|  |  |
| --- | --- |
| Student Name: xxxxxxxxxxxxxxxxxxxxxxxxxx | Old Final Exam |
| Student Roll Number: xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxMax. Time: 3 Hours | **Date: xxxxxxxxxx****Max. Marks: 40** |

**Instructions:**

* Attempt all questions.
* Write in the spaces provided (you can use the back side of the page as well).
* If you feel that the question is not fully specified, state any assumption you need to make in order to solve the problem.
* No extra time will be given.

|  |  |
| --- | --- |
| **Question No.** | **Makes Obtained** |
| **1** |  |
| **2** |  |
| **3** |  |
| **4** |  |
| **5** |  |
| **6** |  |
| **7** |  |
| **8** |  |
|  | **Total** |  |

**Question No. 1 (Quantization) [0.5+0.5+1.5+0.5+1 marks]**

Assuming that a 3-bit ADC channel accepts analog input ranging from -2.5 to 2.5 volts, determine the following:

1. number of quantization levels
2. step size of the quantizer
3. quantization level when the analog voltage is -1.2 volts
4. binary code produced by the ADC
5. quantization error

**Solution:** In this case

$x\_{min}=-2.5$volts and$x\_{max}=2.5$volts, and $m=3$bits.

1. The number of quantization levels are given by

|  |
| --- |
| $$L=2^{3}=2^{3}=8$$ |

1. The step size of the quantizer is given by

$∆=\frac{\left(x\_{max}-x\_{min}\right)}{L}=\frac{\left(2.5-\left(-2.5\right)\right)}{8}=\frac{5}{8}=0.625$volts

1. To find out the quantization level at the given analog voltage, we first calculate the index to the quantization level

$i=round\left(\frac{\left(x-x\_{min}\right)}{∆}\right)=round\left(\frac{\left(-1.2-\left(-2.5\right)\right)}{0.625}\right)=round\left(\frac{1.3}{0.625}\right)=round\left(2.08\right)=2$

Therefore, the quantization level for the analog voltage -1.2 volts is

$x\_{q}=x\_{min}+i×∆=-2.5+2×0.625=-1.25$

1. The binary code corresponding to quantization level $2$ is $010$.
2. The quantization error is given by

$$e\_{q}\left[n\right]=x\left[n\right]-x\_{q}\left[n\right]=-1.2-\left(-1.25\right)=0.05$$

**Question No. 2 (z-Transform) [3+3 marks]**

1. Prove that the convolution in time domain implies convolution in the z-domain, i.e.,

$$Z\left(x\_{1}\left(n\right)\*x\_{1}\left(n\right)\right)=Z\left(x\_{1}\left(n\right)\right) Z\left(x\_{2}\left(n\right)\right)=X\_{1}\left(z\right)X\_{2}\left(z\right) $$

1. Determine the convolution of the following two sequences, using z-transform,

$$x\_{1}\left(n\right)=3δ\left(n\right)+2δ\left(n-1\right)$$

$$x\_{2}\left(n\right)=2δ\left(n\right)-δ\left(n-1\right)$$

**Solution:**

Part (a): According to the definition of the convolution, we have

$$x\left(n\right)=x\_{1}\left(n\right)\*x\_{1}\left(n\right)=\sum\_{k=0}^{\infty }x\_{1}\left(n-k\right)x\_{2}\left(k\right)$$

Taking z-transform

$$X\left(n\right)=Z\left(x\_{1}\left(n\right)\*x\_{1}\left(n\right)\right)=Z\left(\sum\_{k=0}^{\infty }x\_{1}\left(n-k\right)x\_{2}\left(k\right)\right)=\sum\_{n=0}^{\infty }\left(\sum\_{k=0}^{\infty }x\_{1}\left(n-k\right)x\_{2}\left(k\right)\right)z^{-n} $$

It can be written as

$$X\left(n\right)=\sum\_{k=0}^{\infty }x\_{2}\left(k\right) z^{-k}\sum\_{n=0}^{\infty }x\_{1}\left(n-k\right) z^{-(n-k)} $$

Let$ n-k=m$, then the second summation can be written as

$$X\left(n\right)=\sum\_{k=0}^{\infty }x\_{2}\left(k\right) z^{-k}\sum\_{m=-k}^{\infty }x\_{1}\left(m\right) z^{-m} $$

Using causality of both sequences, the second summation can be started from$ m=0$ instead of $m=-k$. Therefore, using the definition of z-transform, we have

$$X\left(n\right)=X\_{1}\left(n\right) X\_{2}\left(n\right) $$

Hence, proved.

