Motion in One Dimension

2.1 Position, Velocity, and Speed

2.2 Instantaneous Velocity and Speed

2.3 Acceleration

2.4 Motion Diagrams

2.5 One-Dimensional Motion with Constant Acceleration

2.6 Freely Falling Objects

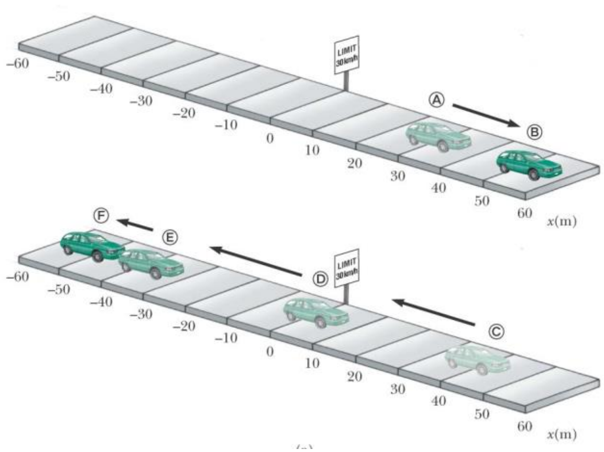
Introduction

- Kinematics is a part of dynamics
  –Describes motion while ignoring the agents that caused the motion
- Motion of an object represents a continuous change in the object's position.
- Types of motion: translational, rotational, and vibrational.
- In this chapter, we consider only motion in one dimension, that is, motion of an object along a straight line.
- A moving object is describe as a *particle* that has mass regardless of its size. (**a particle model**).

2.1 Position, Velocity, and Speed

Position: A particle's position *x* is the location of the particle with respect to a chosen reference point that we can consider to be the origin of a coordinate system.

The motion of a particle is completely known if the particle's position in space is known at all times.

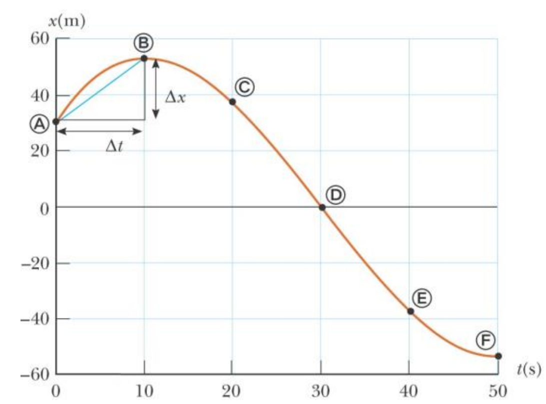


A pictorial representation

Position of the Car at Various Times

Position	t(s)	x(m)
Ⓐ	0	30
Ⓑ	10	52
Ⓒ	20	38
Ⓓ	30	0
Ⓔ	40	−37
Ⓕ	50	−53

A tabular representation



A graphical representation
(a position–time graph)

Displacement:

When a particle moves along the x axis from some initial position x_i to some final position x_f, its **displacement** is

$$\Delta x = x_f - x_i \qquad (2.1)$$

The displacement Δx of a particle is **the change of its position in some time interval.**

Displacement is a vector quantity.

Distance:

The length of a path followed by a particle.

It is scalar quantity

If an athlete runs around a track that is 100 meters long three times, then stops.

Distance= 300 m

Displacement = 0

Vectors and Scalars:

- Vector quantities need both magnitude (size or numerical value) and direction to completely describe them
  - Will use + and – signs to indicate vector directions

- Scalar quantities are completely described by magnitude only

Average Velocity

The average velocity is the particle's displacement Δx divided by the time interval Δt

$$v_{x,\text{avg}} = \bar{v}_x = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i} \qquad \textbf{(2.2)}$$

The dimensions are length / time [L/T]

Unit: meters per second in SI units (m/s)

Direction is the same as the direction of the displacement

Average Speed

Speed:

is the total distance *d* traveled divided by the total time interval required to travel that distance:

$$u = v_{\mathrm{avg}} = \frac{d}{\Delta t}$$

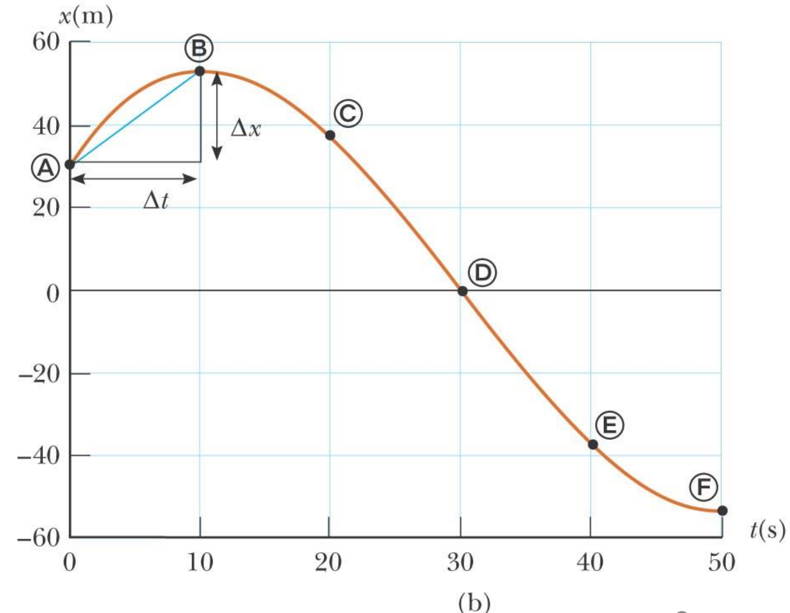
Speed is a scalar quantity

The SI unit of average speed is the same as the unit of average velocity: meters per second (m/s)

The average speed is not (necessarily) the magnitude of the average velocity

Example 2.1

- Find the displacement, average velocity, and average speed of the car in the figure between positions A and F.



Solution

The displacement of the car:

$$\Delta x = x_f - x_i = (-53\text{ m}) - 30\text{ m} = -83\text{ m}$$

The car's average velocity:

$$v_{x,\text{avg}} = \frac{x_f - x_i}{t_f - t_i} = \frac{-53\text{ m} - 30\text{ m}}{50\text{ s} - 0\text{ s}} = -1.7\text{ m/s}$$

The car's average speed:

$$v_{\text{avg}} = \frac{d}{\Delta t} = \frac{(22m + 105m)}{50\text{ s}} = \frac{127\text{ m}}{50\text{ s}} = 2.5\text{ m/s}$$

- Graphical Interpretation of Velocity

- Velocity can be determined from a position-time graph

- The average velocity equals the slope of the line joining the initial and final positions

- a straight line indicates constant velocity

2.2 Instantaneous Velocity and Speed

- We need to know the velocity of a particle at a particular instant in time, rather than the average velocity over a finite time interval.

$$\upsilon_x = \upsilon_{in} = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt} \tag{2.5}$$

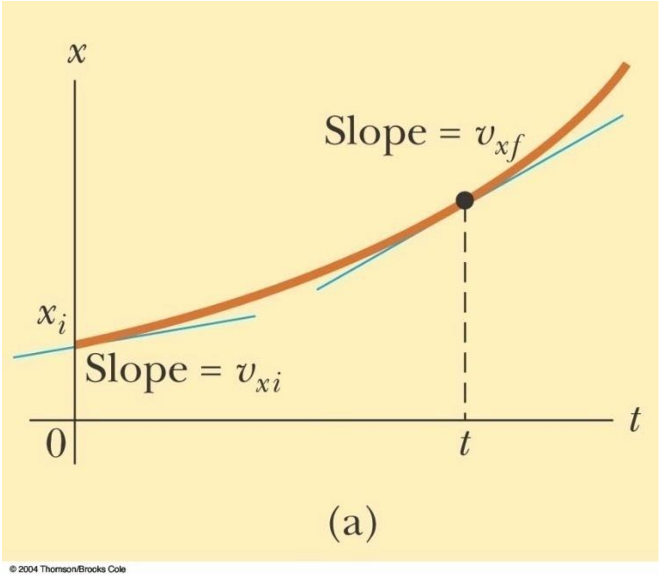
The limit of the average velocity as the time interval becomes infinitesimally short, or as the time interval approaches zero

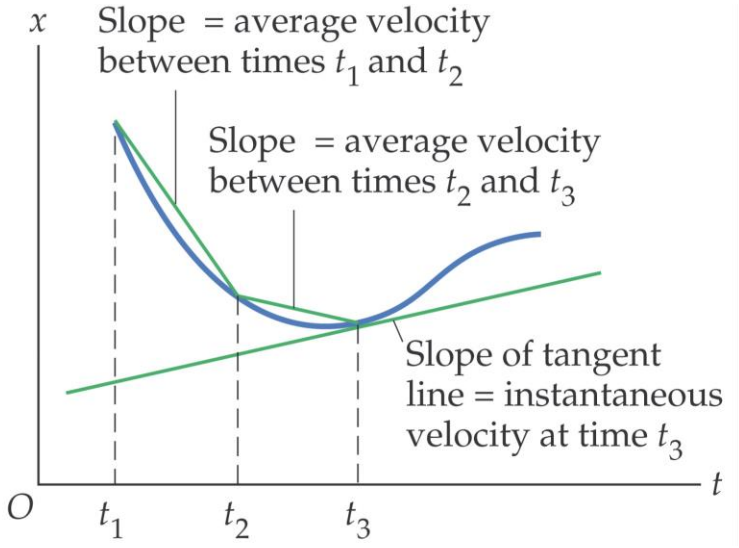
- The instantaneous velocity can be positive, negative, or zero.

