Phys 103
Chapter 2
Motion in One Dimension

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LECTURE OUTLINE

- 2.1 Position, Velocity, and Speed
- 2.2 Instantaneous Velocity and Speed
- 2.3 Acceleration
- 2.5 One-Dimensional Motion with Constant Acceleration
- 2.6 Freely Falling Objects

Introduction

Kinematics:

Describes motion while ignoring the external agents that might have caused or modified the motion.

For now, will consider motion in one dimension

Along a straight line.

Motion represents a continuous change in the position of an object.

Introduction

Types of Motion

Translational :An example is a car traveling on a highway.

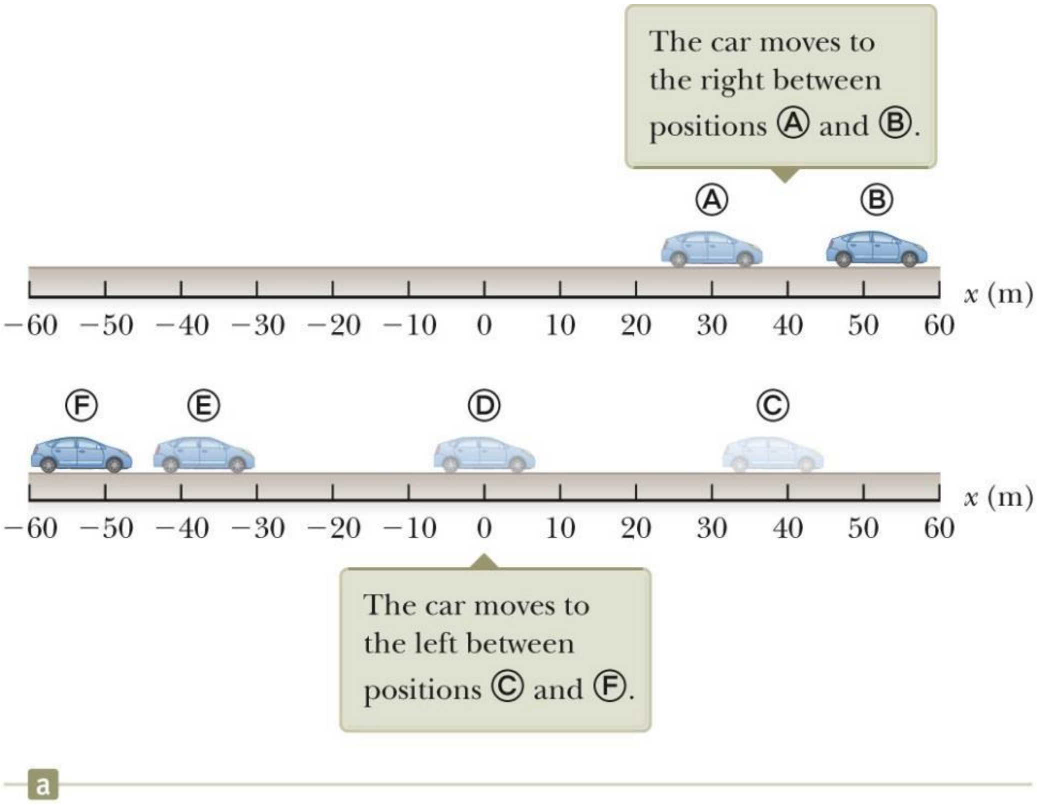
Rotational: An example is the Earth's spin on its axis.

Vibrational: An example is the back-and-forth movement of a pendulum.

2.1 Position, Velocity, and Speed

Position

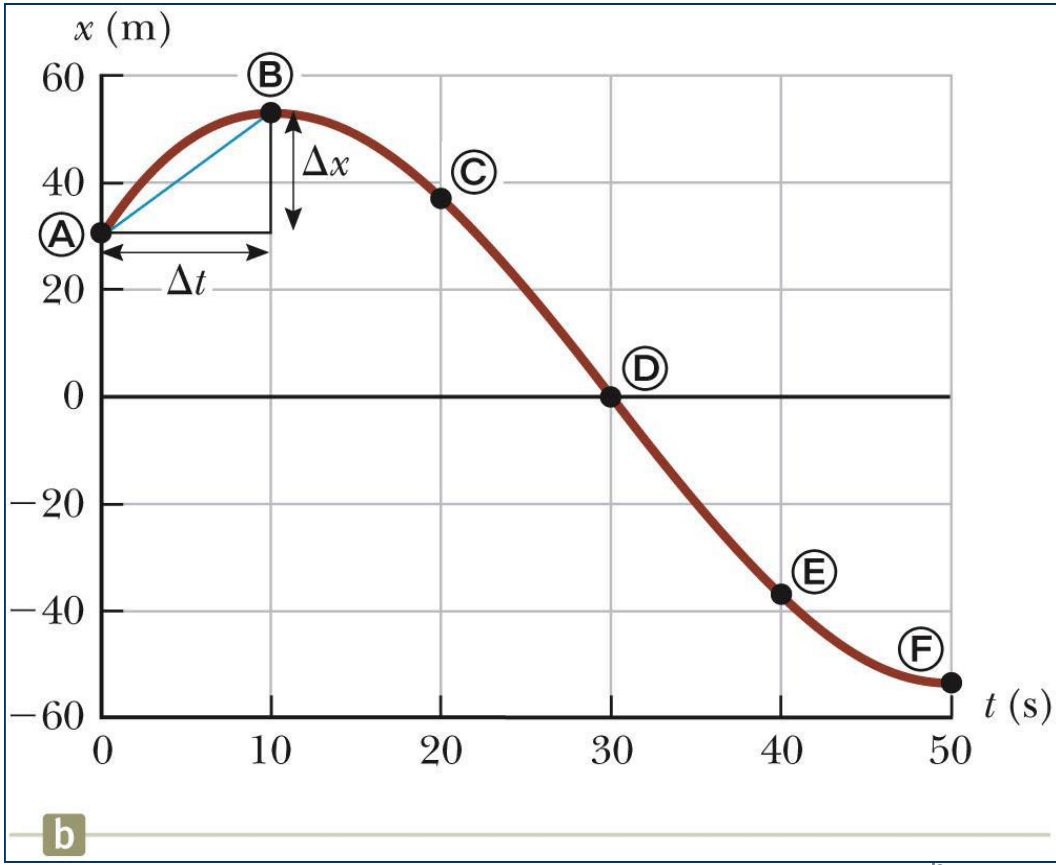
- A particle's position x is the location of the particle with respect to the origin of a coordinate system .



- **Position-Time Graph** : The position-time graph shows the motion of the particle (car). The smooth curve is a guess as to what happened between the data points.

