بِسْمِ اللّه الرحمن الرحيم

College of Science
Dep. Statistics & Operations Research
OR 441 – Modeling and Simulation
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جــامــعــة
الملك سعود
King Saud University

Final Exam
1440-1441 (Dec. 2019) 1st Semester

وَاللَّهُ وَلِيُّ التَّوْفِيقِ

Name		Student Number	

	Q. #1	Q. #2	Q. #3	Q. #4	Q. #5	Q. #6	Total
Score							

Instructions

1. *Show your university ID*
2. *Exam period is 3 hours.*
3. *Exam consists of 6 questions; each question should not take more than 30 min on average.*
4. *The answer of each question is on the same page, use the back of the pages if you need more space.*
5. *Answer all questions and Show all your work in the answer*
6. *Turn off your cell phones*
7. *Do not use your cell phone for calculations*

Question #1 :

Answer the following with _**True**_ or _**False**_ **(**write your answer in the square before the sentence)

	1. The LCG ($X_n = (aX_{n-1} + c)$ mod(m)) it is always possible to have values for: X_0 , a, c and m to generate more than m pseu-dorandom numbers.
	2. Generating random numbers from the empirical input modeling of individual data gives the exact values that appeared in the sample.
	3. To determine the number of runs in simulation with confidence 95% we must determine the accuracy of simulation by choosing $Z_{\alpha/2}$.
	4. We can determine the number of runs by deciding the expected half width of the confidence interval.
	5. DTAT TABLES in Excel are used to generate random numbers from Normal distribution
	6. Generating random numbers from the empirical input modeling of grouped data gives the only values that appeared in the sample.
	7. The sequence of random numbers generated from a given seed is called a random number a _Stream._
	8. In using _Acceptance/Rejection_ method requires at least two U(0,1) random number to give one random follows the function $f(x)$.
	9. The function VLOOKUP in Excel always used to generate numbers from Gamma.
	10. In Excel the Erlang distribution has inverse function using Gamma.
	11. In using _Acceptance/Rejection_ method we must always choose the majorizing g(x) function to be constant.
	12. In simulation of ATM system, the percentage that there are <u>three</u> customers using the ATM is computed by the time average
	13. We use DATA TABLE in Excel to store the measures of simulation for large number of runs.
	14. To generate random numbers from truncated Weibull between [a,b], we use the inverse transform of the original Weibull with the transformation $a + b\ u$
	15. In simulation of ATM system, the expected number of customers in the waiting line for ATM is computed as a simple average: some of observations/number of observations
	16. To generate one value of Erlang (k=2, λ=3) distribution we generate one value from Exponential (λ=3) and multiply it with 2.
	17. We can always use RANDBETWEEN(a,b) to generate integer numbers between (a) and (b) even for any probability distribution
	18. In simulation of ATM system, the expected waiting time of customers in the waiting line is computed as a simple average: some of observations/number of observations
	19. If X_1 , X_2, X_3 are random values generated from Shifted Exponential(λ=3) with shift value δ=5, then X_1 , X_2, X_3 all values must be less than 5.
	20. If X_1 , X_2, X_3 are random values generated from truncated Exponential(λ=3) with between 2 and 5, then X_1 , X_2, X_3 all values must be less than 5.

Question #2 :

Consider the following probability density function:

$$f(x) = \frac{2x}{25} e^{-\frac{x^2}{25}} \quad ; \quad x > 0$$

(a) Compute the CDF of the function $f(x)$.
(b) Using your answer in part **(a)**, Derive an inverse transform algorithm for this distribution.
(c) Let X be the time between breakdowns on a machine. Use the inverse transform in part **(b)** to determine the time of each breakdown. Generate X_n using LCG generator with parameters: $a = 5$, $c = 3$ and $m = 21$ with $X_0 = 0$. Write your answer in the table below.

	1	2	3	4	5	6	7
LCG u_i							
X							
Time of Break-Down#							

Solution:

Question #3 :

Suppose that customers arrive to a pharmacy with time between arrivals follows Integer Uniform distribution between 3 minutes and 8 minutes. Customer can decide to enter the store instead or use the drive through lane. The service time is assumed random in each way. Assume a 75% chance that the arriving customer decides to enter the pharmacy and a 25% chance that use the drive through. If customers enter the pharmacy, then each customer may have 1, 2, or 3 prescriptions for medication (integer uniform). Each medication takes a random amount of time to prepare that follows Exponential distribution with mean time (E[T] = 2 minutes). For example, if the customer has 2 medications and 1st needs 3 min and the 2nd needs 2 min then the service time is (3+2) minutes. Finally, when a customer take drive through he takes a fixed time for service equals 3 minutes.