Instantaneous Velocity, graph

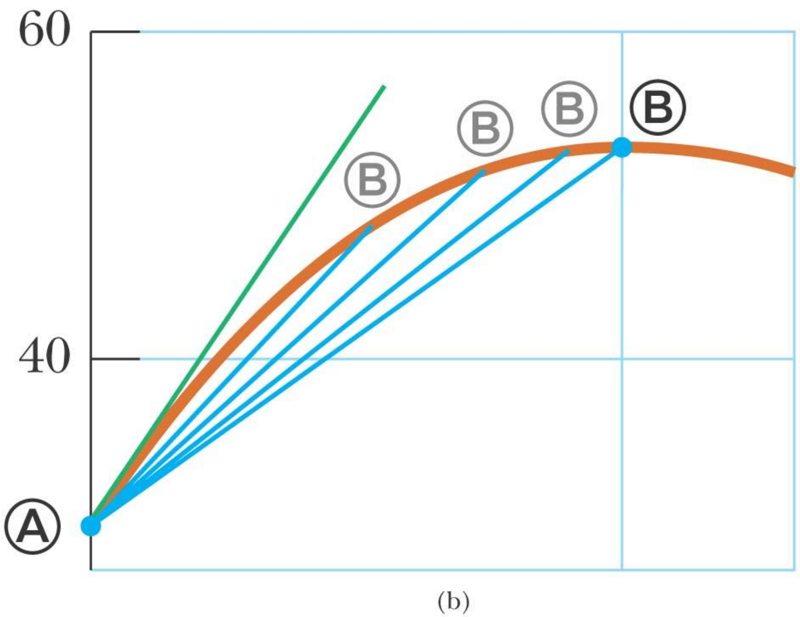
- The slope of the line tangent to the position-vs.-time graph is defined to be the instantaneous velocity at that time

Velocity = instantaneous velocity

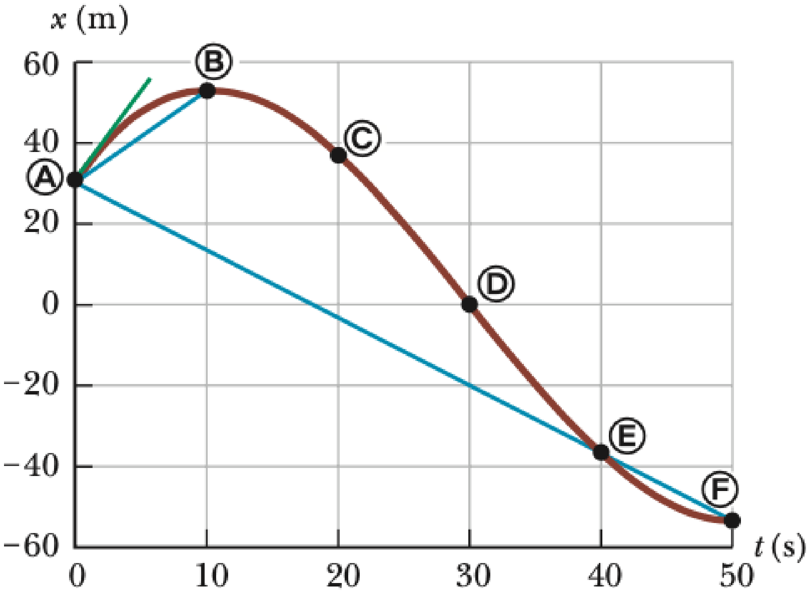




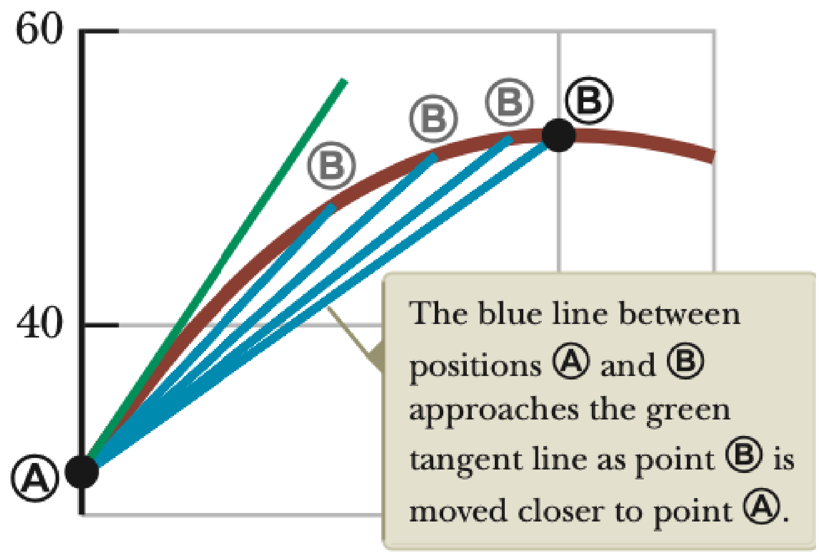
The blue lines show that as Δt gets smaller, they approach the green line



The instantaneous speed of a particle is defined as the magnitude of its instantaneous velocity



x (m)
60
40
20
0
−20
−40
−60
0
10
20
30
40
50
t (s)
A
B
C
D
E
F



60
40
A
B
The blue line between positions A and B approaches the green tangent line as point B is moved closer to point A.

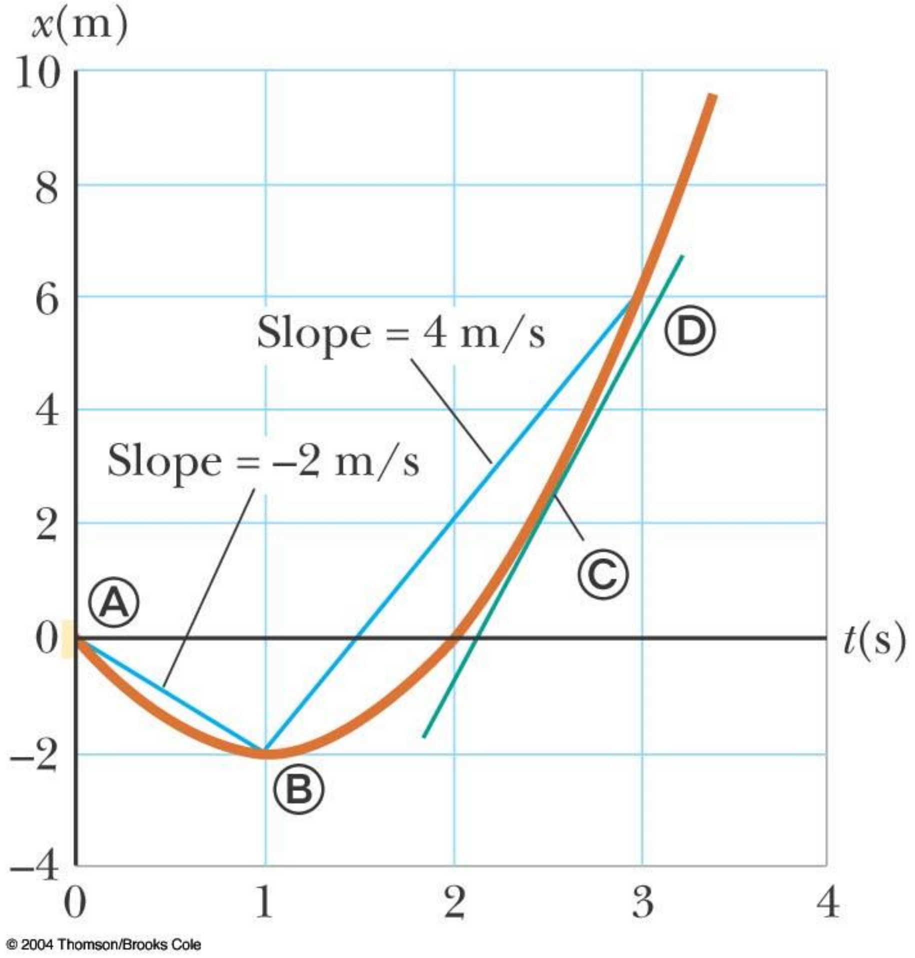
Example 2.3:

A particle moves along the x axis. Its position varies with time according to the expression $x = -4t + 2t^2$ where x is in meters and t is in seconds. The position–time graph for this motion is shown in the following figure.

(A) Determine the displacement of the particle in the time intervals $t = 0$ to $t = 1$ s and $t = 1$ s to $t = 3$ s.

(B) Calculate the average velocity during these two time intervals.

(C) Find the instantaneous velocity of the particle at $t = 2.5$ s



Note that the particle moves in the negative x direction for the first second of motion, is momentarily at rest at the moment t = 1 s, and moves in the positive *x* direction at times *t* > 1 s

Solution

i) In the time intervals $t_i = 0$ to $t_f = 1\ s$:

A) The displacement of the particle:

$$\Delta x_{A \to B} = x_f - x_i = x_B - x_A$$
$$= [-4(1) + 2(1)^2] - [-4(0) + 2(0)^2]$$
$$= \boxed{-2\ \text{m}}$$

B) The average velocity:

$$\bar{v}_{x(A \to B)} = \frac{\Delta x_{A \to B}}{\Delta t} = \frac{-2\ \text{m}}{1\ \text{s}} = \boxed{-2\ \text{m/s}}$$

C)The instantaneous velocity of the particle at *t = 2.5 s*

By measuring the slope of the green line at t = 2.5 s in the Figure, we find that

$$v_x = +6\ \text{m/s}$$

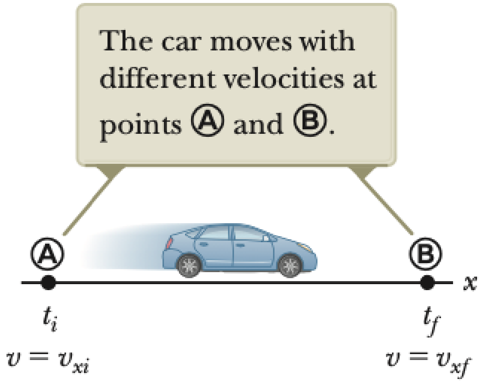
$$v_x = \frac{dx}{dt} = -4 + 4t = -4 + 4(2.5) = +6\ \text{m/s}$$

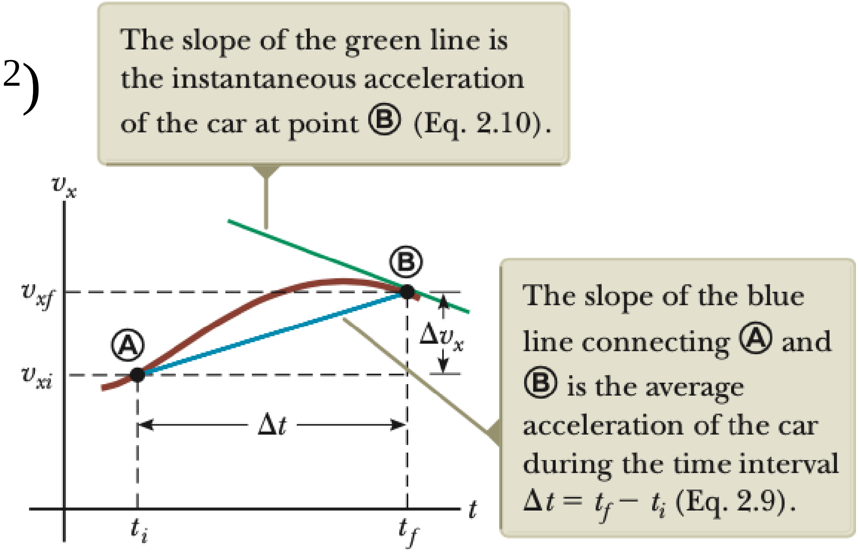
2.3 Acceleration

- The **average acceleration** $a_{x,\text{avg}}$ of the particle is defined as the *change* in velocity Δv_x divided by the time interval Δt during which that change occurs:

$$a_{x,\text{avg}} = \frac{\Delta v_x}{\Delta t} = \frac{v_{xf} - v_{xi}}{t_f - t_i} \qquad \textbf{(2.6)}$$

- The SI unit of acceleration (m/s^2)





Instantaneous Acceleration

is defined as the limit of the average acceleration as Δt approaches zero.