Table 2.1 Position of the Car at Various Times

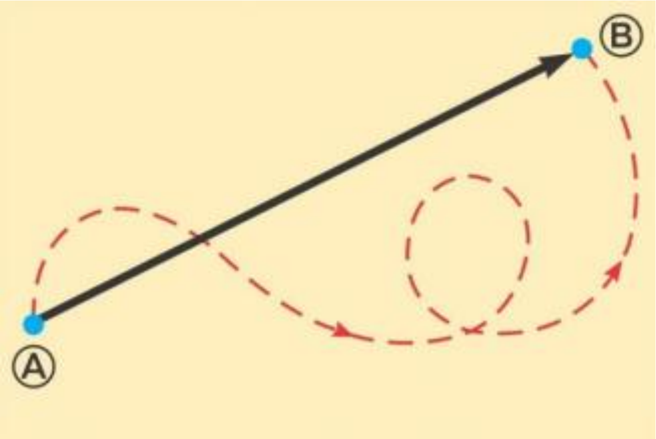
Position	t (s)	x (m)
Ⓐ	0	30
Ⓑ	10	52
Ⓒ	20	38
Ⓓ	30	0
Ⓔ	40	−37
Ⓕ	50	−53



Displacement vs. Distance

Displacement (الازاحة) : the change in position during some time interval, represented as $\Delta x = x_f - x_i$

- SI unit is meter (m)
- Δx can be positive or negative



distance (المسافة): the length of a path followed by a particle

- SI unit is meter (m)
- Distance is always positive

- **Displacement**

The displacement of a particle is defined as its change in position in some time interval.

$$\Delta x = x_f - x_i \begin{cases} \Delta x > 0 \text{ or } x_f > x_i \text{ :motion to the right } \rightarrow + \\ \Delta x < 0 \text{ or } x_f < x_i \text{ :motion to the left } \leftarrow - \\ \Delta x = 0 \text{ or } x_f = x_i \text{ :object returned to} \\ \text{its initial position,or there was no motion} \end{cases}$$

Displacement is an example of a vector quantity.

(requires the specification of both direction and magnitude)

- **Distance**

Distance is the length of a path followed by a particle.

Distance is an example of a scalar quantity.

Average Velocity

The average velocity (السرعة المتوسطة) of particle is defined as the particle's displacement Δx divided by the time interval Δt during which that displacement occurs

$$v_{x,avg} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{\Delta t}$$

- The average velocity is vector quantity
- The dimensions are length/time [L/T]
- The SI unit is (m/s)

(The average velocity of a particle moving in one dimension can be positive or negative, depending on the sign of the displacement , a vector quantity)

Average Speed is the total distance d traveled divided by the total time interval required to travel that distance

$$v_{avg} = \frac{d}{\Delta t}$$

- The average speed is scalar quantity
- The dimensions are length/time [L/T]
- The SI unit is (m/s)

The average speed of particle is a scalar quantity

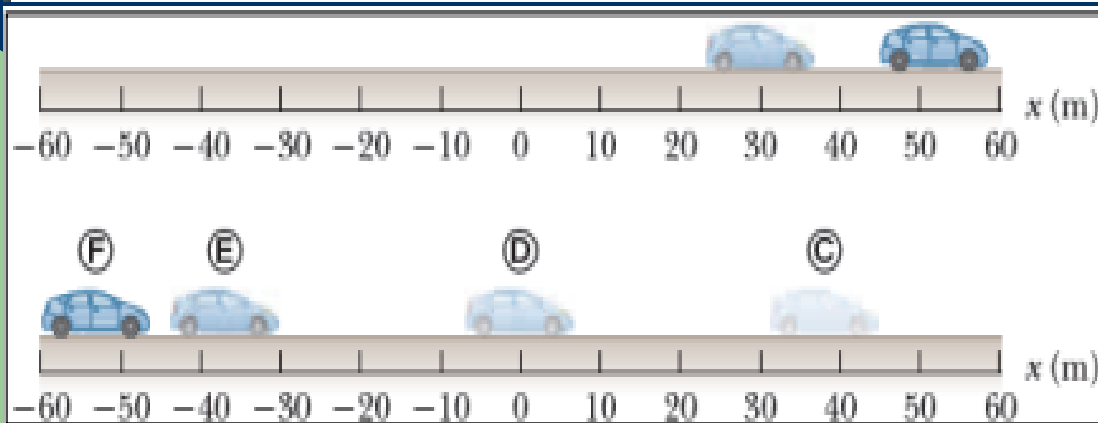
The average speed is not (necessarily) the magnitude of the average velocity

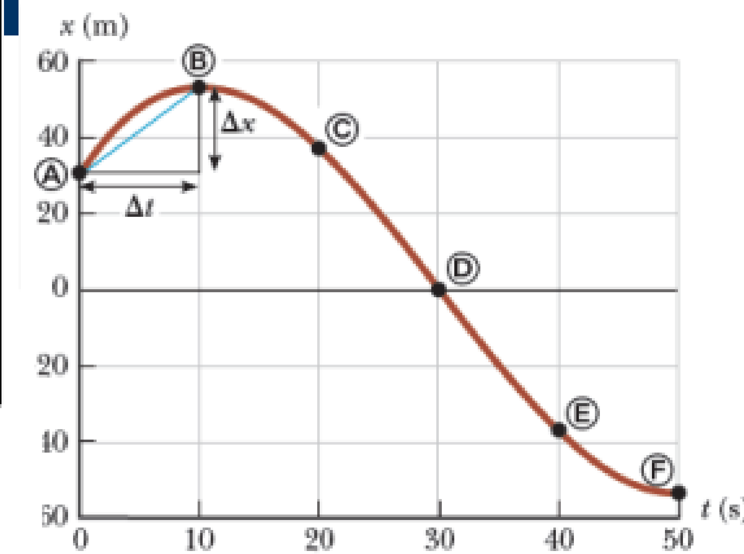
Example 2.1 Calculating the Average Velocity and Speed

Find the displacement, average velocity, and average speed of the car in Figure 2.1a between positions Ⓐ and Ⓕ.