Part (b): Taking the z-transform of the two sequences, we have

$$X\_{1}\left(z\right)=Z\left(3δ\left(n\right)+2δ\left(n-1\right)\right)=3+2z^{-1}$$

$$X\_{2}\left(z\right)=Z\left(2δ\left(n\right)-δ\left(n-1\right)\right)=2-z^{-1}$$

Using the z-transform property for convolution of two sequences, we have

$$X\left(z\right)=X\_{1}\left(z\right) X\_{2}\left(z\right)=\left(3+2z^{-1}\right)\left(2-z^{-1}\right)$$

Therefore,

$$X\left(z\right)=6+z^{-1}-2z^{-2}$$

Taking inverse z-transform of both sides (and using shift theorem), we have

$$x\left(n\right)=Z^{-1}\left(X\left(z\right)\right)=6Z^{-1}\left(1\right)+Z^{-1}\left(z^{-1}\right)-2Z^{-1}\left(z^{-2}\right)=6δ\left(n\right)+δ\left(n-1\right)-2δ\left(n-2\right)$$

*(Answer)*

**Question No. 3 (Fast Fourier Transform)**

**[4+1 marks]**

Given a sequence$ x\left(n\right)$ for$ 0\leq n\leq 3$, where $x\left(0\right)=1, x\left(1\right)=2, x\left(2\right)=3$, and$ x\left(3\right)=4$,

1. Determine its DFT$ X\left(k\right)$ using decimation-in-frequency FFT method ?
2. Determine the number of complex multiplication in doing part (a).

**Solution:**

1. According to the FFT (decimation-in-frequency method) for calculating DFT of a give sequence

$$DFT\left\{x\left(n\right) with N points\right\}=\left\{\begin{matrix}DFT\left\{a\left(n\right) with \frac{N}{2} points\right\}\\DFT\left\{b\left(n\right) W\_{N}^{n} with \frac{N}{2} points\right\}\end{matrix}\right.$$

where, $W\_{N}^{n}=e^{-j2πn/N}=cos \left(\frac{2πn}{N}\right)+j sin \left(\frac{2πn}{N}\right)$ and

|  |
| --- |
| $$a\left(n\right)=x\left(n\right)+x\left(n+\frac{N}{2}\right), for n=0, 1, 2,…,\frac{N}{2}-1$$ |
| $$b\left(n\right)=x\left(n\right)-x\left(n+\frac{N}{2}\right), for n=0, 1, 2,…,\frac{N}{2}-1$$ |

The DFT of the given sequence, using FFT (decimation-in-frequency method) is given in the following diagram:

Therefore, the DFT coefficients are $X\left(0\right)=10, X\left(1\right)=-2+2j, X\left(2\right)=-2$, and$ X\left(3\right)=-2-2j$.

1. Number of complex multiplications in $DFT=\frac{N}{2} log\_{2} \left(N\right)=\frac{4}{2} log\_{2} \left(4\right)=4$.

**Question No. 4 [5 marks]**

A relaxed (zero initial conditions) DSP system is described by the difference equation

$$y\left(n\right)+0.1y\left(n-1\right)-0.2y\left(n-2\right)=x\left(n\right)+x(n-1)$$

Determine the impulse response$ y\left(n\right)$ due to the impulse sequence$ x\left(n\right)=δ(n)$.

**Solution:**

Taking z-transform of both sides of the given equation, we get

|  |  |
| --- | --- |
| $$Z\left(y\left(n\right)\right)+0.1Z\left(y\left(n-1\right)\right)-0.2Z\left(y\left(n-2\right)\right)=Z\left(x\left(n\right)\right)+Z\left(x(n-1)\right)$$ | (1) |