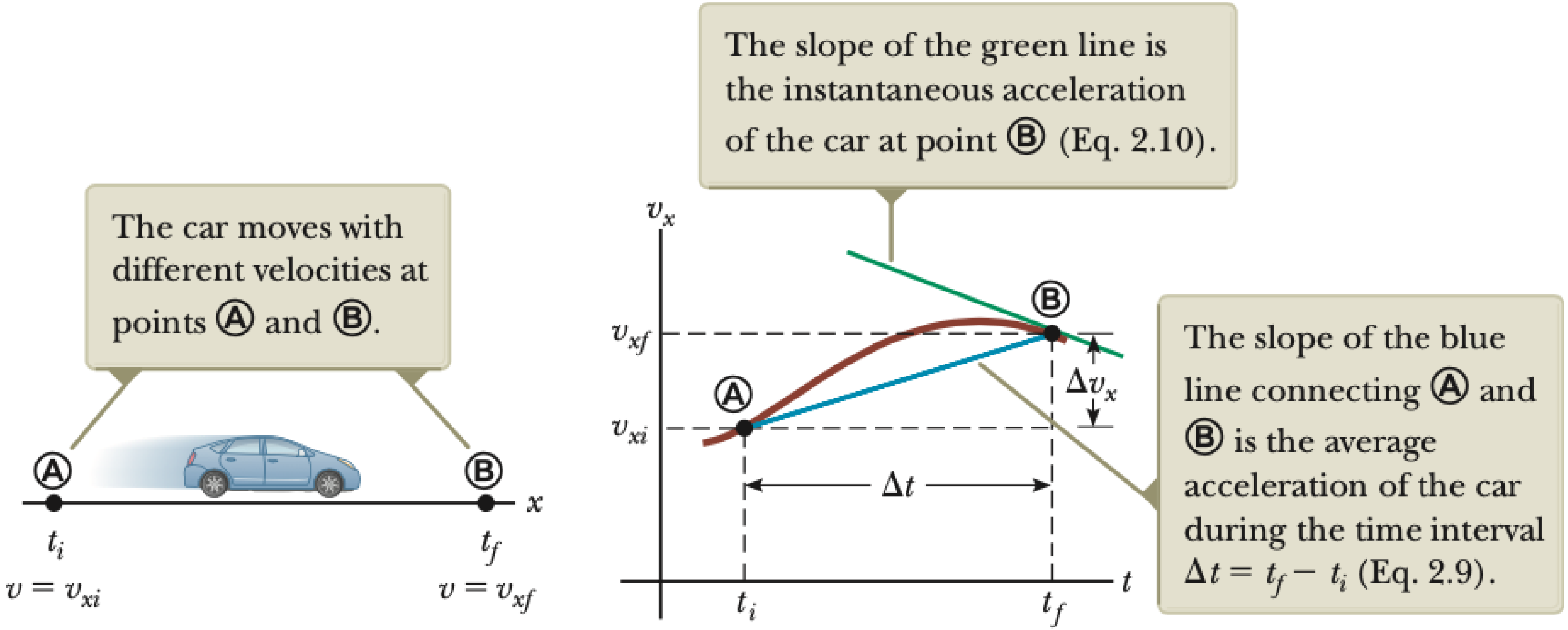
$$a_x \equiv \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t} = \frac{dv_x}{dt} \qquad \textbf{(2.7)}$$

$$\Downarrow$$

$$a_x = \frac{dv_x}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2}$$

That is, in one-dimensional motion, the acceleration equals the *second derivative* of *x* with respect to time.

Instantaneous Acceleration -- graph

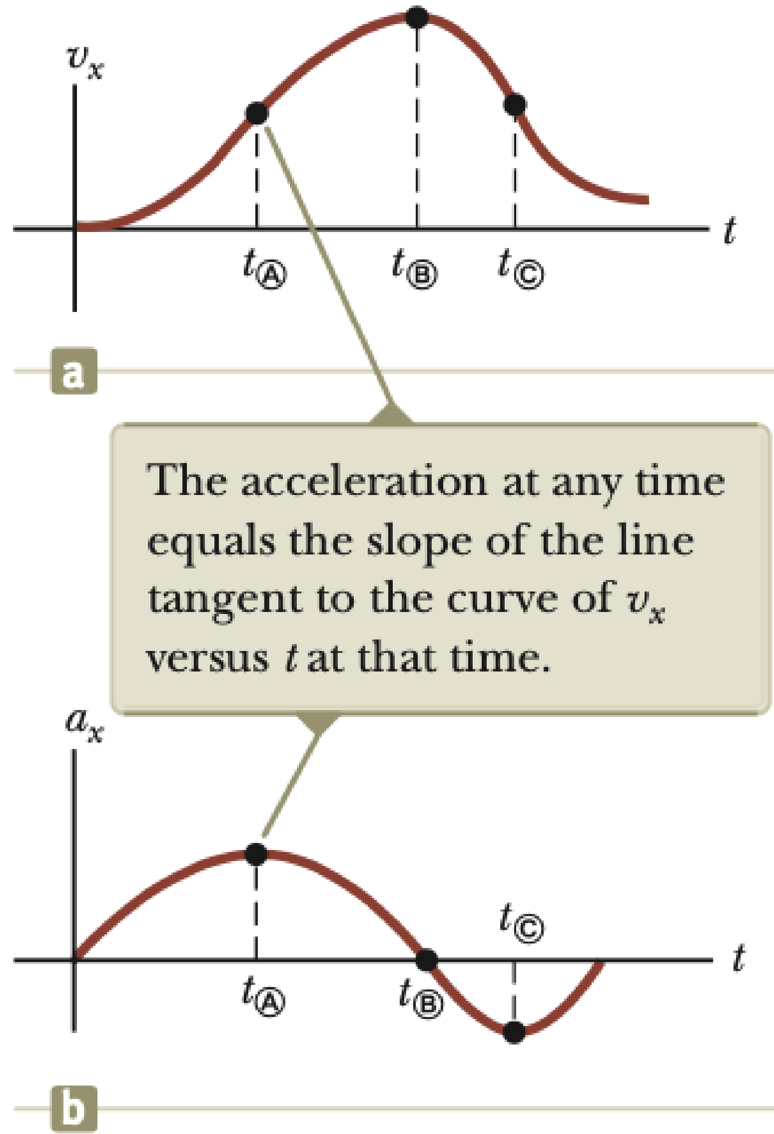


- The slope of the velocity vs. time graph is the acceleration
- The green line represents the instantaneous acceleration
- The blue line is the average acceleration

The acceleration can be positive or negative (vector).

Negative acceleration does not necessarily mean that an object is slowing down.

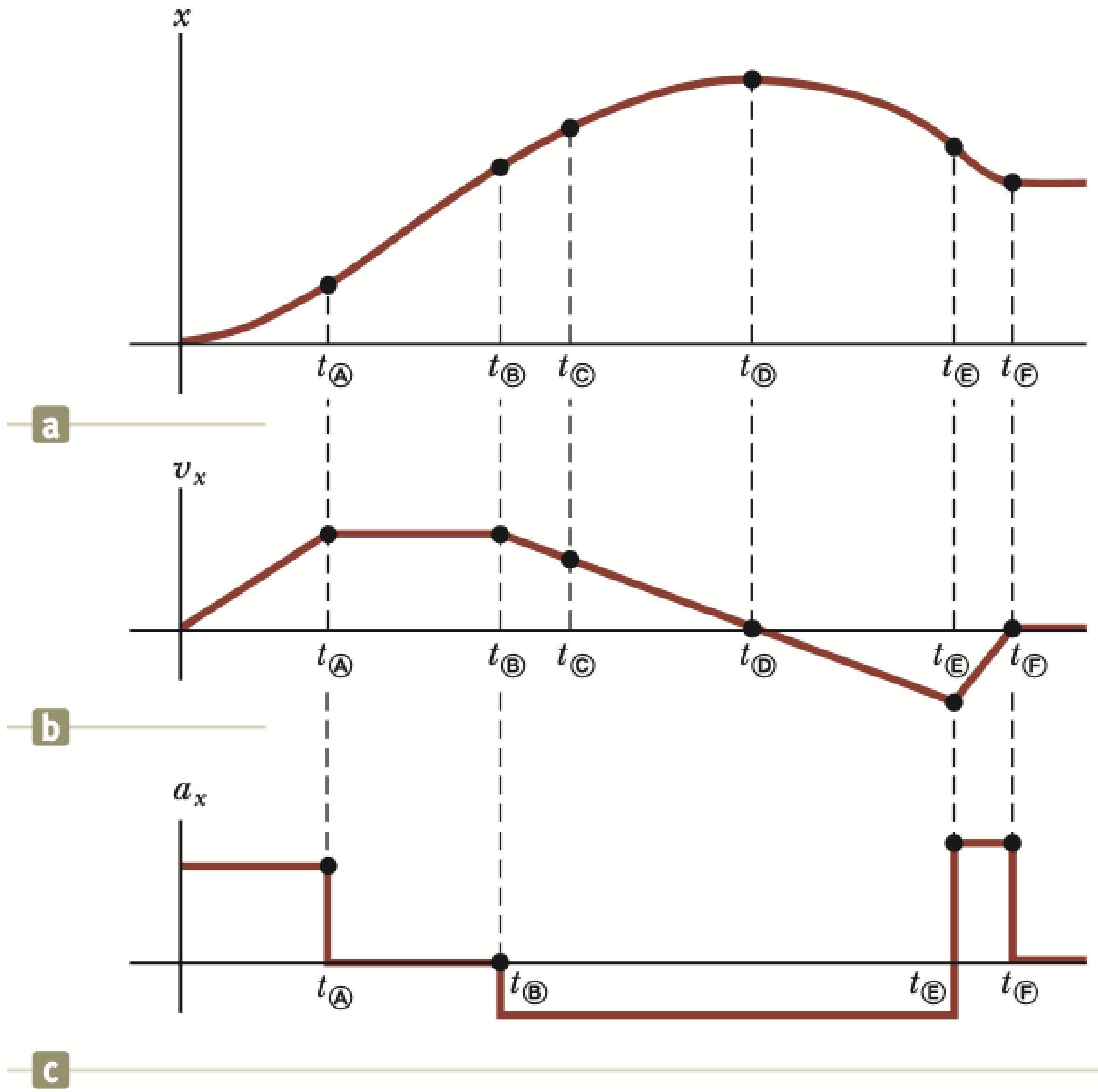
When the object's velocity and acceleration are in the same direction, the object is speeding up. On the other hand, when the object's velocity and acceleration are in opposite directions, the object is slowing down.