SOLUTION

Consult Figure 2.1 to form a mental image of the car and its motion. We model the car as a particle. From the position–time graph given in Figure 2.1b, notice that $x_{Ⓐ} = 30$ m at $t_{Ⓐ} = 0$ s and that $x_{Ⓕ} = -53$ m at $t_{Ⓕ} = 50$ s.





$$\Delta x = x_{Ⓕ} - x_{Ⓐ} = -53 \text{ m} - 30 \text{ m} = -83 \text{ m}$$

$$v_{x,\text{avg}} = \frac{x_{Ⓕ} - x_{Ⓐ}}{t_{Ⓕ} - t_{Ⓐ}}$$

$$= \frac{-53 \text{ m} - 30 \text{ m}}{50 \text{ s} - 0 \text{ s}} = \frac{-83 \text{ m}}{50 \text{ s}} = -1.7 \text{ m/s}$$

$$v_{\text{avg}} = \frac{127 \text{ m}}{50 \text{ s}} = 2.5 \text{ m/s}$$

2.2 Instantaneous Velocity and Speed

- **Instantaneous Velocity**

The limit of the average velocity as the time interval becomes infinitesimally short, or as the time interval approaches zero.

$$v_x = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

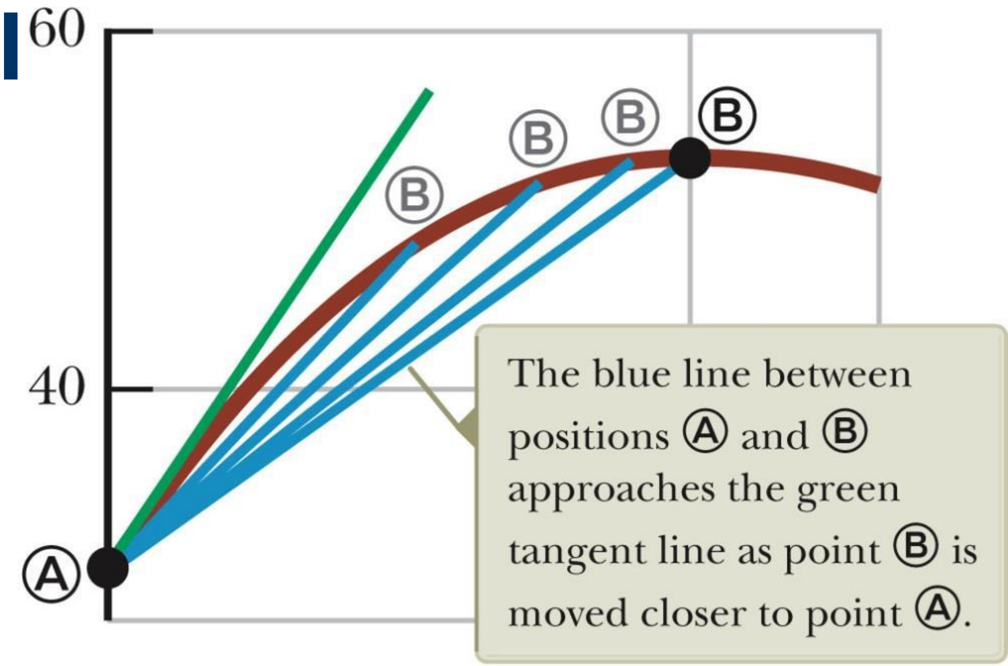
$v_x = 0$ (at maxima or minima) particle is momentarily at rest

this limit is called the *derivative* of *x* with respect to *t*

The instantaneous velocity indicates what is happening at every point of time.

The instantaneous velocity is the slope of the line tangent to the x vs. t curve.

This would be the green line. The light blue lines show that as $\Delta \boldsymbol{t}$ gets smaller, they approach the green line.



The slope of a graph of physical data represents the ratio of change in the quantity represented on the vertical axis to the change in the quantity represented by the horizontal axis.

Instantaneous Speed

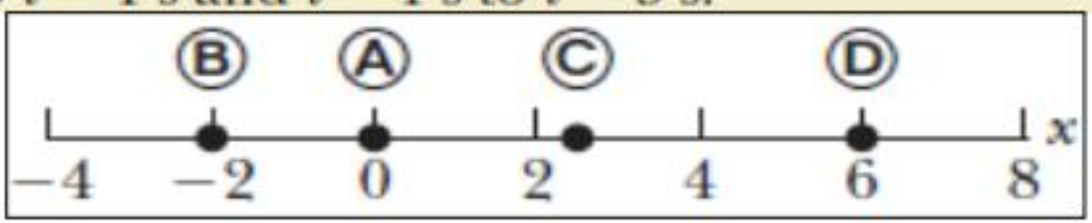
The instantaneous speed is the magnitude of the instantaneous velocity.
The instantaneous speed has no direction associated with it and hence carries no algebraic sign**.**

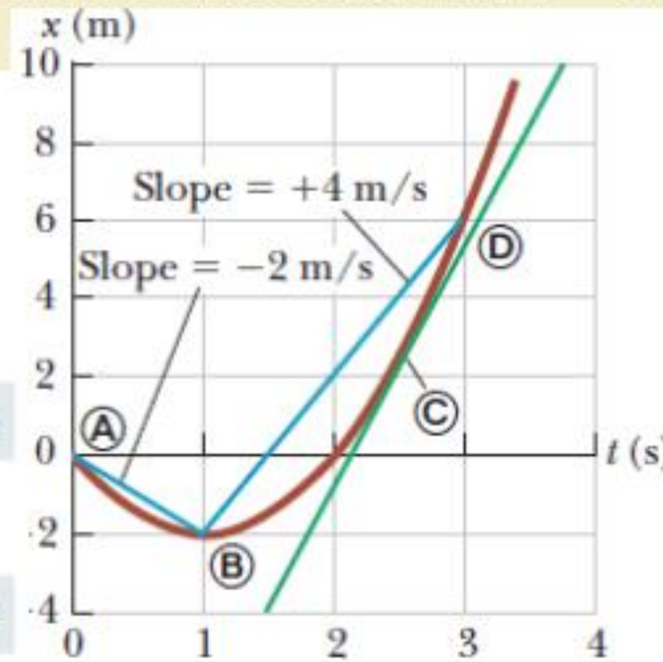
we use the word velocity to designate instantaneous velocity. When it is average velocity we are interested in, we shall always use the adjective *average*.

Example 2.3 Average and Instantaneous Velocity

A particle moves along the x axis. Its position varies with time according to the expression $x = -4t + 2t^2$, where x is in meters and t is in seconds.[3] The position–time graph for this motion is shown in Figure 2.4a. Because the position of the particle is given by a mathematical function, the motion of the particle is completely known, unlike that of the car in Figure 2.1. Notice that the particle moves in the negative x direction for the first second of motion, is momentarily at rest at the moment $t = 1$ s, and moves in the positive x direction at times $t > 1$ s.

