV(n) line is for CO-B and the third PV(n) is for CO-C.

The value of the portfolio is the sum of PV(n) of CO-A, CO-B and CO-C. (for example, 190,000+190,009.5+216075.6)

And take the summation of all portfolio and divide it by no. of days which is 10 and the answer is **693,255.941**.

(c) Form a data table in Excel to simulate the daily value of the portfolio PV(n) for one year. Then compute the Average monthly APV(n) with 95% confidence interval and draw the empirical distribution of the PV(n) from the data table (fix your results in the data table). Put your results in an Excel sheet and <Past> the screen shot of Excel Data Table for first 20 days with numbers and with cell functions (Ctrl + ~).

(d) Now consider the case where the distribution for the daily change of each share is given as a continuous variable as in the table below. Also, the investor will maintain the equal value for each share as in the table. Using Excel simulation, build a data table for one month (30 day) what is the simulated value of the portfolio by the end of the month. Put your results in an Excel sheet and <Past> the screen shot of model simulation table for first 20 days with numbers and with cell functions (Ctrl + ~). Send the excel file with your answer sheet.

	Current Portfolio		Percentage of Daily Price Change is Normal distribution	
	Total value	**Price Per Share**	**Mean μ**	**Standard deviation σ**
CO-A	200,000	32 SR	3%	5%
CO-B	200,010	15 SR	6%	10%
CO-C	200,070	95 SR	8%	15%

(e) Find the expected **monthly** value of the portfolio in part (d) for 100 months. Give the average monthly value and the 95% confidence interval. Put your results in an Excel sheet and <Past> the screen shot of model simulation table for first 20 days with numbers and with cell functions (Ctrl + ~). Send the excel file with your answer sheet.

Question #2:

Rubric: 28 Points	
+3 Points—(3)	Attempt to write something on Inverse F(x) -- part (a)
+4 Points—(7)	writing The Correct and detailed Inverse F(X)-- part (a)
+4 Points—(11)	Attempt to do short simulation by hand-- part (b)
+3 Points—(14)	Writing the correct calculations for Simulation by hand-- part (b)
+4 Points—(18)	Attempt to do Inverse for f(N) by Excel-- part (c)
+2 Points—(20)	Writing the correct Inverse for f(N) by Excel-- part (c)
+4 Points—(24)	Attempt to do the Excel simulation -- part (d)
+4 Points—(28)	Complete and correct output of Excel-- Part (d)
Total	

Consider the **CDF** function for the random variable X:

$$F(x) = \begin{cases} 0 & ; \quad x < 1 \\ \dfrac{(x-1)^2}{15} & ; \ 1 \le x \le 4 \\ 1 - \dfrac{(6-x)^2}{10} & ; \ 4 < x \le 6 \\ 1 & ; \quad x > 6 \end{cases}$$

Answer the following:

(a) Derive an inverse transform algorithm for this distribution, and write the detailed algorithm for simulating from F(x).

Question 2

2a) $F(x) = u$

$\frac{(x-1)^2}{15} = u$

$15u = (x-1)^2$

$x-1 = \pm\sqrt{15u}$

$x = 1 \pm \sqrt{15u}$ $\quad x \in [1,4]$

$\Rightarrow x = 1 + \sqrt{15u}$

$1 \le x \le 4$

$1 \le 1 + \sqrt{15u} \le 4$

$0 \le \sqrt{15u} \le 3$

$0 \le 15u \le 9$

$0 \le u \le \frac{9}{15}$

$0 \le u \le 0.6$

$0 \le u \le 0.6$

$F(x) = u$

$1 - \frac{(6-x)^2}{10} = u$

$\frac{(6-x)^2}{10} = 1-u$

$(6-x)^2 = 10(1-u)$

$6-x = \pm\sqrt{10(1-u)}$

$x = 6 \pm \sqrt{10(1-u)}$

$4 \le x \le 6$

$\Rightarrow x = 6 - \sqrt{10(1-u)} = F^{-1}(u)$

$$\boxed{x = F^{-1}(u) = \begin{cases} 1+\sqrt{15u}, & 0 \le u \le 0.6 \\ 6-\sqrt{10(1-u)}, & 0.6 < u \le 1 \end{cases}}$$

(b) Let X be the time between arrival of buses in a given station (in minutes). Using the U[0,1] random number in the following table, using the inverse transform in part **(b)** to determine the arrival time of the 1st ten buses. (Use uniforms as needed)

From U1 if 0<U1<0.6 we use X=$1+\sqrt{15u_1}$.