We have

$$Z\left(y\left(n\right)\right)=Y(z)$$

$$Z\left(x\left(n\right)\right)=X(z)$$

Using shift theorem, we have

$$Z\left(x\left(n-1\right)\right)=z^{-1}X(z)$$

Also we can apply sift theorem for$ y$ in case of zero initial conditions, i.e.,

$$Z\left(y\left(n-1\right)\right)=z^{-1}Y(z)$$

$$Z\left(y\left(n-2\right)\right)=z^{-2}Y(z)$$

Putting these values in Equation (1), we have

|  |  |
| --- | --- |
| $$Y(z)+0.1z^{-1}Y(z)-0.2z^{-2}Y(z)=X(z)+z^{-1}X(z)$$ |  |
| $$⟹ Y\left(z\right)\left(1+0.1z^{-1}-0.2z^{-2}\right)=X\left(z\right)\left(1+z^{-1}\right)$$ |  |

As$ x\left(n\right)=δ(n)$ therefore, (from Table 1), $X\left(z\right)=1.$ The above equation can now be written as

|  |  |
| --- | --- |
| $$Y\left(z\right)=\frac{\left(1+z^{-1}\right)}{\left(1+0.1z^{-1}-0.2z^{-2}\right)}$$ |  |

Multiplying both the numerator and the denominator with$z^{2}$ we get

|  |  |
| --- | --- |
| $$Y\left(z\right)=\frac{z\left(z+1\right)}{\left(z^{2}+0.1z-0.2\right)}$$ |  |

The denominator can be factorized as

|  |  |
| --- | --- |
| $$Y\left(z\right)=\frac{z\left(z+1\right)}{\left(z^{2}+0.5z-0.4z-0.2\right)}=\frac{z\left(z+1\right)}{\left(z(z+0.5)-0.4(z+0.5)\right)}=\frac{z\left(z+1\right)}{(z+0.5)(z-0.4)}$$ |  |
| $$⟹ \frac{Y\left(z\right)}{z}=\frac{\left(z+1\right)}{(z+0.5)(z-0.4)}$$ | (2) |

The right hand side of the above equation is a proper rational polynomial, with the denominator polynomial having distinct poles, therefore, it can be written into partial fractions as

|  |  |
| --- | --- |
| $$\frac{Y\left(z\right)}{z}=\frac{A}{(z+0.5)}+\frac{B}{(z-0.4)}$$ | (3) |

To find out the unknown constants$ A$ and$ B$, we use:

$$A=\left[\left(z+0.5\right)×\frac{X\left(z\right)}{z}\right]\_{z=-0.5}=\left[\left(z+0.5\right)×\frac{\left(z+1\right)}{\left(z+0.5\right)\left(z-0.4\right)}\right]\_{z=-0.5}=\left[\frac{\left(z+1\right) }{\left(z-0.4\right)}\right]\_{z=-0.5}=\frac{\left(-0.5+1\right)}{\left(-0.5-0.4\right)}=\frac{0.5}{-0.9}=-0.5556$$

$$B=\left[\left(z-0.4\right)×\frac{X\left(z\right)}{z}\right]\_{z=0.4}=\left[\left(z-0.4\right)×\frac{\left(z+1\right)}{\left(z+0.5\right)\left(z-0.4\right)}\right]\_{z=0.4}=\left[\frac{\left(z+1\right) }{\left(z+0.5\right)}\right]\_{z=0.4}=\frac{\left(0.4+1\right)}{\left(0.4+0.5\right)}=\frac{1.4}{0.9}=1.5556$$

Equation (3) becomes:

|  |  |
| --- | --- |
| $$\frac{Y\left(z\right)}{z}=\frac{-0.5556}{(z+0.5)}+\frac{1.5556}{(z-0.4)}$$ |  |
| $$Y\left(z\right)=\frac{-0.5556 z}{(z+0.5)}+\frac{1.5556 z}{(z-0.4)}$$ |  |

Taking inverse z-transform of both sides

|  |  |
| --- | --- |
| $$y\left(n\right)=Z^{-1}\left(Y\left(z\right)\right)=Z^{-1}\left(\frac{-0.5556 z}{(z+0.5)}\right)+Z^{-1}\left(\frac{1.5556 z}{(z-0.4)}\right)=\left(-0.5556\right)Z^{-1}\left(\frac{ z}{(z-(-0.5))}\right)+\left(1.5556\right)Z^{-1}\left(\frac{ z}{(z-0.4)}\right)=\left(-0.5556\right)\left(-0.5\right)^{n} u\left(n\right)+\left(1.5556\right)\left(0.4\right)^{n} u(n)$$ |  |