(A) Determine the displacement of the particle in the time intervals $t = 0$ to $t = 1$ s and $t = 1$ s to $t = 3$ s.





$$\Delta x_{Ⓐ\to Ⓑ} = x_f - x_i = x_Ⓑ - x_Ⓐ$$

$$= [-4(1) + 2(1)^2] - [-4(0) + 2(0)^2] = -2 \text{ m}$$

$$\Delta x_{Ⓑ\to Ⓓ} = x_f - x_i = x_Ⓓ - x_Ⓑ$$

$$= [-4(3) + 2(3)^2] - [-4(1) + 2(1)^2] = +8 \text{ m}$$

(B) Calculate the average velocity during these two time intervals.

$$v_{x,\text{avg}\,(Ⓐ\to Ⓑ)} = \frac{\Delta x_{Ⓐ\to Ⓑ}}{\Delta t} = \frac{-2\text{ m}}{1\text{ s}} = -2\text{ m/s}$$

$$v_{x,\text{avg}\,(Ⓑ\to Ⓓ)} = \frac{\Delta x_{Ⓑ\to Ⓓ}}{\Delta t} = \frac{8\text{ m}}{2\text{ s}} = +4\text{ m/s}$$

(C) Find the instantaneous velocity of the particle at t = 2.5 s.

$$v_x = \frac{10\text{ m} - (-4\text{ m})}{3.8\text{ s} - 1.5\text{ s}} = +6\text{ m/s}$$

Another solution:

$$v_x = \frac{dx}{dt} = -4 + 4t$$

$$v_x(at\ t = 2.5\ s) = -4 + 4(2.5) = -4 + 10 = +6\text{ m/s}$$

2.3 Acceleration

- **Average Acceleration**

- The average acceleration $\overline{a_x}$ of the particle is defined as the *change* in velocity Δv_x divided by the time interval Δt during which that change occurs:

$$\overline{a_x} = \frac{\Delta v_x}{\Delta t} = \frac{v_{xf} - v_{xi}}{t_f - t_i}$$

- Because the dimensions of velocity are L/T and the dimension of time is T, acceleration has dimensions of length divided by time squared, or L/T2. The SI unit of acceleration is meters per second squared (m/s2).

In one dimension, positive and negative can be used to indicate direction.

- **Instantaneous acceleration**

Instantaneous acceleration define as the limit of the average acceleration as $\Delta \boldsymbol{t}$ approaches zero.

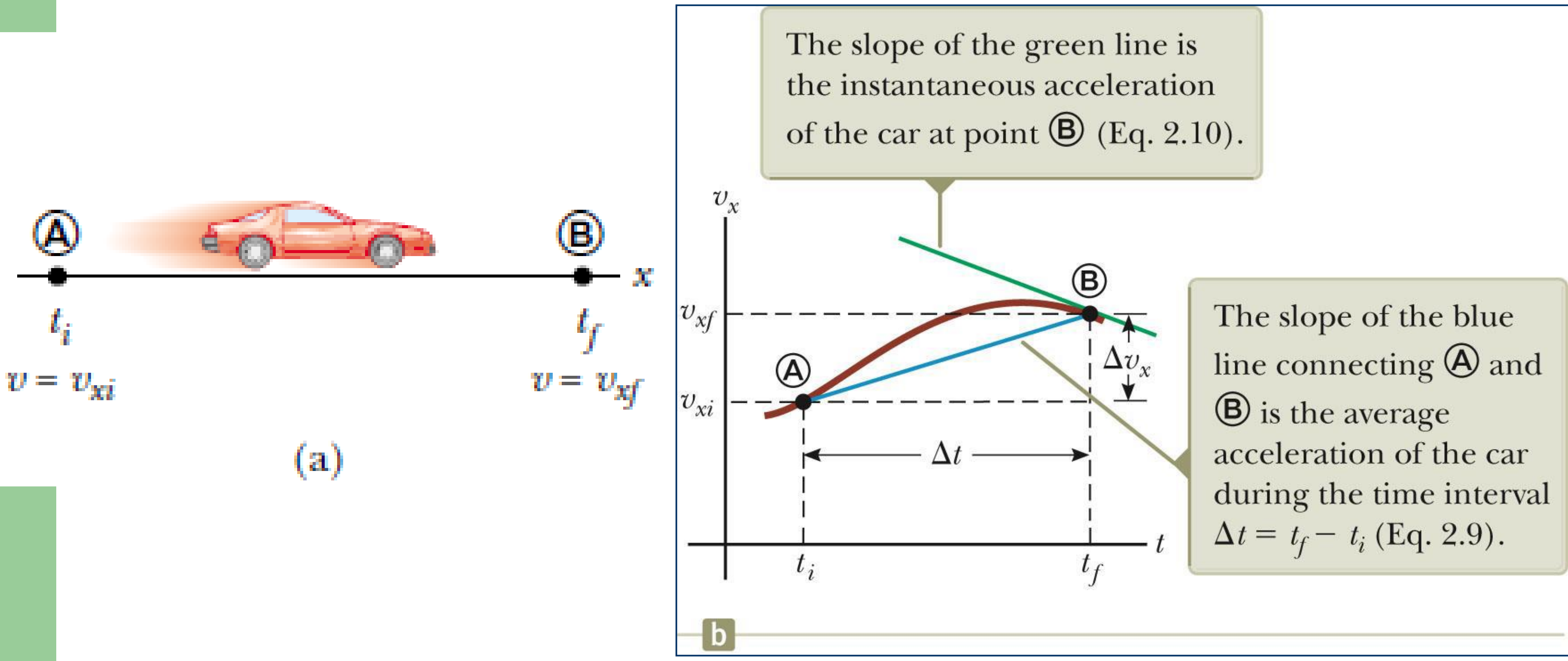
$$a_x = \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t} = \frac{dv_x}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2 x}{dt^2}$$

That is, the instantaneous acceleration equals the derivative of the velocity with respect to time, which by definition is the slope of the velocity–time graph.

The term acceleration will mean instantaneous acceleration. If average acceleration is wanted, the word average will be included.

- **Instantaneous acceleration**

The slope of the velocity-time graph is the acceleration. The green line represents the instantaneous acceleration. The blue line is the average acceleration.



2.3 Acceleration

Example 2.5 Average and Instantaneous Acceleration

The velocity of a particle moving along the x axis varies in time according to the expression $v_x = (40 - 5t^2)$ m/s, where t is in seconds.

(A) Find the average acceleration in the time interval $t = 0$ to $t = 2.0$ s.

Solution Figure 2.8 is a v_x-t graph that was created from the velocity versus time expression given in the problem statement. Because the slope of the entire v_x-t curve is negative, we expect the acceleration to be negative.