Position–time graph for an object moving along the x axis.

The velocity–time graph for the object is obtained by measuring the slope of the position–time graph at each instant.

The acceleration–time graph for the object is obtained by measuring the slope of the velocity–time graph at each instant.

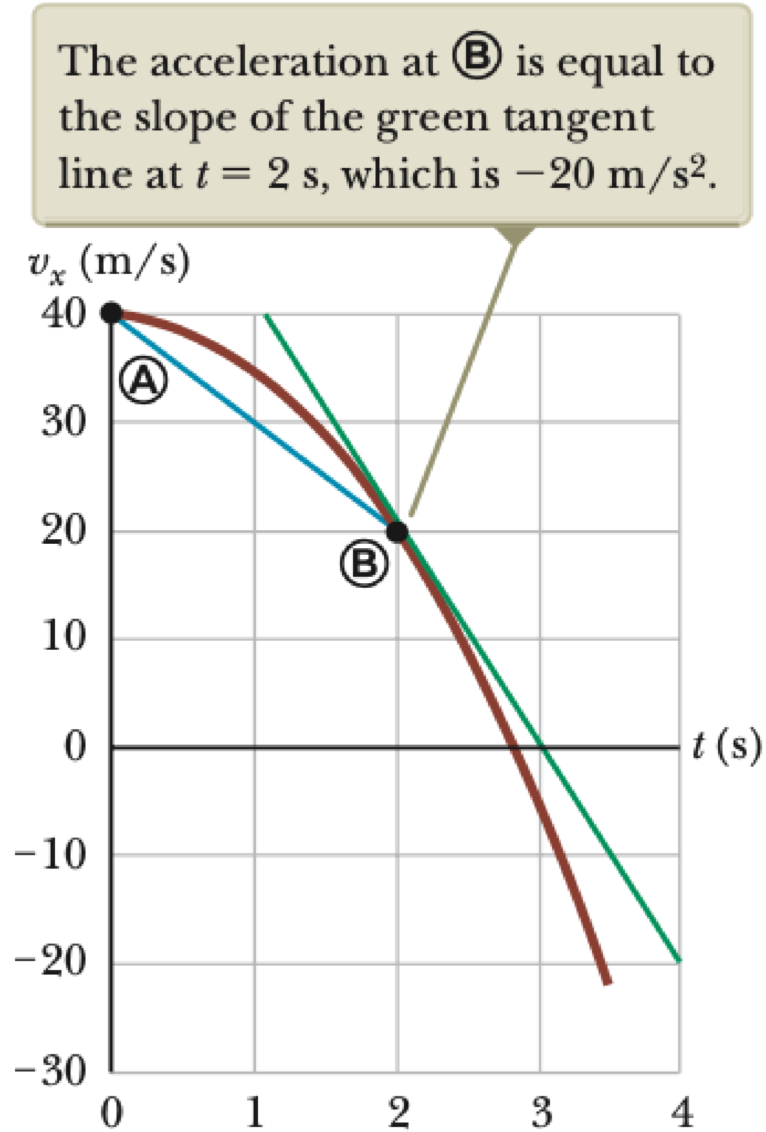


Example: 2.5

The velocity of a particle moving along the *x* axis varies in time according to the expression $\upsilon = (40 - 5t^2)$ m/s, where t is in seconds.

(a) Find the average acceleration in the time interval $t = 0$ *to* $t = 2.0$ *s*.

(b) Determine the acceleration at $t = 2.0$ *s*.


The acceleration at Ⓑ is equal to the slope of the green tangent line at $t = 2$ s, which is $-20\ \mathrm{m/s^2}$.
v_x (m/s)
40
30
20
10
0
−10
−20
−30
Ⓐ
Ⓑ
t (s)
0
1
2
3
4

Solution

(a) Find the velocities using at t =0 and t = 2.0s using υ =40-5t^2

$\upsilon_{t=2.0s}$ =40 - 5(2.0)^2 = +20 m/s

$\upsilon_{t=0s}$ = 40 - 5(0)^2 = +40 m/s

Thee average acceleration in the time interval t =0 *to* t = 2.0 s:

$$a_{x,\text{avg}} = \frac{\upsilon_{xf} - \upsilon_{xi}}{t_f - t_i} = \frac{20 - 40}{2.0 - 0} = -10 \text{ m/s}^2$$

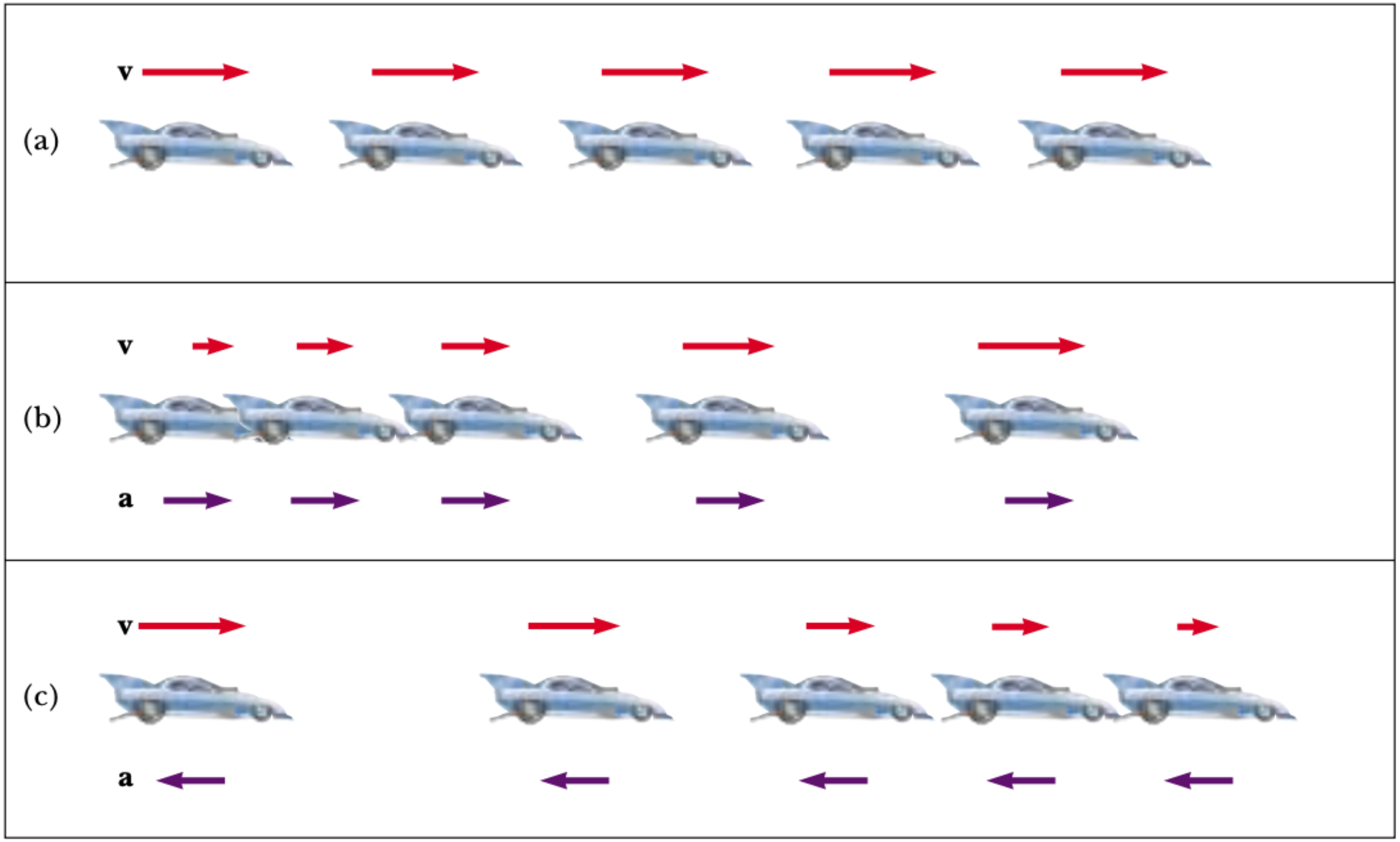
(b) The acceleration at t = 2.0 s:

$$x = At^n$$

$$a_x = \lim_{\Delta t \to 0} \frac{\Delta \upsilon_x}{\Delta t} = \frac{d\upsilon_x}{dt} = -10t = -20 \text{ m/s}^2$$

$$\frac{dx}{dt} = nAt^{n-1}$$

2.4 Motion Diagrams



Active Figure 2.9 (a) Motion diagram for a car moving at constant velocity (zero acceleration). (b) Motion diagram for a car whose constant acceleration is in the direction of its velocity. The velocity vector at each instant is indicated by a red arrow, and the constant acceleration by a violet arrow. (c) Motion diagram for a car whose constant acceleration is in the direction *opposite* the velocity at each instant.

2.5 One-Dimensional Motion with Constant Acceleration

In this lecture, we will derive the kinematic equations for motion of a particle under constant acceleration

$$a_{x,\text{avg}} = \frac{v_{xf} - v_{xi}}{t_f - t_i}$$

If we replace $a_{x,\text{avg}}$ by a_x
and take $t_i = 0$, $t_f = t$, we find that

$$a_x = \frac{v_{xf} - v_{xi}}{t - 0}$$

$$\boxed{v_{xf} = v_{xi} + a_x t} \quad \text{(for constant } a_x \text{)} \qquad \textbf{(2.9)}$$

We can express the average velocity in any time interval of the initial velocity and final velocity as

$$v_{x,\text{avg}} = \frac{v_{xf} + v_{xi}}{2} \qquad \text{(for constant } a_x \text{)} \qquad \textbf{(2.10)}$$

To obtain the position of an object as a function of time

$$\Delta x = x_f - x_i \qquad\qquad \Delta t = t_f - t_i = t - 0 = t$$

$$v_{x,\text{avg}} = \frac{\Delta x}{\Delta t} \quad \Longrightarrow \quad \Delta x = v_{x,\text{avg}}\,\Delta t$$

$$x_f - x_i = v_{x,\text{avg}}\,t = \frac{v_{xf} + v_{xi}}{2}\,t$$

$$\boxed{x_f = x_i + \frac{1}{2}(v_{xf} + v_{xi})t} \quad \text{(for constant } a_x\text{)} \qquad \textbf{(2.11)}$$

This equation provides the final position of the particle at time t in terms of the initial and final velocities.