Thus the output signal is

$$y(n)=\left(-0.5556\right)\left(-0.5\right)^{n} u\left(n\right)+\left(1.5556\right)\left(0.4\right)^{n} u(n)$$

**Question No. 5 [5 marks]**

**Given the following digital system with a sampling rate of 8000 Hz,**

$$y\left(n\right)= x\left(n\right)-0.5y\left(n-1\right)$$

**Determine the frequency response of the system.**

**Solution**

**Taking the z-transform of the both sides of the difference equation, we get**

$$Y\left(z\right)= X\left(z\right)-0.5 z^{-1} Y\left(z\right)$$

$$⟹ Y\left(z\right)+0.5 z^{-1} Y\left(z\right)= X\left(z\right)$$

$$⟹ \left(1+0.5 z^{-1}\right) Y\left(z\right)= X\left(z\right)$$

**Therefore, the transfer function of the system us given by**

$$H\left(z\right)=\frac{Y\left(z\right)}{X\left(z\right)}=\frac{1}{\left(1+0.5 z^{-1}\right)} $$

**To find out the frequency response of the system, we replace**$ z$ **with**$ e^{jΩ}$**. This leads to**

$$H\left(e^{jΩ}\right)=\frac{1}{\left(1+0.5 e^{-jΩ}\right)}$$

**This can be written as**

$$H\left(e^{jΩ}\right)=\frac{1}{\left(1+0.5 cos\left(Ω\right)-j0.5sin\left(Ω\right)\right)}$$

**Therefore, the magnitude frequency response and phase response are given by**

$$\left|H\left(e^{jΩ}\right)\right|=\left|\frac{1}{\left(1+0.5 cos\left(Ω\right)-j0.5sin\left(Ω\right)\right)}\right|=\frac{1}{\sqrt{\left(1 +0.5 cos\left(Ω\right)\right)^{2}+\left(-0.5sin\left(Ω\right)\right)^{2}}}$$

**And**

$$∠H\left(e^{jΩ}\right)=-tan\left(\frac{-0.5sin\left(Ω\right)}{1 +0.5 cos\left(Ω\right)}\right)$$

**This is an example of a highpass filter.**

**Question No. 6 [5 marks]**

**Given a second-order transfer function**

$$H\left(z\right)=\frac{0.5 \left(1-z^{-1}\right)}{\left(1+1.3 z^{-1}+0.36 z^{-2}\right)}$$

**Perform the filter realizations and write the difference equations using the following realizations**

1. **Cascade form via the first-order sections**

**Solution**

**Part (1): For the cascade realization, the transfer function is written in the product form. This is achieved by factorization of the numerator and the denominator polynomials. The given transfer function is**

$$H\left(z\right)=\frac{0.5\left(1-z^{-2}\right) }{1+1.3 z^{-1}+0.36 z^{-2}}$$

**The numerator polynomial can be factorized as**

$$B\left(z\right)= 0.5\left(1-z^{-2}\right)=0.5 \left(1-z^{-1}\right)\left(1+z^{-1}\right)$$

**The denominator polynomial can be factorized as**

$$A\left(z\right)= 1+1.3 z^{-1}+0.36 z^{-2}=1+0.4 z^{-1}+0.9 z^{-1}+0.36 z^{-2}=1\left(1+0.4 z^{-1}\right)+0.9z^{-1}\left(1+0.4 z^{-1}\right)=\left(1+0.4 z^{-1}\right)\left(1+0.9z^{-1}\right) $$

**Therefore, the transfer function can be written as**

$$H\left(z\right)=\frac{0.5 \left(1-z^{-1}\right)\left(1+z^{-1}\right) }{\left(1+0.4 z^{-1}\right)\left(1+0.9z^{-1}\right)}$$

**Or**

$$H\left(z\right)=\left(\frac{ 0.5-0.5 z^{-1} }{1+0.4 z^{-1}}\right)\left(\frac{ 1+z^{-1} }{1+0.9z^{-1}}\right)=H\_{1}\left(z\right)∙H\_{2}\left(z\right)$$