We find the velocities at $t_i = t_A = 0$ and $t_f = t_B = 2.0$ s by substituting these values of t into the expression for the velocity:

$$v_{xA} = (40 - 5t_A^2)\ \text{m/s} = [40 - 5(0)^2]\ \text{m/s} = +40\ \text{m/s}$$

$$v_{xB} = (40 - 5t_B^2)\ \text{m/s} = [40 - 5(2.0)^2]\ \text{m/s} = +20\ \text{m/s}$$

Therefore, the average acceleration in the specified time interval $\Delta t = t_B - t_A = 2.0$ s is

$$\bar{a}_x = \frac{v_{xf} - v_{xi}}{t_f - t_i} = \frac{v_{xB} - v_{xA}}{t_B - t_A} = \frac{(20 - 40)\ \text{m/s}}{(2.0 - 0)\ \text{s}}$$

$$= -10\ \text{m/s}^2$$

The negative sign is consistent with our expectations—namely, that the average acceleration, which is represented by the slope of the line joining the initial and final points on the velocity–time graph, is negative.

(B) Determine the acceleration at $t = 2.0$ s.

2.3 Acceleration

Solution The velocity at any time t is $v_{xi} = (40 - 5t^2)$ m/s and the velocity at any later time $t + \Delta t$ is

$$v_{xf} = 40 - 5(t + \Delta t)^2 = 40 - 5t^2 - 10t\,\Delta t - 5(\Delta t)^2$$

Therefore, the change in velocity over the time interval Δt is

$$\Delta v_x = v_{xf} - v_{xi} = [-10t\,\Delta t - 5(\Delta t)^2] \text{ m/s}$$

Dividing this expression by Δt and taking the limit of the result as Δt approaches zero gives the acceleration at *any* time t:

$$a_x = \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t} = \lim_{\Delta t \to 0} (-10t - 5\Delta t) = -10t \text{ m/s}^2$$

Therefore, at $t = 2.0$ s,

$$a_x = (-10)(2.0) \text{ m/s}^2 = \boxed{-20 \text{ m/s}^2}$$

Because the velocity of the particle is positive and the acceleration is negative, the particle is slowing down.

Note that the answers to parts (A) and (B) are different. The average acceleration in (A) is the slope of the blue line in Figure 2.8 connecting points Ⓐ and Ⓑ. The instantaneous acceleration in (B) is the slope of the green line tangent to the curve at point Ⓑ. Note also that the acceleration is *not* constant in this example. Situations involving constant acceleration are treated in Section 2.5.

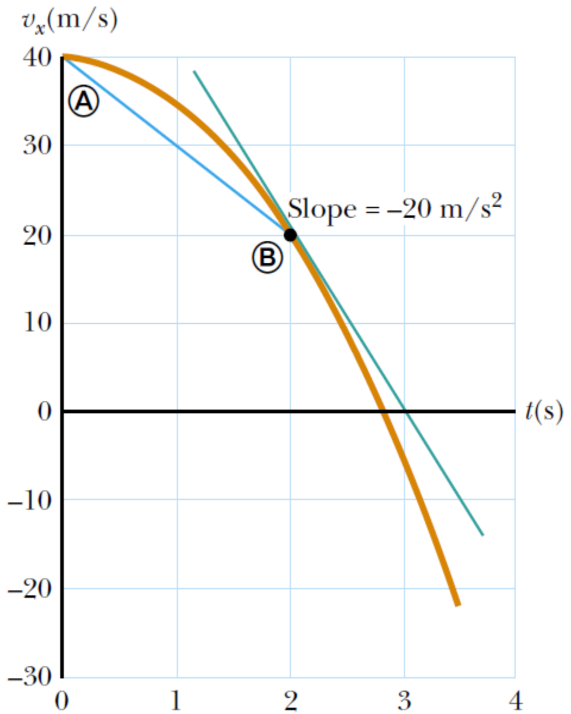


Figure 2.8 (Example 2.5) The velocity–time graph for a particle moving along the x axis according to the expression $v_x = (40 - 5t^2)$ m/s. The acceleration at $t = 2$ s is equal to the slope of the green tangent line at that time.

Example 2.6 Average and Instantaneous Acceleration

The velocity of a particle moving along the x axis varies according to the expression $v_x = 40 - 5t^2$, where v_x is in meters per second and t is in seconds.

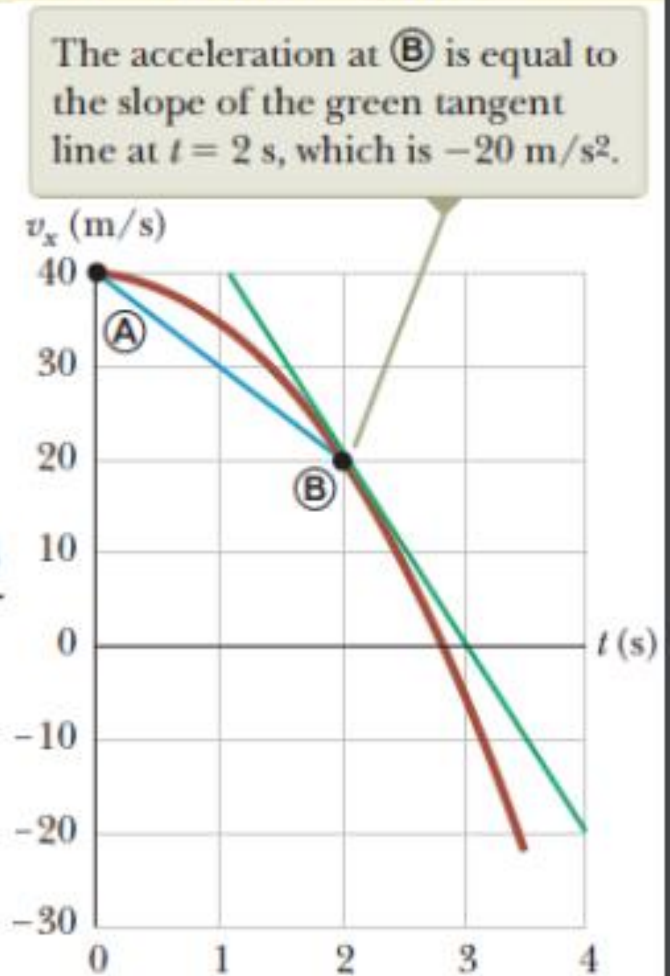
(A) Find the average acceleration in the time interval $t = 0$ to $t = 2.0$ s.

$$v_{x(A)} = 40 - 5t_{(A)}^2 = 40 - 5(0)^2 = +40 \text{ m/s}$$

$$v_{x(B)} = 40 - 5t_{(B)}^2 = 40 - 5(2.0)^2 = +20 \text{ m/s}$$

$$a_{x,\text{avg}} = \frac{v_{xf} - v_{xi}}{t_f - t_i} = \frac{v_{x(B)} - v_{x(A)}}{t_{(B)} - t_{(A)}} = \frac{20 \text{ m/s} - 40 \text{ m/s}}{2.0 \text{ s} - 0 \text{ s}}$$

$$= -10 \text{ m/s}^2$$



The negative sign is consistent with our expectations: the average acceleration, represented by the slope of the blue line joining the initial and final points on the velocity–time graph, is negative.