From Equation 2.9

$$v_{xf} = v_{xi} + a_x t$$

into Equation 2.11:

$$x_f = x_i + \frac{1}{2}(v_{xf} + v_{xi})t$$

$$x_f = x_i + \frac{1}{2}((v_{xi} + a_x t) + v_{xi})t$$

$$x_f = x_i + \frac{1}{2}v_{xi}t + \frac{1}{2}a_x t^2 + \frac{1}{2}v_{xi}t$$

$$\boxed{x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2}$$ (for constant a_x) **(2.12)**

- We can obtain an expression for the final velocity that does not contain time as a variable by substituting the value of t from Equation 2.9

$$v_{xf} = v_{xi} + a_x t \quad \Rightarrow \quad t = \frac{v_{xf} - v_{xi}}{a_x}$$

into Equation 2.11:

$$x_f = x_i + \frac{1}{2}(v_{xf} + v_{xi})t \qquad x_f = x_i + \frac{1}{2}(v_{xf} + v_{xi})\left(\frac{v_{xf} - v_{xi}}{a_x}\right)$$

$$x_f = x_i + \frac{1}{2}\left(\frac{{v_{xf}}^2 - {v_{xi}}^2}{a_x}\right)$$

$${v_{xf}}^2 = {v_{xi}}^2 + 2a_x(x_f - x_i) \quad \text{(for constant } a_x \text{)} \qquad \textbf{(2.13)}$$

kinematic equations for motion of a particle under a constant acceleration

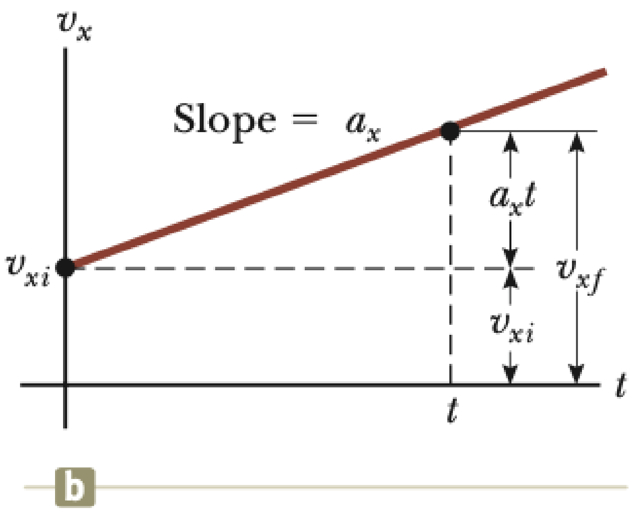
Kinematic Equations for Motion of a Particle Under Constant Acceleration	
Equation	**Information Given by Equation**
$v_{xf} = v_{xi} + a_x t$	Velocity as a function of time
$x_f = x_i + \frac{1}{2}(v_{xi} + v_{xf})t$	Position as a function of velocity and time
$x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2$	Position as a function of time
$v_{xf}^2 = v_{xi}^2 + 2a_x(x_f - x_i)$	Velocity as a function of position

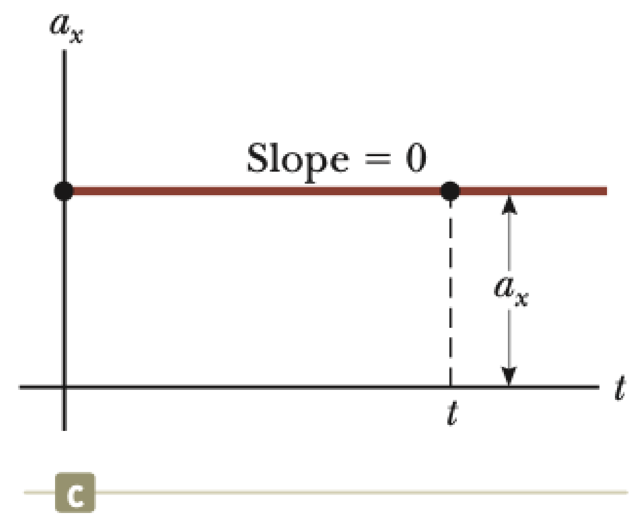
Graphical Look at Motion – velocity – time curve

- The slope gives the acceleration

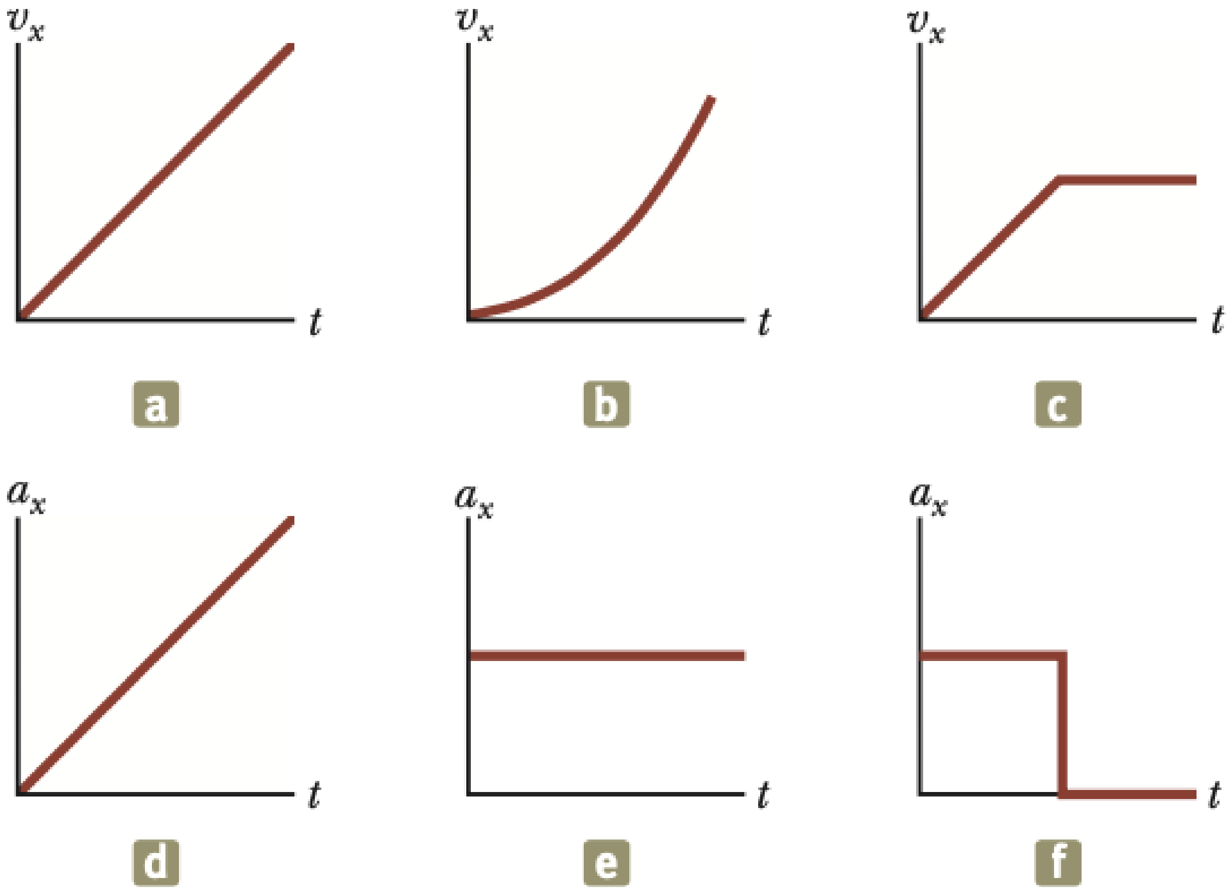
$$v_{xf} = v_{xi} + a_x t$$

- The straight line indicates a constant acceleration





Quick Quiz 2.5 In Figure 2.11, match each v_x-t graph on the left with the a_x-t graph on the right that best describes the motion.



Example 2.7:

A jet lands on an aircraft carrier at 140 mi/h (~ 63 m/s).

(A) What is its acceleration (assumed constant) if it stops in 2.0 s due to an arresting cable that snags the airplane and brings it to a stop?

(B) If the jet touches down at position $x_i=0$, what is the final position of the plane?

Solution

(A) The acceleration

$$a_x = \frac{v_{xf} - v_{xi}}{t} \approx \frac{0 - 63\ \text{m/s}}{2.0\ \text{s}}$$

$$= \boxed{-32\ \text{m/s}^2}$$

(B) The final position of the jet is

$$x_f = x_i + \tfrac{1}{2}(v_{xi} + v_{xf})t = 0 + \tfrac{1}{2}(63\ \text{m/s} + 0)(2.0\ \text{s}) = \boxed{63\ \text{m}}$$

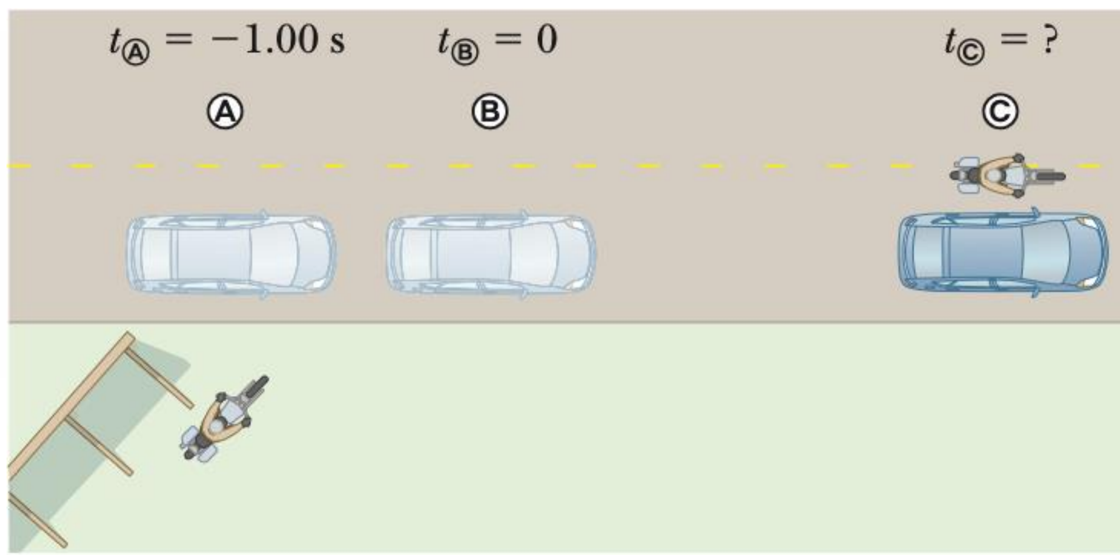
Example 2.8:

A car traveling at a constant speed of 45.0 m/s passes a trooper hidden behind a billboard. One second after the speeding car passes the billboard, the trooper sets out from the billboard to catch it, accelerating at a constant rate of 3.00 m/s^2. How long does it take her to overtake the car?

$$v_{x\,\text{car}} = 45.0 \text{ m/s}$$

$$a_{x\,\text{car}} = 0$$

$$a_{x\,\text{trooper}} = 3.00 \text{ m/s}^2$$



Solution

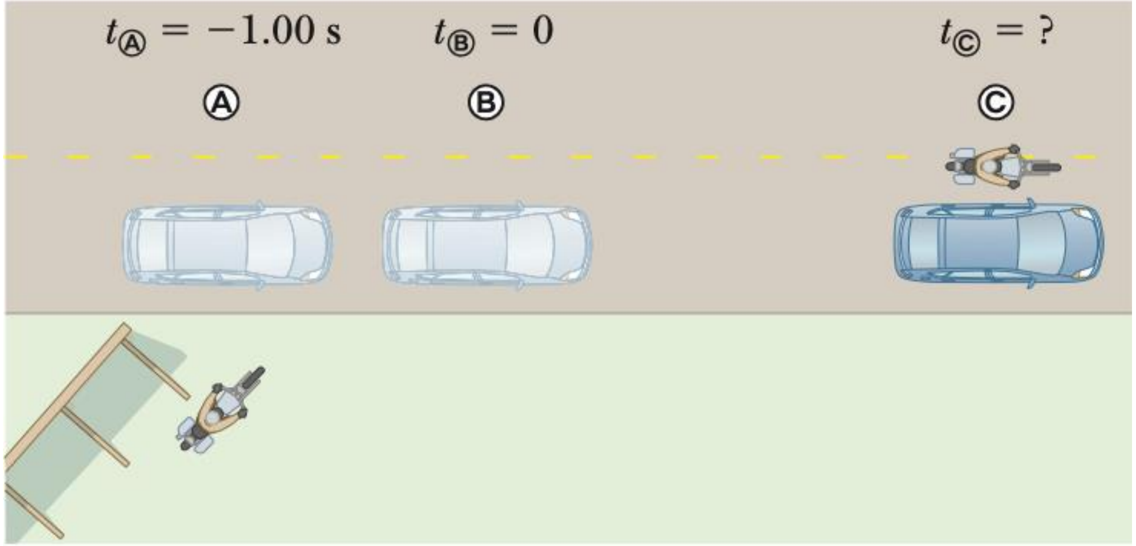
First, we write expressions for the position of each vehicle as a function of time.

Using the particle under constant velocity model.

At point (B), $t = 0$, $x_{(B)} = 45$ m

$$x_{\text{car}} = x_{(B)} + v_{x\,\text{car}}t$$

$$x_{\text{car}} = 45.0 \text{ m}$$



The trooper starts from rest at $t_{(B)} = 0$ and accelerates at $a_x = 3.00\ \text{m/s}^2$ away from the origin.

$$x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2$$

$$x_{\text{trooper}} = 0 + (0)t + \frac{1}{2}a_x t^2$$

At position (c):

$$x_{\text{car}} = x_{\text{trooper}}$$

$$x_{(B)} + v_{x\ \text{car}}t = \frac{1}{2}a_x t^2$$

$$\frac{1}{2}a_x t^2 - v_{x\ \text{car}}t - x_{(B)} = 0$$

$$v_{xf} = v_{xi} + a_x t$$

$$v_{x,\text{avg}} = \frac{v_{xi} + v_{xf}}{2}$$

$$x_f = x_i + \tfrac{1}{2}(v_{xi} + v_{xf})t$$

$$x_f = x_i + v_{xi}t + \tfrac{1}{2}a_x t^2$$

$$v_{xf}^2 = v_{xi}^2 + 2a_x(x_f - x_i)$$

Solve the quadratic equation for the time at which the trooper catches the car

$$t = \frac{v_{x\,\text{car}} \pm \sqrt{v_{x\,\text{car}}^2 + 2a_x x_{Ⓑ}}}{a_x}$$

$$t = \frac{v_{x\,\text{car}}}{a_x} \pm \sqrt{\frac{v_{x\,\text{car}}^2}{a_x^{\,2}} + \frac{2x_{Ⓑ}}{a_x}}$$

$$t = \frac{45.0\ \text{m/s}}{3.00\ \text{m/s}^2} + \sqrt{\frac{(45.0\ \text{m/s})^2}{(3.00\ \text{m/s}^2)^2} + \frac{2(45.0\ \text{m})}{3.00\ \text{m/s}^2}} = \boxed{31.0\ \text{s}}$$

2.6 Freely Falling Objects

- **A freely falling object is any object moving freely under the influence of gravity alone, regardless of its initial motion.**
- **Any freely falling object experiences an acceleration directed downward, regardless of its initial motion** (upward or downward or released from rest).
- In the absence of air resistance, all objects dropped near the Earth's surface fall toward the Earth with the same constant acceleration (g) under the influence of the Earth's gravity
- At the Earth's surface, the value of g is approximately 9.80 m/s^2 (decreases with increasing altitude)
- For freely falling objects, the motion is in the vertical direction and the acceleration is downward

$a_y = -g = -9.80\ \mathrm{m/s^2}$.

2.7 The Sign of *g*

Keep in mind that g is a *positive number*—it is tempting to substitute $-9.80\ \mathrm{m/s^2}$ for g, but resist the temptation. Downward gravitational acceleration is indicated explicitly by stating the acceleration as $a_y = -g$.

2.8 Acceleration at the Top of The Motion

It is a common misconception that the acceleration of a projectile at the top of its trajectory is zero. While the velocity at the top of the motion of an object thrown upward momentarily goes to zero, *the acceleration is still that due to gravity* at this point. If the velocity and acceleration were both zero, the projectile would stay at the top!

Quick Quiz 2.6 A ball is thrown upward. While the ball is in free fall, does its acceleration (a) increase (b) decrease (c) increase and then decrease (d) decrease and then increase (e) remain constant?

Quick Quiz 2.7 After a ball is thrown upward and is in the air, its speed (a) increases (b) decreases (c) increases and then decreases (d) decreases and then increases (e) remains the same.

Equation
$v_{xf} = v_{xi} + a_x t$
$x_f = x_i + \frac{1}{2}(v_{xi} + v_{xf})\,t$
$x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2$
$v_{xf}{}^2 = v_{xi}{}^2 + 2a_x(x_f - x_i)$

The kinematic equations in y direction:

$$v_{yf} = v_{yi} + a_y t \tag{1}$$

$$y_f = y_i + \frac{1}{2}(v_{yf} + v_{yi})t \tag{2}$$

$$y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2 \tag{3}$$

$$v_{yf}{}^2 = v_{yi}{}^2 + 2a_y(y_f - y_i) \tag{4}$$

Equation
$v_{xf} = v_{xi} + a_x t$
$x_f = x_i + \frac{1}{2}(v_{xi} + v_{xf})t$
$x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2$
${v_{xf}}^2 = {v_{xi}}^2 + 2a_x(x_f - x_i)$

The kinematic equations for free falling object:

$$v_{yf} = v_{yi} - gt \quad (1)$$

$$y_f = y_i + \frac{1}{2}(v_{yf} + v_{yi})t \quad (2)$$

$$y_f = y_i + v_{yi}t - \frac{1}{2}gt^2 \quad (3)$$

$${v_{yf}}^2 = {v_{yi}}^2 - 2g(y_f - y_i) \quad (4)$$

Note that:

When an object is dropped

Initial velocity is zero

Let up be positive

Use the kinematic equations

Acceleration is $a = -\,g = -9.80\ \text{m/s}^2$

When an object is thrown downward

$a = g = -9.80 \text{m/s}^2$

Initial velocity $\neq 0$

Initial velocity will be negative

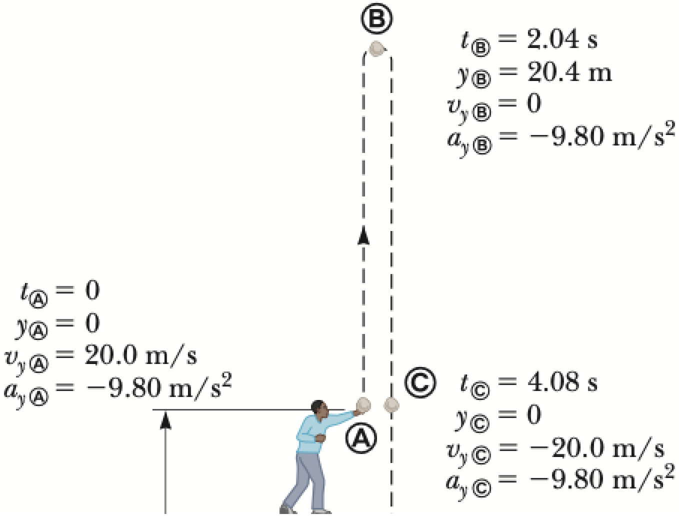
If an object is thrown upward

Initial velocity is upward (positive)

The instantaneous velocity at the maximum height is zero

The velocity will change sign after the object reaches its highest point

The acceleration $a = g = -9.80$ m/s^2 everywhere in the motion

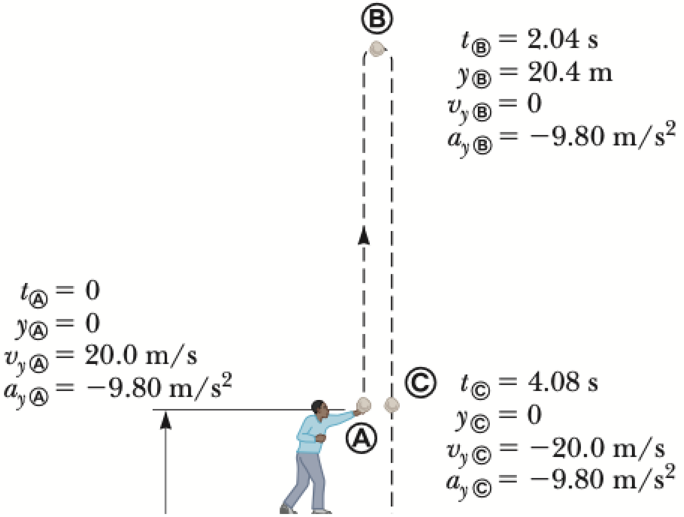


An object is thrown upward (2)