1) Write the steps and functions for generating the time between arrivals.

2) Using the table and the uniform numbers below, generate the arrival times of 8 customer's pharmacy in minutes.

Cust. #	1	2	3	4	5	6	7	8
U(0,1)	0.389	0.536	0.649	0.609	0.779	0.913	0.842	0.058
Time between								
Arrival Time								

3) Write the steps and functions for generating the choices of each arrival (In-Pharmacy/Drive-through).

4) Using the table and the uniform numbers below, Generate the choice of 8 customers (Let: In-Pharmacy = 1 and Drive-through = 2).

Cust. #	1	2	3	4	5	6	7	8
U(0,1)	0.823	0.450	0.027	0.017	0.670	0.965	0.027	0.468
Choice								

5) Write the steps and functions for generating the number of medication for each customer (1 or 2 or 3).

6) Using the table and the uniform numbers below, generate the choice of the number of medication for each customers

Cust. #	1	2	3	4	5	6	7	8
U(0,1)	0.285	0.769	0.308	0.752	0.929	0.712	0.281	0.709
No. of Medic.								

7) Write the steps and functions for generating Service Times (in minutes).

8) Using the table and the uniform numbers below and your answers in part **(d)** and part **(f)**, generate the service time of each customer.

Cust. #	1	2	3	4	5	6	7	8
Choice Part (d)								
No. of Medic. Part (f)								
$U_1(0,1)$	0.780	0.693	0.157	0.834	0.097	0.460	0.872	0.415
$U_2(0,1)$	0.457	0.429	0.808	0.468	0.627	0.527	0.881	0.588
$U_3(0,1)$	0.566	0.332	0.863	0.732	0.281	0.306	0.067	0.281
Service Time								

Remark: *for each customer use uniform as needed*

9) When customer enter the pharmacy for medication there is only on server and a waiting line for service. So, customer have to wait in line when the server is busy. In the table below, write the details of ***all customers who enter the pharmacy only***.

Cust. #	Arrival time	No. of Medic.	Service time	Service start	Cust. Wait ??	Wait Time	Departure time	Server Idle time
1								
2								
3								
4								
5								
6								
7								
8								

10) From the table above, compute the average waiting time?

11) From the table above, compute the probability of idle server?

Question #4:

Customers arrive to a minimarket according to a random process. The arriving customers come to a single server checkout counter after they finish shopping. It is assumed that the checkout sever takes a random amount of time to finish the checkout for a customer. The following data was collected.

Cust. #	Time between arrivals	Arrival time (min)	Service time (min)	Service Start Time (min)	Cust. Wait?	Wait Time (min)	EXIT time (min)	Cashier Idle Time (min)
1	8.00	8.00	9.00	8.00	0	0	17.00	8.00
2	7.00	15.00	5.00	17.00	1	2.00	22.00	0.00
3	2.00	17.00	3.00	22.00	1	5.00	25.00	0.00
4	17.00	34.00	5.00	34.00	0	0.00	39.00	9.00
5	5.00	39.00	1.00	39.00	0	0.00	40.00	0.00
6	3.00	42.00	14.00	42.00	0	0.00	56.00	2.00
7	8.00	50.00	4.00	56.00	1	6.00	60.00	0.00
8	13.00	63.00	2.00	63.00	0	0.00	65.00	3.00
9	9.00	72.00	11.00	72.00	0	0.00	83.00	7.00
10	2.00	74.00	2.00	83.00	1	9.00	85.00	0.00

1. From the simulation run, what is the percentage of customers who wait in line?
2. Let Y be a random variable of the customer (Wait or No Wait), use the data above to write the algorithm to generate only the variable Y?
3. What is the probability that the cashier is IDLE during the simulation time?
4. If you want to simulate only (the cashier is idle or not), use the data above to write the algorithm to generate only this variable?
5. What is the average number of arrivals in one hour from simulation?
6. If you want to model the service time as Erlang distribution with parameters α and β cust../min what are the parameters for the service time.
7. If you want to model the arrival process as Poisson distribution, using Moment Matching Method, write the algorithm and the parameters used in the model.
8. Using the output data for the first 6 customers only, compute the probability that there are 1,2,3 in the system?
9. Given that the customer wait in line, what is the average waiting time and the standard deviation?
10. From (9), if you want to do simulation this system to get the average waiting time with confidence interval of half-width E less than 1, we should run the simulation for how many customers to get this accuracy with $\alpha = 0.05$ and $Z_{\alpha/2} = 1.96$.