(B) Determine the acceleration at $t = 2.0$ s.

$$v_{xf} = 40 - 5(t + \Delta t)^2 = 40 - 5t^2 - 10t\,\Delta t - 5(\Delta t)^2$$

$$\Delta v_x = v_{xf} - v_{xi} = -10t\,\Delta t - 5(\Delta t)^2$$

$$a_x = \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t} = \lim_{\Delta t \to 0}(-10t - 5\,\Delta t) = -10t$$

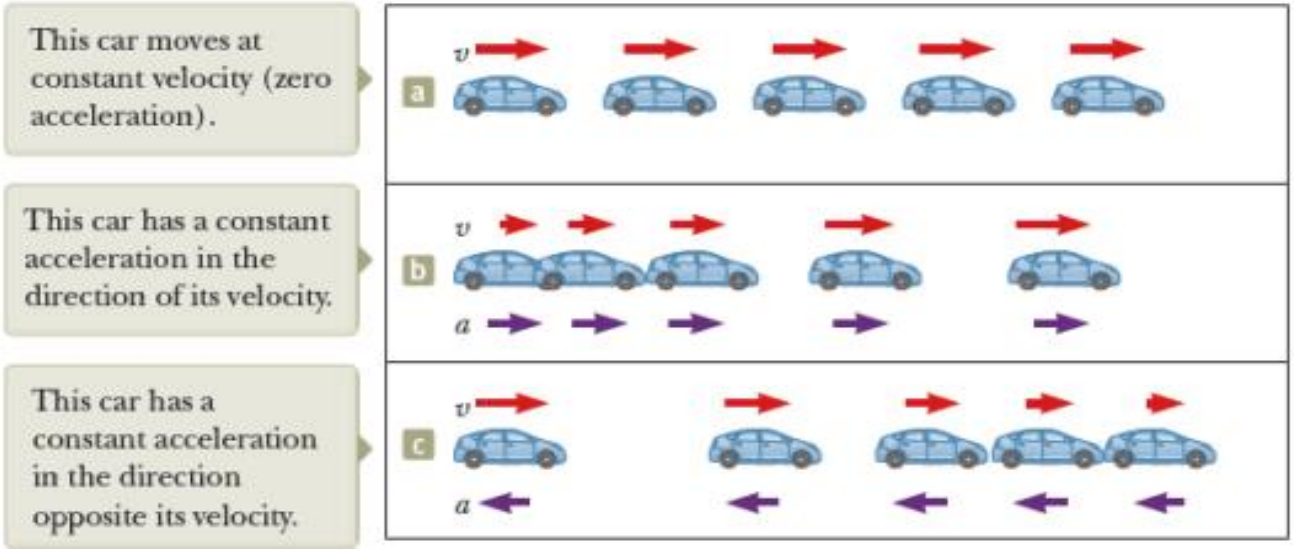
$$a_x = (-10)(2.0)\ \text{m/s}^2 = \boxed{-20\ \text{m/s}^2}$$

Because the velocity of the particle is positive and the acceleration is negative at this instant, the particle is slowing down.

Notice that the answers to parts (A) and (B) are different. The average acceleration in part (A) is the slope of the blue line in Figure 2.9 connecting points Ⓐ and Ⓑ. The instantaneous acceleration in part (B) is the slope of the green line tangent to the curve at point Ⓑ. Notice also that the acceleration is *not* constant in this example. Situations involving constant acceleration are treated in Section 2.6.

Acceleration and Velocity, Directions

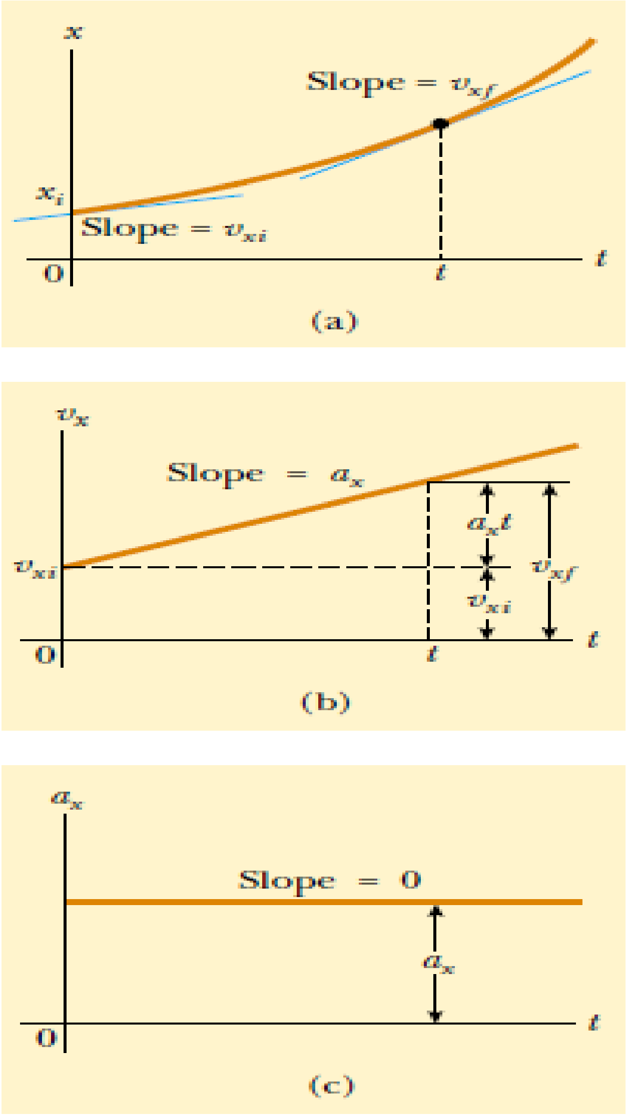
- When an object's velocity and acceleration (driving force) are in the same direction, the object is speeding up. In this case the acceleration is positive.
- When an object's velocity and acceleration (driving force) are in the opposite direction, the object is slowing down. In this case the acceleration is negative.