The motion may be symmetrical

$$t_{up} = t_{down}$$

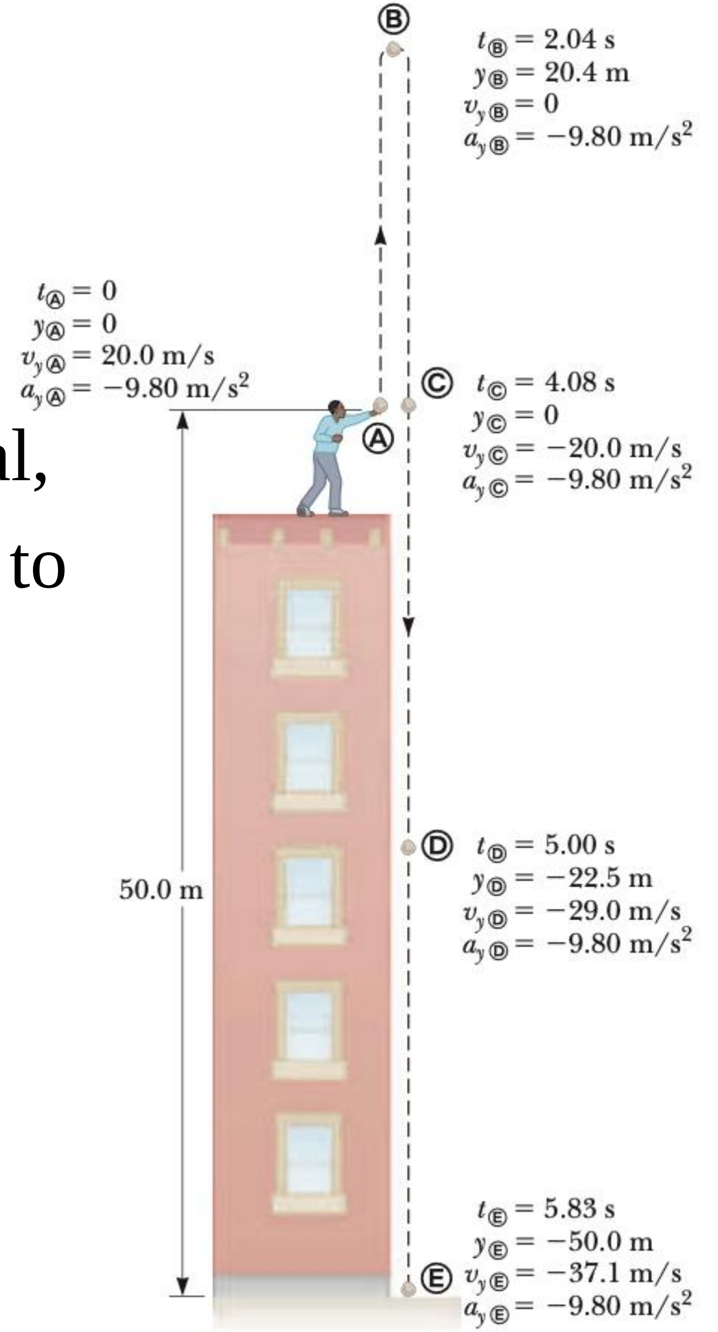
$$\upsilon = -\upsilon_i \text{ at point c}$$



An object is thrown upward (3)

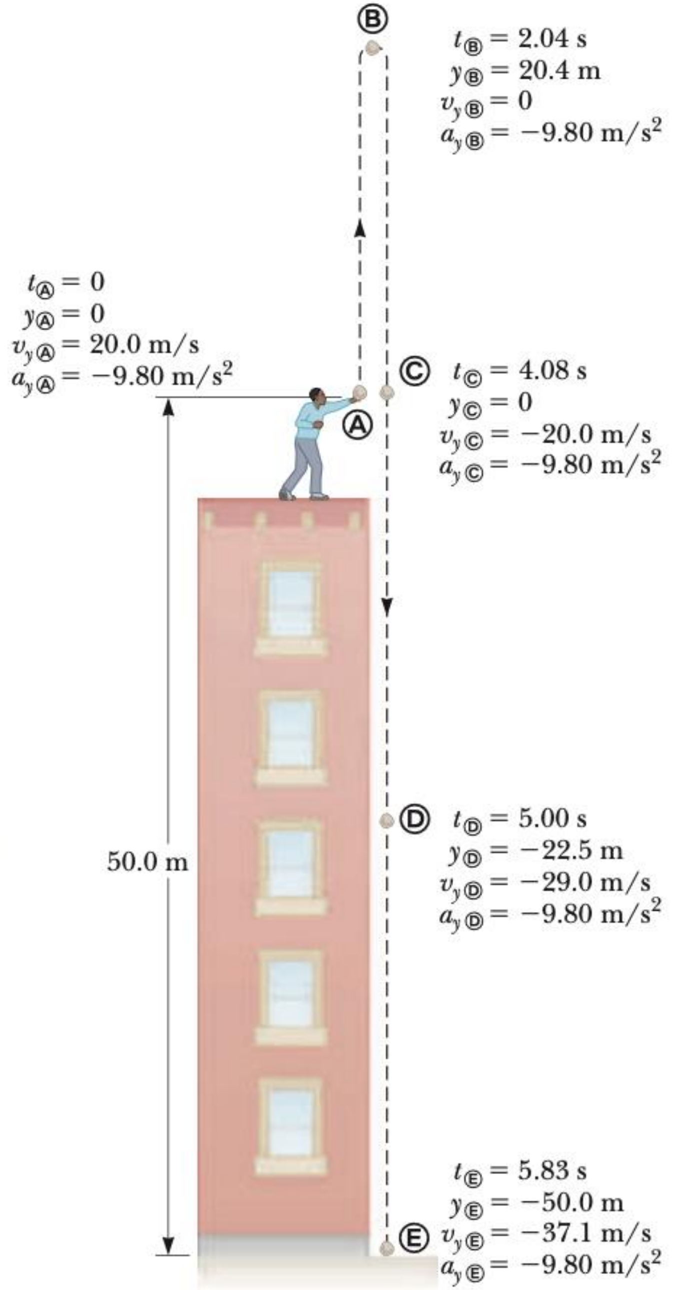
The motion may not be symmetrical,

break the motion into various parts to simplify the problem



Example 2.12 Not a Bad Throw for a Rookie!

A stone thrown from the top of a building is given an initial velocity of 20.0 m/s straight upward. The building is 50.0 m high, and the stone just misses the edge of the roof on its way down, as shown in Figure 2.14. Using $t_A = 0$ as the time the stone leaves the thrower's hand at position Ⓐ, determine **(A)** the time at which the stone reaches its maximum height, **(B)** the maximum height, **(C)** the time at which the stone returns to the height from which it was thrown, **(D)** the velocity of the stone at this instant, and **(E)** the velocity and position of the stone at $t = 5.00$ s.



Solution

(A)

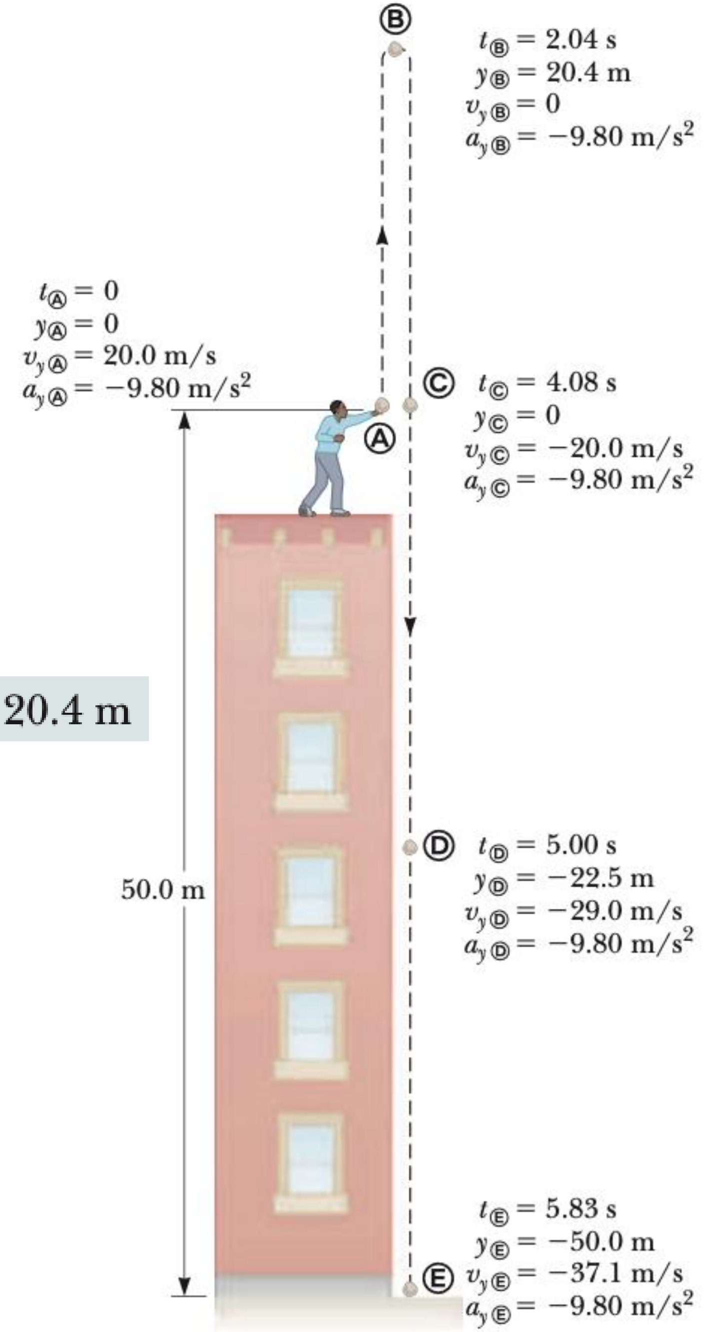
$$v_{yf} = v_{yi} + a_y t \quad \rightarrow \quad t = \frac{v_{yf} - v_{yi}}{a_y}$$

$$t = t_{Ⓑ} = \frac{0 - 20.0\text{ m/s}}{-9.80\text{ m/s}^2} = \boxed{2.04\text{ s}}$$

(B) the maximum height

$$y_{\max} = y_{Ⓑ} = y_{Ⓐ} + v_{xⒶ}t + \tfrac{1}{2}a_y t^2$$

$$y_{Ⓑ} = 0 + (20.0\ \mathrm{m/s})(2.04\ \mathrm{s}) + \tfrac{1}{2}(-9.80\ \mathrm{m/s^2})(2.04\ \mathrm{s})^2 = \boxed{20.4\ \mathrm{m}}$$