If 0.6<U1<1 we use X=$6-\sqrt{10(1-u_1)}$

	1	2	3	4	5	6	7	8	9	10
$U_1[0,1]$	0.485	0.389	0.601	0.374	0.808	0.527	0.533	0.532	0.122	0.854
$U_2[0,1]$	0.328	0.708	0.027	0.653	0.283	0.113	0.662	0.701	0.169	0.283
X	3.697	3.415	4.002	3.369	4.617	3.812	3.828	3.825	2.352	4.793
Arrival Time of	3.697	7.112	11.114	14.483	19.1	22.912	26.74	30.565	32.917	37.71

(c) Assume that all buses are of the same size with maximum of 20 seats. Let N be the number of passengers on board in each bus arrived. Where N is random variable Geometric distribution with minimum 5 and max 20. Write the inverse function of the N, and write the detailed algorithm for simulating from *f*(N):

$$f(N=n) = (1-0.9)(0.9)^n \quad ; \quad n = 0, 1, 2, \ldots$$

2c) $f(N=n) = (1-0.9)(0.9)^n$;

$$g(n) = \frac{f(n)}{F(b)-F(a)} \qquad a \le x \le b$$

$$g(n) = \frac{f(n)}{F(20)-F(5)} \qquad 5 \le x \le 20$$

$$G(n) = \frac{F(n)-F(5)}{F(20)-F(5)}$$

$F(20) = P(N \le 20) = 0.890$

$F(5) = P(N \le 5) = 0.468$

$$G(n) = \frac{F(n) - \cancel{0.89}\ 0.468}{0.890 - 0.468}$$

$$G(n) = \frac{1-(0.9)^{n+1} - 0.468}{0.890 - 0.468}$$

$G(n) = u$

$$\frac{1-(0.9)^{n+1} - 0.468}{0.890 - 0.468} = u$$

$$1-(0.9)^{n+1} - 0.468 = 0.422u$$

$$(0.9)^{n+1} = 0.532 - 0.422u$$

$$n+1 = \frac{\ln(0.532 - 0.422u)}{\ln(0.9)}$$

$$n = \left\lfloor \frac{\ln(0.532 - 0.422u)}{\ln(0.9)} \right\rfloor - 1 = G^{-1}(u)$$

d) Get $u \sim U_{[0,1]}$

Return $G^{-1}(u)$

(d) Using Excel, data table in Excel and your answers, **run a simulation model for 1000 bus** to estimate average number of passengers arrived to the bus station per hour with the 95% confidence interval (fix the results in Excel for the data table). Put your results in an Excel sheet and <Past> the screen shot of the model simulation table for first 20 days with numbers and with cell functions (Ctrl + ~). Send the excel file with the answer sheet.

Average =(SUM(pass. Bus#1:pass. Bus#1000)/Bus#1000 arrival Time)*60

	1	2	3	4	5	6	7
Average Number of Passengers in an Hour:	160.04	169.77	162.48	164.58	160.07	164.85	165.09
Lower Bound	159.79	169.52	162.23	164.31	159.82	164.59	164.83
Upper Bound	160.29	170.03	162.73	164.84	160.32	165.10	165.34

Question #3:

Rubric: 35 Points	
+2 Points—(2)	part (a)--Attempt to write something on CDF of F(x)
+2 Points—(4)	part (a)--writing The Correct and detailed CDF of F(X)
+3 Points—(7)	part (b)--Attempt to write something on Inverse of F(x)
+2 Points—(9)	part (b)--writing The Correct and detailed Inverse of F(X)

+4 Points—(13)	part (c)-- Attempt to do short simulation by hand
+3 Points—(16)	part (c)--Writing the correct calculations for Simulation by hand
+4 Points—(20)	part (d)--Attempt to write something on Inverse of F(x) call time
+2 Points—(22)	part (d)--writing The Correct and detailed Inverse of F(X) call time
+3 Points—(25)	part (e)--writing the Correct Estimate of the percentage of calls answered (-1)
+3 Points—(28)	part (f)--writing the Correct Estimate of the answered calls per hour (-1)
+3 Points—(31)	Part(g)—Attempting to answer buy a new line
+2 Points—(33)	Part(g)—Correct answer
+4 Points—(37)	part (h) -- Attempt to do the Excel simulation
+3 Points—(40)	part (h) --Complete and correct output of Excel
Total	