Question #5:

A student takes a rout to the university that takes an integer uniform time between 20 min and 30 min if there is nothing stopes him in his way to the university. However, there are two traffic lights and an intersection that may be congested as shown in the graph.
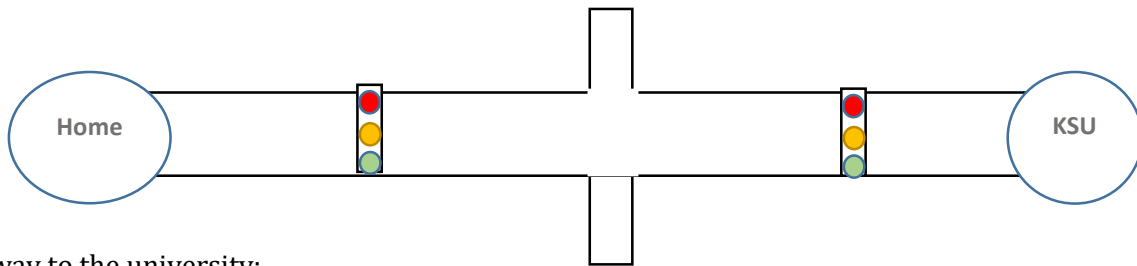
In his way to the university:

- If the student stops at the 1st traffic light (with probability 0.6), then he will spend a random time that follows the distribution function.

$$p_X(x) = \begin{cases} 0.2, & x = 1 \\ 0.3, & x = 2 \\ 0.5, & x = 4 \end{cases}$$

- If the intersection is congested (with probability 0.4) then he will spend a random time that follows an exponential distribution with mean 10 min.
- If the student stops at the 2nd traffic light (with probability 0.3), then he will spend a random time that follows the Erlang distribution with parameters $\alpha = 2$ and $\beta = 0.25$.
1. Write the steps of the simulation for this student.
2. Use your output to model the distribution function of the travel time as binomial distribution.
3. Make a simulation for 5 days for this students to give the data for:
   - Time to the university
   - Waiting time at the 1st traffic light, if the student stops.
   - Delay at the intersection if there is congestion.
   - Waiting time at the 2nd traffic light, if the student stops.

Day 1	Day 2	Day 3	Day 4	Day 5
0.909	0.635	0.077	0.309	0.114
0.228	0.809	0.456	0.590	0.767
0.787	0.724	0.458	0.254	0.127
0.140	0.135	0.153	0.536	0.126
0.277	0.887	0.698	0.394	0.823
0.063	0.099	0.116	0.270	0.882
0.272	0.707	0.013	0.611	0.577
0.271	0.362	0.179	0.934	0.316

Question #6:

Customers arrive to a sweet-shop. Number of slices purchased from Chees-Cake depends on the gender of the customer (Male or Female). The distribution of number of slices purchased by Male is Binomial ($n=3, p=0.35$) and number of slices Female purchased by is given as follows:

Females	
# Slices	Prob.
1	0.25
2	0.35
3	0.40

Solution:

(a) Past data the manager estimates that 40% of the customers are Male and 60% of the customers are Female Write the steps and functions for simulation of this system.

(b) Using the table and the uniform numbers below, do simulation for 8 customers

Cust. #	1	2	3	4	5	6	7	8
U(0,1)	0.389	0.536	0.649	0.609	0.779	0.913	0.842	0.058
Gender								
U(0,1)	0.823	0.450	0.027	0.017	0.670	0.965	0.027	0.468
Number of Slices								

(c) The percentage of male customers depends on number of male arrived before. If total number of male customers arrived is (M), then the probability of next arrival (number N) to be male is max{0.3,M/N}. For example if you want to generate for customer number 5 and there are 3 male customers arrived before him, then customer 5 will be male with probability max{0.3, 3/5}. Write the steps and functions for simulation of this system.

(d) Using the table and the uniform numbers below, do simulation for 8 customers

Cust. #	1	2	3	4	5	6	7	8
U(0,1)	0.389	0.536	0.649	0.609	0.779	0.913	0.842	0.058
Prob. Male								
Gender								
U(0,1)	0.823	0.450	0.027	0.017	0.670	0.965	0.027	0.468
Number of Slices								