**Thus, in this case**

$$H\_{1}\left(z\right)=\frac{ 0.5-0.5 z^{-1} }{1+0.4 z^{-1}}$$

$$H\_{2}\left(z\right)=\frac{ 1+z^{-1} }{1+0.9z^{-1}}$$

**Each one of**$ H\_{1}\left(z\right)$ **and**$ H\_{2}\left(z\right)$ **can be realized in direct form I or direct form II. Overall, we get the cascaded realization with two sections. It should be noted that there could be other forms for**$ H\_{1}\left(z\right)$ **and**$ H\_{2}\left(z\right)$**, for example, we could have taken** $H\_{1}\left(z\right)=\frac{ 1+z^{-1} }{1+0.4 z^{-1}}$**,** $H\_{2}\left(z\right)=\frac{ 0.5-0.5 z^{-1} }{1+0.9z^{-1}}$**, to yield the same**$ H\left(z\right)$**. Using the former** $H\_{1}\left(z\right)$ **and**$ H\_{2}\left(z\right)$**, and using direct form II realizations for the two cascaded sections, we get the following difference equations:**

**Section 1: (**$H\_{1}\left(z\right)=\frac{ 0.5-0.5 z^{-1} }{1+0.4 z^{-1}}$**)**

$$w\_{1}\left(n\right)=x\left(n\right)- 0.4 w\left(n-1\right)$$

$$y\_{1}\left(n\right)=0.5 w\_{1}\left(n\right)- 0.5 w\_{1}\left(n-1\right)$$

**Section 2: (**$H\_{2}\left(z\right)=\frac{ 1+z^{-1} }{1+0.9z^{-1}}$**)**

$$w\_{2}\left(n\right)=y\_{1}\left(n\right)- 0.9 w\_{2}\left(n-1\right)$$

$$y\left(n\right)= w\_{2}\left(n\right)+w\_{2}\left(n-1\right)$$

****

**Question No. 7 [3+2+1 marks]**

1. **Calculate the filter coefficients for a 5-tap FIR bandpass filter with a lower cutoff frequency of 2000 Hz and an upper cutoff frequency of 2400 at a sampling rate of 8000 Hz.**
2. **Determine the transfer function and the difference equation.**
3. **Find out the frequency response of the filter.**

**Solution**

**Part (a): We first determine the normalized cutoff frequencies**

$Ω\_{L}=\frac{2πf\_{L}}{f\_{s}}=2π×\frac{2000}{8000}=0.5π$ **radians**

$Ω\_{H}=\frac{2πf\_{H}}{f\_{s}}=2π×\frac{2400}{8000}=0.6π$ **radians**

**In this case**$ 2M+1=5$**, therefore, from Table7.1**

$$h\left(n\right)=\left\{\begin{matrix}\frac{Ω\_{H}-Ω\_{L}}{π},&n=0\\\frac{sin\left(Ω\_{H} n\right)}{n π}-\frac{sin\left(Ω\_{L} n\right)}{n π},&-2\leq n\leq 2\end{matrix}\right.$$

**The non-causal FIR coefficients are**

$$h\left(0\right)=\frac{Ω\_{H}-Ω\_{L}}{π}=\frac{0.6π-0.5π}{π}=0.1$$

$$h\left(1\right)=\frac{sin\left(Ω\_{H}×1\right)}{π×1}-\frac{sin\left(Ω\_{L}×1\right)}{π×1}=\frac{sin\left(0.6π\right)}{π}-\frac{sin\left(0.5π\right)}{π}=-0.01558$$

$$h\left(2\right)=\frac{sin\left(Ω\_{H}×2\right)}{π×2}-\frac{sin\left(Ω\_{L}×2\right)}{π×2}=\frac{sin\left(1.2π\right)}{2π}-\frac{sin\left(1.0π\right)}{2π}=-0.09355$$

**Using the symmetry property**

$$h\left(-1\right)=h\left(1\right)=-0.01558$$

$$h\left(-2\right)=h\left(2\right)=-0.09355$$

**Thus the filter coefficients are obtained by delaying by**$ M=2$ **samples, as**

$$b\_{0}=h\left(0-2\right)=h\left(-2\right)=-0.09355$$

$$b\_{1}=h\left(1-2\right)=h\left(-1\right)=-0.01558$$

$$b\_{2}=h\left(2-2\right)=h\left(0\right)=0.1$$

$$b\_{3}=h\left(3-2\right)=h\left(1\right)=-0.01558$$

$$b\_{4}=h\left(4-2\right)=h\left(2\right)=-0.09355$$

**Filter Coefficients**

**Part (b): Therefore, the transfer function in this case is**

$$H\left(z\right)=b\_{0}+b\_{1}z^{-1}+b\_{2}z^{-2}+b\_{3}z^{-3}+b\_{4}z^{-4}=-0.09355-0.01558 z^{-1}+0.1z^{-2}-0.01558z^{-3}-0.09355 z^{-4}$$