Consider the following probability density function:

$$f(x) = \begin{cases} 0.5x - 1 & 2 \le x \le 4 \\ 0 & \text{otherwise} \end{cases}$$

(a) Compute the CDF of the function *f*(x).

a) $F(x) = \int_{\cancel{0}2}^{x} f(t)\,dt$

$= \int_{2}^{x} (0.5t - 1)\,dt = \left. \frac{0.5t^2}{2} - t \right]_2^x$

$= 0.25x^2 - x - (-1)$

$F(x) = 1 + 0.25x^2 - x$

$$F(x) = \begin{cases} 0, & x < 2 \\ 1 + 0.25x^2 - x, & 2 \le x \le 4 \\ 1, & x > 4 \end{cases}$$

(b) Using your answer in part **(a)**, Derive an inverse transform algorithm for this distribution.

$$F(x) = u$$

$$1 + 0.25x^2 - x = u$$

$$0.25x^2 - x + (1-u) = 0$$

By the quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{1 \pm \sqrt{1 - 4(0.25)(1-u)}}{2(0.25)}$$

$$2 \le x \le 4$$

$$x = 2 \pm 2\sqrt{1-(1-u)}$$

$$x = 2 \pm 2\sqrt{u} \quad \Rightarrow \quad x = 2 + 2\sqrt{u}$$

(c) Let X be the time between calls to a call center. Using U[0,1] random number in the following table, use the inverse transform in part **(b)** to simulate the call arrival time of each customer.

Call #	1	2	3	4	5	6	7	8	9	10
U[0,1]	0.013	0.116	0.681	0.951	0.202	0.328	0.708	0.027	0.653	0.283
X	2.228	2.681	3.650	3.950	2.899	3.145	3.683	2.329	3.616	3.006
Arrival Time	2.228	4.909	8.560	12.51	15.409	18.554	22.237	24.566	28.182	31.188

(d) Assume that the call center has one line. Simulate the call time spend on the line assuming that the call takes a random amount of time that follows the shifted *pdf*:

$$f(x) = \frac{2}{x^3} \quad ; \quad x > 1 \quad \text{with shift parameter = 1 min.}$$

$$F(x) = \int_1^x \frac{2}{t^3}\,dt \rightarrow F(x) = 1 - \frac{1}{x^2} \; ; x > 1$$

$$F(x) = u + \delta$$

$$1 - \frac{1}{x^2} = u \rightarrow X = \sqrt{\frac{1}{1-u}} + 1$$

Call #	1	2	3	4	5	6	7	8	9	10
Arrival Time	2.228	4.909	8.560	12.51	15.409	18.554	22.237	24.566	28.182	31.188
U[0,1]	0.370	0.143	0.978	0.067	0.232	0.052	0.691	0.173	0.276	0.137

Call Time	2.260	2.080	7.742	2.035	2.141	2.027	2.799	2.100	2.175	2.076
Call ended	4.48	6.98	16.29	14.5	17.5	20.5	25.02	26.5	30.33	33.3
Accept / Lost	A	A	A	L	L	A	A	L	A	A

(e) Estimate the percentage of calls answered from simulation output. Show all your answers on paper.

Call#	Arr Time	Call Time	Call Finish	call answerd
1	2.23	2.26	4.49	1
2	4.91	2.08	6.99	1
3	8.56	7.74	16.30	1
4	12.51	2.04	14.55	0
5	15.41	2.14	17.55	0
6	18.55	2.03	20.58	1
7	22.24	2.80	25.04	1
8	24.57	2.10	26.67	0
9	28.18	2.18	30.36	1
10	31.19	2.08	33.26	1
		percentage	0.70	

Call Finish(i) = Arrival time(i) + Call Time(i) ; i=1,2,….,10

First call of course will be answered, and from 2nd call till the 10th call we Used IF STATEMENT.

If the call finish before the coming arrival call then it will be answer, value = 1

If the call finish after the coming arrival call then it will be lost, value = 0

(i.e. IF(Call Finish(i)<Arrival Call(i+1),1,0)).

Hence, as we see from above the answered calls was 7 of the total which is 10

$$P(call\ answered) = \frac{Call\ answered}{Total\ calls} = 0.70$$

(f) Estimate average number of answered calls per hour from simulation output. Show all your answers on paper.