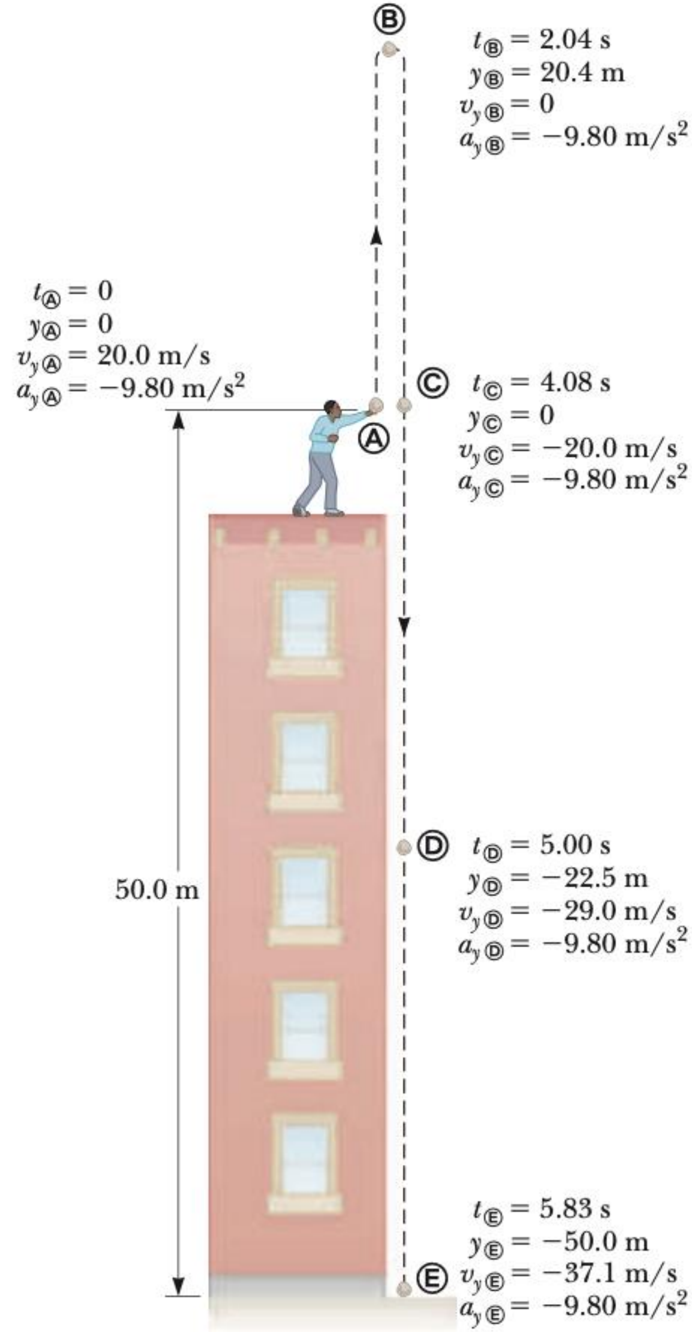
(C) the time at which the stone returns to the height from which it was thrown

$$y_C = y_A + v_{yA}t + \tfrac{1}{2}a_y t^2$$
$$0 = 0 + 20.0t - 4.90t^2$$

This is a quadratic equation and so has two solutions for $t = t_C$. The equation can be factored to give

$$t(20.0 - 4.90t) = 0$$

One solution is $t = 0$, corresponding to the time the stone starts its motion. The other solution is $t = 4.08$ s, which is the solution we are after. Notice that it is double the value we calculated for t_B.



(D) the velocity of the stone at this instant (C)

$$v_{yC} = v_{yA} + a_y t = 20.0\ \text{m/s} + (-9.80\ \text{m/s}^2)(4.08\ \text{s})$$

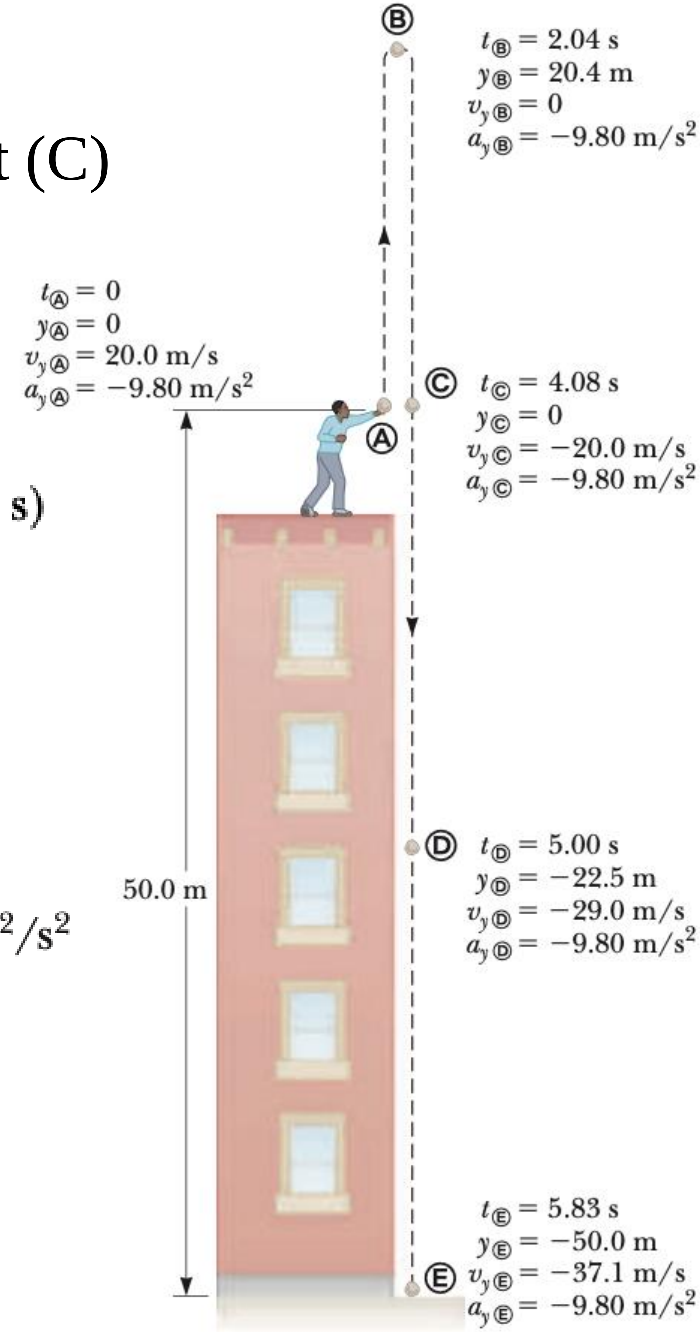
$$= \boxed{-20.0\ \text{m/s}}$$

or

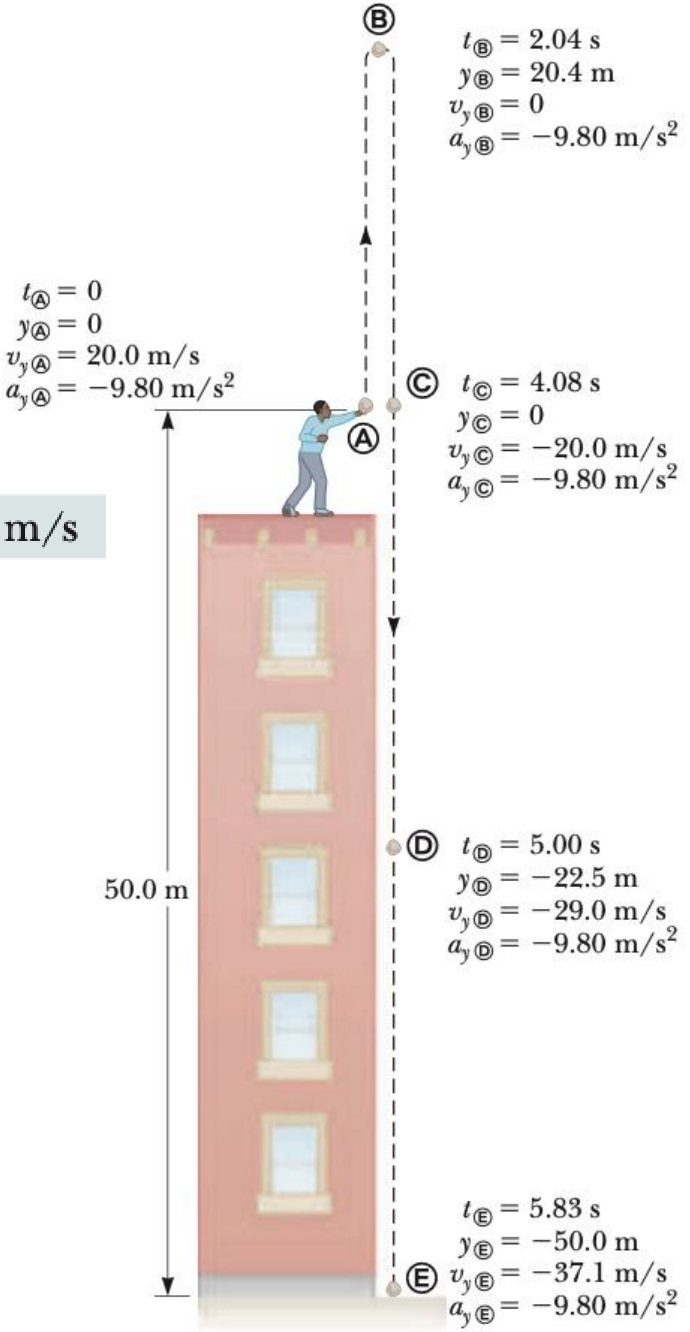
$$v_{y©}{}^2 = v_{yⒶ}{}^2 + 2a_y(y_© - y_Ⓐ)$$

$$v_{y©}{}^2 = (20.0\ \text{m/s})^2 + 2(-9.80\ \text{m/s}^2)(0 - 0) = 400\ \text{m}^2/\text{s}^2$$

$$v_{y©} = \boxed{-20.0\ \text{m/s}}$$



(E) the velocity and position of the stone at t = 5.00 s.



$$v_{yⒹ} = v_{yⒶ} + a_y t = 20.0\text{ m/s} + (-9.80\text{ m/s}^2)(5.00\text{ s}) = \boxed{-29.0\text{ m/s}}$$

$$\begin{aligned} y_Ⓓ &= y_Ⓐ + v_{yⒶ} t + \tfrac{1}{2} a_y t^2 \\ &= 0 + (20.0\text{ m/s})(5.00\text{ s}) + \tfrac{1}{2}(-9.80\text{ m/s}^2)(5.00\text{ s})^2 \\ &= \boxed{-22.5\text{ m}} \end{aligned}$$

Exercises

Problems: 4, 5, 11, 15, 16, 20, 21, 22, 23, 25, 27, 28, 29, 32, 33, 40, 42, 43, 46, 48, 51, 52