**The difference equation is**

$$y\left(n\right)=-0.09355 x\left(n\right)-0.01558 x\left(n-1\right)+0.1 x\left(n-2\right)-0.01558 x\left(n-3\right)-0.09355 x\left(n-4\right)$$

**Part (c): The frequency response of the filter is**

$$H\left(e^{jΩ}\right)=-0.09355-0.01558 e^{-jΩ}+0.1 e^{-2jΩ}-0.01558 e^{-3jΩ}-0.09355 e^{-4jΩ}$$

**Question No. 8 [4 marks]**

**Given the B and A coefficients of a filter as below:**

**B = [-0.09355 -0.01558 0.1 -0.01558 -0.09355]**

**A = [1]**

**Write down a MATLAB program to compute the magnitude frequency response and phase response of this filter. If the sampling rate is 8000 Hz, then write down MATLAB code to plot the magnitude in dB and phase response in degrees as a function of frequency in Hz.**

**Solution**

**The magnitude frequency response and the phase response are computed by the MATLAB program shown below along with the code for the desired plots are given in the following.**

**[response, w] = freqz([-0.09355 -0.01558 0.1 -0.01558 -0.09355], ...**

 **[1], 512);**

**magnitude = abs(response);**

**magnitude2 = 20\*log10(magnitude);**

**phase2 = 180\*unwrap(angle(response))/pi;**

**figure,**

**subplot(2,1,1); plot(w, magnitude2,'LineWidth',2); grid on;**

**xlabel('Frequency (rad)');**

**ylabel('Magnitude response(dB)');**

**subplot(2,1,2); plot(w, phase2,'LineWidth',2); grid on;**

**xlabel('Frequency (rad)');**

**ylabel('Phase response(degrees)');**

**Table 5.1 Table of z-transform pairs (for causal sequences)**

|  |  |  |  |
| --- | --- | --- | --- |
| Line No. | Signal$$x\left(n\right), n\geq 0$$ | z-Transform$$Z\left(x\left(n\right)\right)=X(z)$$ | Region of Convergence |
| 1 | $$x(n)$$ | $$\sum\_{n=0}^{\infty }x\left(n\right) z^{-n}$$ |  |
| 2 | $$δ(n)$$ | **1** | **Entire z-plane** |
| 3 | $$a u(n)$$ | $$\frac{az}{z-1}$$ | $$\left|z\right|>1$$ |
| 4 | $$n u(n)$$ | $$\frac{z}{\left(z-1\right)^{2}}$$ | $$\left|z\right|>1$$ |
| 5 | $$n^{2} u(n)$$ | $$\frac{z(z+1)}{\left(z-1\right)^{3}}$$ | $$\left|z\right|>1$$ |
| 6 | $$a^{n} u(n)$$ | $$\frac{z}{z-a}$$ | $$\left|z\right|>\left|a\right|$$ |
| 7 | $$e^{-na} u(n)$$ | $$\frac{z}{z-e^{-a}}$$ | $$\left|z\right|>e^{-a}$$ |
| 8 | $$n a^{n} u(n)$$ | $$\frac{az}{\left(z-a\right)^{2}}$$ | $$\left|z\right|>\left|a\right|$$ |
| 9 | $$sin\left(an\right) u(n)$$ | $$\frac{z sin\left(a\right)}{z^{2}-2z cos\left(a\right)+1}$$ | $$\left|z\right|>\left|1\right|$$ |
| 10 | $$cos\left(an\right) u(n)$$ | $$\frac{z \left(z-cos\left(a\right)\right)}{z^{2}-2z cos\left(a\right)+1}$$ | $$\left|z\right|>\left|1\right|$$ |
| 11 | $$a^{n} sin\left(bn\right) u(n)$$ | $$\frac{ \left[a sin\left(b\right)\right] z}{z^{2}-\left[2a cos\left(b\right)\right]z +a^{2}}$$ | $$\left|z\right|>\left|a\right|$$ |
| 12 | $$a^{n} cos\left(bn\right) u(n)$$ | $$\frac{z \left[z-a cos\left(b\right)\right] }{z^{2}-\left[2a cos\left(b\right)\right]z +a^{2}}$$ | $$\left|z\right|>\left|a\right|$$ |
| 13 | $$e^{-an} sin\left(bn\right) u(n)$$ | $$\frac{ \left[e^{-a} sin\left(b\right)\right] z}{z^{2}-\left[2e^{-a} cos\left(b\right)\right]z +e^{-2a}}$$ | $$\left|z\right|>e^{-a}$$ |
| 14 | $$e^{-an} cos\left(bn\right) u(n)$$ | $$\frac{z \left[z-e^{-a} cos\left(b\right)\right] }{z^{2}-\left[2e^{-a} cos\left(b\right)\right]z +e^{-2a}}$$ | $$\left|z\right|>e^{-a}$$ |
| 15 | $2\left|A\right|\left|P\right|^{n} cos\left(nθ+ϕ\right) u\left(n\right)$ **where**$ P$ **and**$ A$ **are complex constants defined by**$P=\left|P\right|∠θ$**,** $A=\left|A\right|∠ϕ$ | $$\frac{Az}{z-P}+\frac{A^{\*}z}{z-P^{\*}}$$ |  |