The average number is $\frac{Answered\ call*60}{Total\ Simulation\ time} = \frac{7*60}{33.26} = \mathbf{12.63}$

(g) The management decided to buy a new line. Show that the new line will affect the Expected average number of answered calls using simulation in (c) and (d). Show all your answers.

$$\frac{Total\ calls*60}{Total\ Simulation\ Time} = \frac{10*60}{33.26} = \mathbf{18.04}$$

(h) Assume that the call center has one line and takes <u>only 20 calls per day</u>. Build a <u>**five**</u> different date table using Excel with 100 days for each table and compute:

a. percentage of calls answered with confidence 95% interval.

b. number of answered calls per hour with confidence 95% interval.

measure	Ave.	STD.	LL-95%	UL-95%
Ave. percentage calls answered				
Ave. number calls answered per hour				

Your results will be something like:

	measure	Ave.	STD.	LL-95%	UL-95%
Data Table #1 outputs	Ave. percentage calls answered				
	Ave. number calls answered per hour				
Data Table #2 outputs	Ave. percentage calls answered				
	Ave. number calls answered per hour				
Data Table #3 outputs	Ave. percentage calls answered				
	Ave. number calls answered per hour				
Data Table #4 outputs	Ave. percentage calls answered				
	Ave. number calls answered per hour				
Data Table #5 outputs	Ave. percentage calls answered				
	Ave. number calls answered per hour				

Finally write the summary for all 5 data tables:

	Ave.	STD.	LL-95%	UL-95%
Ave. percentage calls answered from all data tables				
Ave. number calls answered per hour from all data tables				

Send the excel file with your answers sheet

Question #4: Extra

Rubric: 20 Points	
+2 Points—(2)	part (a)--Attempt to write something on X(n) and Y(n)
+1 Points—(3)	part (a)--writing The Correct and detailed X(n) and Y(n)
+2 Points—(5)	part (b)--Attempt to write something on Define all parameters
+1 Points—(6)	part (b)--writing The Correct and detailed Define all parameters
+2 Points—(8)	part (c)--Attempt to write something on sim confirmed booking seats

+1 Points—(9)	part (c)--writing The Correct and detailed sim confirmed booking seats
+2 Points—(11)	Part (d) Attempting to compute average revenue per flight and the 95% C.I.
+2 Points—(13)	Part (d) Attempting to compute average number of actual passengers boarded and 95% C.I.
+2 Points—(15)	Part (d) Attempting to compute The empirical CDF of the revenue
+2 Points—(17)	Part (d) Attempting to compute The empirical CDF for the actual passengers traveled on the airplane
+3 Points—(20)	Part (d) Correct and organized solutions
Total	

SimAir is a local airline company that runs small jet airplanes between some local cities. Each airplane can take a maximum of 100 passengers. Past data shows that, for any flight, the number of confirmed booking seats follows a shifted binomial distribution with shift 60 and mean 30. In addition, the number of confirmed booking passengers may miss the flight for some reason and do not show at the time of departure. The number of "*no-show*" passengers is a random integer that range between zero and 20% of the confirmed bookings. For every confirmed booking SimAir charge 750 SR. If the passenger did not show up he for his flight, he gets 50% refund. Also, any *unbooked* empty seat cost SimAir 250 SR.

(a) Write the total revenue for SimAir from flight (n) given the variables defined as follows:

- X(n): be the number of confirmed booking seats
- Y(n): be the number of no-show passengers.

(b) Define all parameters needed for simulation in Excel in the following cells location:

	A	B	C	D
1				
2				
3				
4				
5				
6				

You can use screen shot from Excel for the same cells.

(c) Using the parameters you write in part **(a),** write in the following cell the Excel function that you will use to simulate number of confirmed booking seats. *Your answered will be corrected based the cell location you used in (a).*

	A
10	

(d) Using the parameters you write in part **(a),** write in the following cell the Excel function that you will use to simulate number of no-show passengers. *Your answered will be corrected based the cell location you used in (a).*

	B
10	

(e) Using data table in Excel and your answers, **run a simulation model for 1000 flight** to provide the following outputs (*fix the result of data table in Excel*):

- The average revenue per flight and the 95% confidence interval
- The average number of actual passengers boarded the airplane with the 95% confidence interval
- The empirical CDF of the revenue
- The empirical CDF for the actual passengers traveled on the airplane.