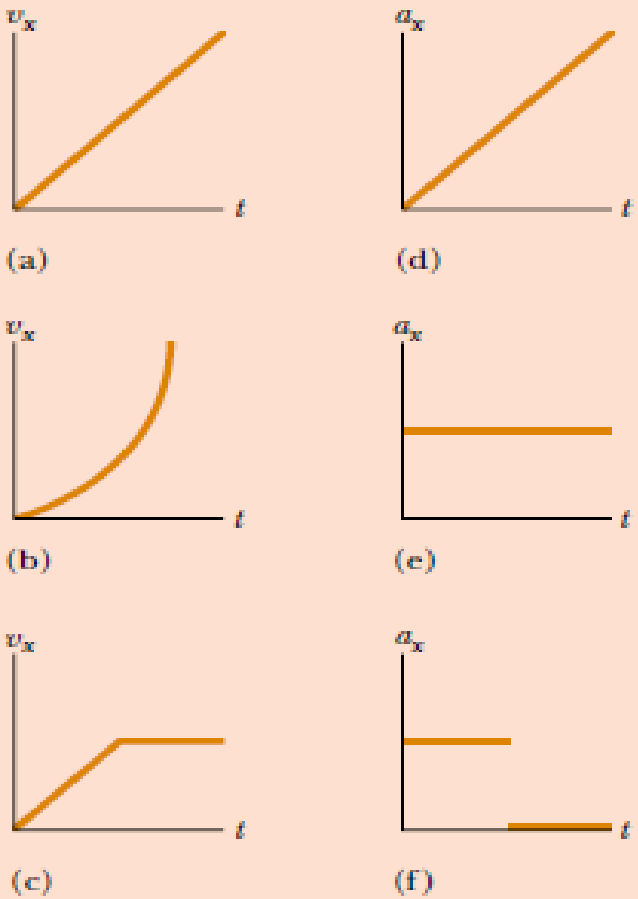
2.5 One-Dimensional Motion with Constant Acceleration

If the acceleration of a particle varies in time, its motion can be complex and difficult to analyze.

- **Simple type of 1-D motion is that in which the acceleration is constant. When this is the case, the average acceleration over any time interval is numerically equal to the instantaneous acceleration a_x at any instant within the interval, and the velocity changes at the same rate throughout the motion.**



Quick Quiz 2.5 In Figure 2.11, match each v_x-t graph on the left with the a_x-t graph on the right that best describes the motion.



Active Figure 2.11 (Quick Quiz 2.5) Parts (a), (b), and (c) are v_x-t graphs of objects in one-dimensional motion. The possible accelerations of each object as a function of time are shown in scrambled order in (d), (e), and (f).

At the Active Figures link at http://www.pse6.com, you can practice matching appropriate velocity vs. time graphs and acceleration vs. time graphs.

Kinematic Equations

- The kinematic equations can be used with any particle to solve any problem involving one-dimensional motion with a constant acceleration

Kinematic Equations for Motion of a Particle Under Constant Acceleration

Equation	Information Given by Equation
$v_{xf} = v_{xi} + a_x t$	Velocity as a function of time
$x_f = x_i + \frac{1}{2}(v_{xi} + v_{xf})\,t$	Position as a function of velocity and time
$x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2$	Position as a function of time
${v_{xf}}^2 = {v_{xi}}^2 + 2a_x(x_f - x_i)$	Velocity as a function of position

Kinematic Equations

- The equation: $v_{x,f} = v_{x,i} + a_x t$ determine an object's velocity at any time ***t*** when we know its initial velocity and its acceleration
- The equation: $x_f = x_i + v_{x,i}t + \frac{1}{2}a_x t^2$ gives final position in terms of velocity and acceleration
- The equation: $v_{x,f}{}^2 = v_{x,i}{}^2 + 2a_x(x_f - x_i)$ gives final velocity in terms of acceleration and displacement

Example 2.7: Carrier Landing

A jet lands on an aircraft carrier at a speed of 63 m/s.
(A) What is its acceleration (assumed constant) if it stops in 2.0 s due to an arresting cable that snags the jet and brings it to a stop?

$$a_x = \frac{v_{xf} - v_{xi}}{t} \approx \frac{0 - 63\ \mathrm{m/s}}{2.0\ \mathrm{s}}$$

$$= -32\ \mathrm{m/s^2}$$

(A) If the jet touches down at position $x_i = 0$, what is its final position?

$$x_f = x_i + \tfrac{1}{2}(v_{xi} + v_{xf})t = 0 + \tfrac{1}{2}(63\ \mathrm{m/s} + 0)(2.0\ \mathrm{s}) = 63\ \mathrm{m}$$

Exercise 1

A truck covers 40.0 m in 8.50 s while smoothly slowing down to a final speed of 2.80 m/s. (a) Find its original speed. (b) Find its acceleration.

2.6 Freely Falling Objects

A freely falling object is any object moving freely under the influence of gravity alone, regardless of its initial motion. Objects thrown upward or downward and those released from rest are all falling freely once they are released. Any freely falling object experiences an acceleration directed downward, regardless of its initial motion.

- At the Earth's surface, the value of g is approximately 9.80 m/s2.
- we always choose a_y=-g=-9.80 m/s2