**Shift Theorem:** $ Z\left(x\left(n-m\right)\right)=z^{-m} Z\left(x\left(n\right)\right)$

**Table 7.1: Summary of ideal impulse responses for standard FIR filters**

|  |  |
| --- | --- |
| Filter Type | Ideal Impulse Response$h\left(n\right)$ (non-causal FIR coefficients) |
| Lowpass | $$h\left(n\right)=\left\{\begin{matrix}\frac{Ω\_{c}}{π},&n=0\\\frac{sin\left(Ω\_{c} n\right)}{πn}&-M\leq n\leq M\end{matrix}\right.$$ |
| Highpass | $$h\left(n\right)=\left\{\begin{matrix}\frac{π-Ω\_{c}}{π},&n=0\\-\frac{sin\left(Ω\_{c} n\right)}{πn}&-M\leq n\leq M\end{matrix}\right.$$ |
| Bandpass | $$h\left(n\right)=\left\{\begin{matrix}\frac{Ω\_{H}-Ω\_{L}}{π},&n=0\\\frac{sin\left(Ω\_{H} n\right)}{πn}-\frac{sin\left(Ω\_{L} n\right)}{πn}&-M\leq n\leq M\end{matrix}\right.$$ |
| Bandstop | $$h\left(n\right)=\left\{\begin{matrix}\frac{π-Ω\_{H}+Ω\_{L}}{π},&n=0\\-\frac{sin\left(Ω\_{H} n\right)}{πn}+\frac{sin\left(Ω\_{L} n\right)}{πn}&-M\leq n\leq M\end{matrix}\right.$$ |
| Causual FIR filter coefficients: shifting$ h\left(n\right)$ to the right by samples.Transfer Function:$$H\left(z\right)=b\_{0}+b\_{1}z^{-1}+b\_{2}z^{-2}+\cdots +b\_{2M}z^{-2M}$$where$ b\_{n}=h\left(n-M\right) for n=0,1,2,\cdots ,2$  |

**Table 7.7: Filter length estimation using window (**$∆f=\left|f\_{stop}-f\_{pass}\right|/f\_{s}$**)**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Window Type | Window Function$ w\left(n\right)$, $-M\leq n\leq M$ | Window Length | Passband Ripple (dB) | Stopband Attenuation (dB) |
| Rectangular | $$1$$ | $$N=0.9/∆f$$ | **0.7416** | **21** |
| Hanning | $$0.5+0.5 cos\left(\frac{nπ}{M}\right)$$ | $$N=3.1/∆f$$ | **0.0546** | **44** |
| Hamming | $$0.54+0.46 cos\left(\frac{nπ}{M}\right)$$ | $$N=3.3/∆f$$ | **0.0194** | **53** |
| Blackman | $$0.42+0.5 cos\left(\frac{nπ}{M}\right)+0.08 cos\left(\frac{2nπ}{M}\right)$$ | $$N=5.5/∆f$$ | **0.0017** | **74** |