Free fall equations

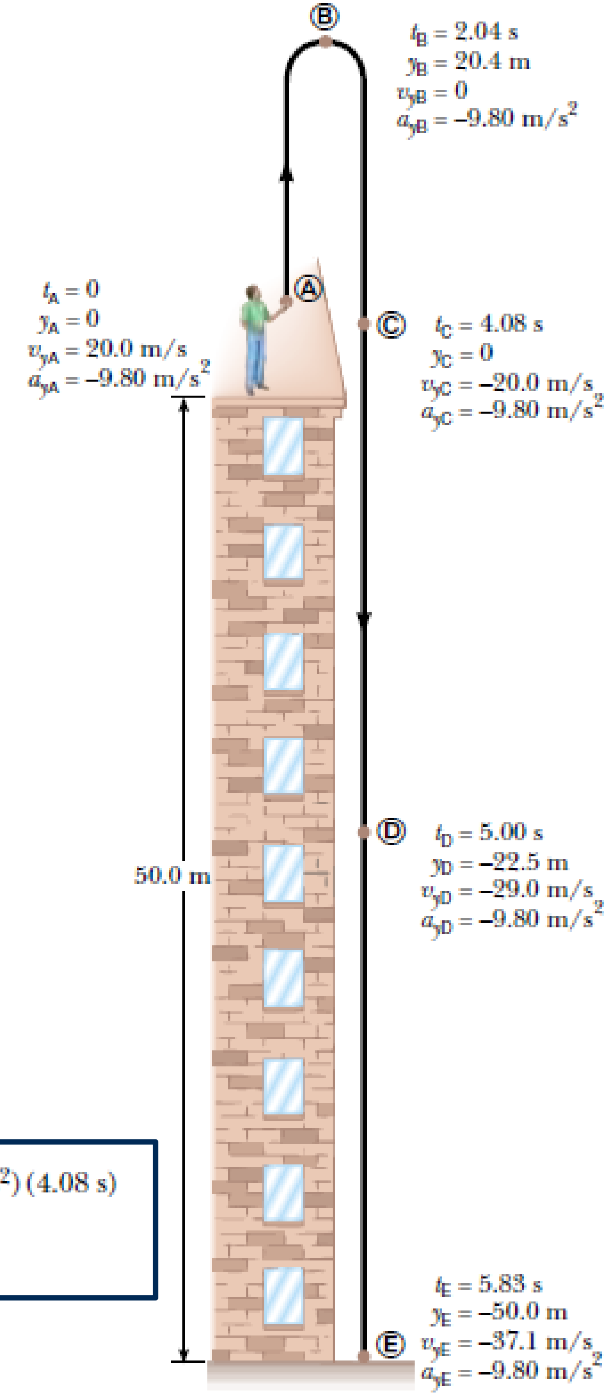
$$v_{yf} = v_{yi} - \mathrm{g}t$$

$$\Delta y = v_{yi}t - \frac{1}{2}\mathrm{g}\,t^2$$

$$v_{yf}{}^2 = v_{yi}{}^2 - 2\mathrm{g}(\Delta y)$$

Example 2.12 Not a Bad Throw for a Rookie!

A stone thrown from the top of a building is given an initial velocity of 20.0 m/s straight upward. The building is 50.0 m high, and the stone just misses the edge of the roof on its way down, as shown in Figure 2.14. Using $t_A = 0$ as the time the stone leaves the thrower's hand at position Ⓐ, determine **(A)** the time at which the stone reaches its maximum height, **(B)** the maximum height, **(C)** the time at which the stone returns to the height from which it was thrown, **(D)** the velocity of the stone at this instant, and **(E)** the velocity and position of the stone at $t = 5.00$ s.



(A)

$$0 = 20.0 \text{ m/s} + (-9.80 \text{ m/s}^2)t$$

$$t = t_B = \frac{20.0 \text{ m/s}}{9.80 \text{ m/s}^2} = \boxed{2.04 \text{ s}}$$

(B)

$$y_{max} = y_B = y_A + v_{xA}t + \tfrac{1}{2}a_y t^2$$

$$y_B = 0 + (20.0 \text{ m/s})(2.04 \text{ s}) + \tfrac{1}{2}(-9.80 \text{ m/s}^2)(2.04 \text{ s})^2$$

$$= \boxed{20.4 \text{ m}}$$

(C)

$$y_C = y_A + v_{yA}t + \tfrac{1}{2}a_y t^2$$

$$0 = 0 + 20.0t - 4.90t^2$$

$$\boxed{t = 4.08 \text{ s,}}$$

(D)

$$v_{yC} = v_{yA} + a_y t = 20.0 \text{ m/s} + (-9.80 \text{ m/s}^2)(4.08 \text{ s})$$

$$= \boxed{-20.0 \text{ m/s}}$$

Example 2.12 Not a Bad Throw for a Rookie!

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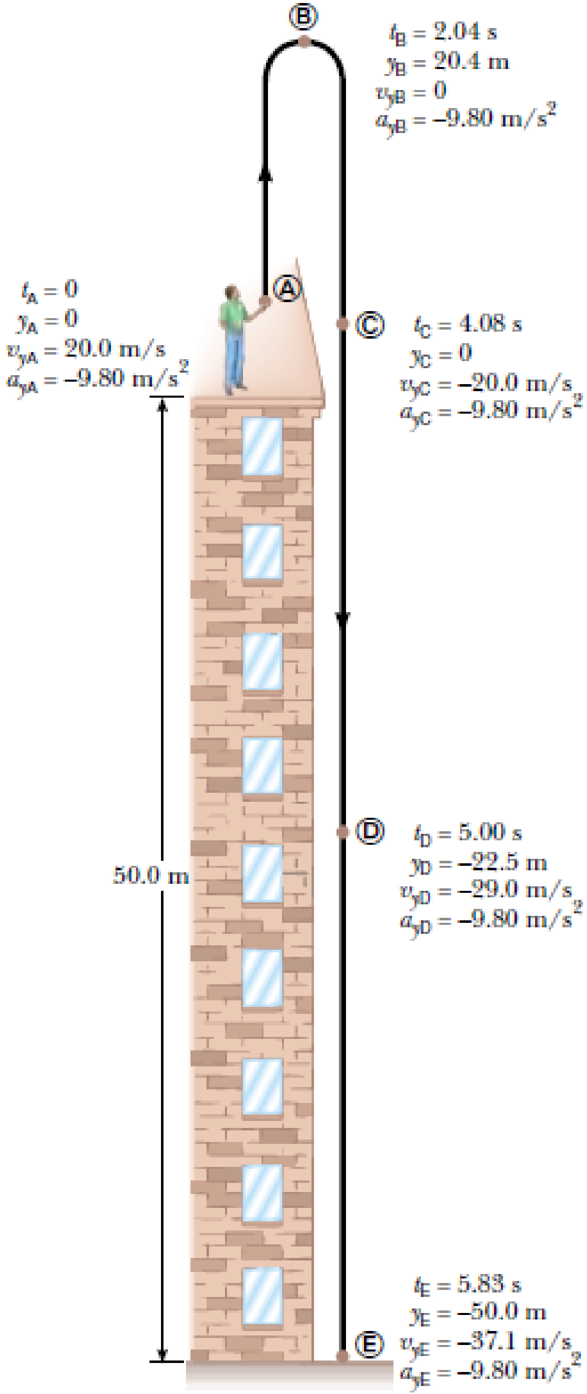

(E)

$$v_{yD} = v_{yB} + a_y t = 0 \text{ m/s} + (-9.80 \text{ m/s}^2)(2.96 \text{ s})$$
$$= -29.0 \text{ m/s}$$

$$v_{yD} = v_{yA} + a_y t = 20.0 \text{ m/s} + (-9.80 \text{ m/s}^2)(5.00 \text{ s})$$
$$= -29.0 \text{ m/s}$$

$$y_D = y_C + v_{yC} t + \tfrac{1}{2} a_y t^2$$
$$= 0 + (-20.0 \text{ m/s})(5.00 \text{ s} - 4.08 \text{ s}) + \tfrac{1}{2}(-9.80 \text{ m/s}^2)(5.00 \text{ s} - 4.08 \text{ s})^2$$
$$= -22.5 \text{ m}$$

Example : John throws the ball straight upward and after 1 second it reaches its maximum height then it does free fall motion which takes 2 seconds. Calculate the maximum height and velocity of the ball before it crashes the ground. ($g=10m/s^2$)