Chapter 5: The Laws of Motion

The Laws of Motion

In Chapters 2 and 4, we have studied motion of particles without considering what causes or influences that motion.

The two main factors we need to consider are

-the *forces* acting on an object

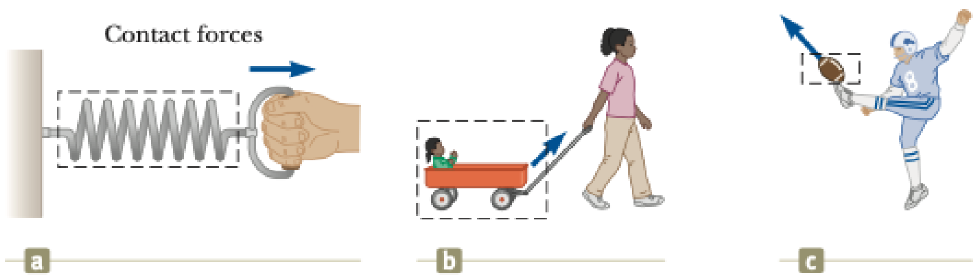
-the *mass* of the object.

This chapter discusses what causes the motion (Dynamics): The three basic laws of motion, were formulated more than three centuries ago by Isaac Newton.

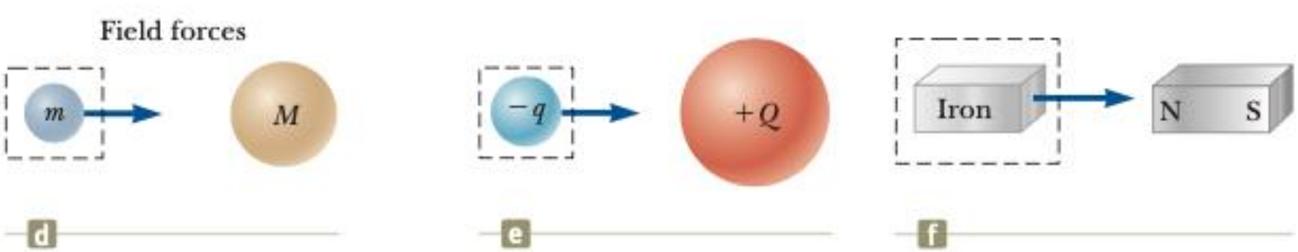
5.1 The Concept of Force

Forces can be either

1. *contact forces* involving physical contact between two objects (e.g. when you pull a spring)



2. *field forces* which do not involve physical contact (e.g. electric forces between charged objects)



The *fundamental* forces in nature are all field forces:

(1) *gravitational forces* between objects,

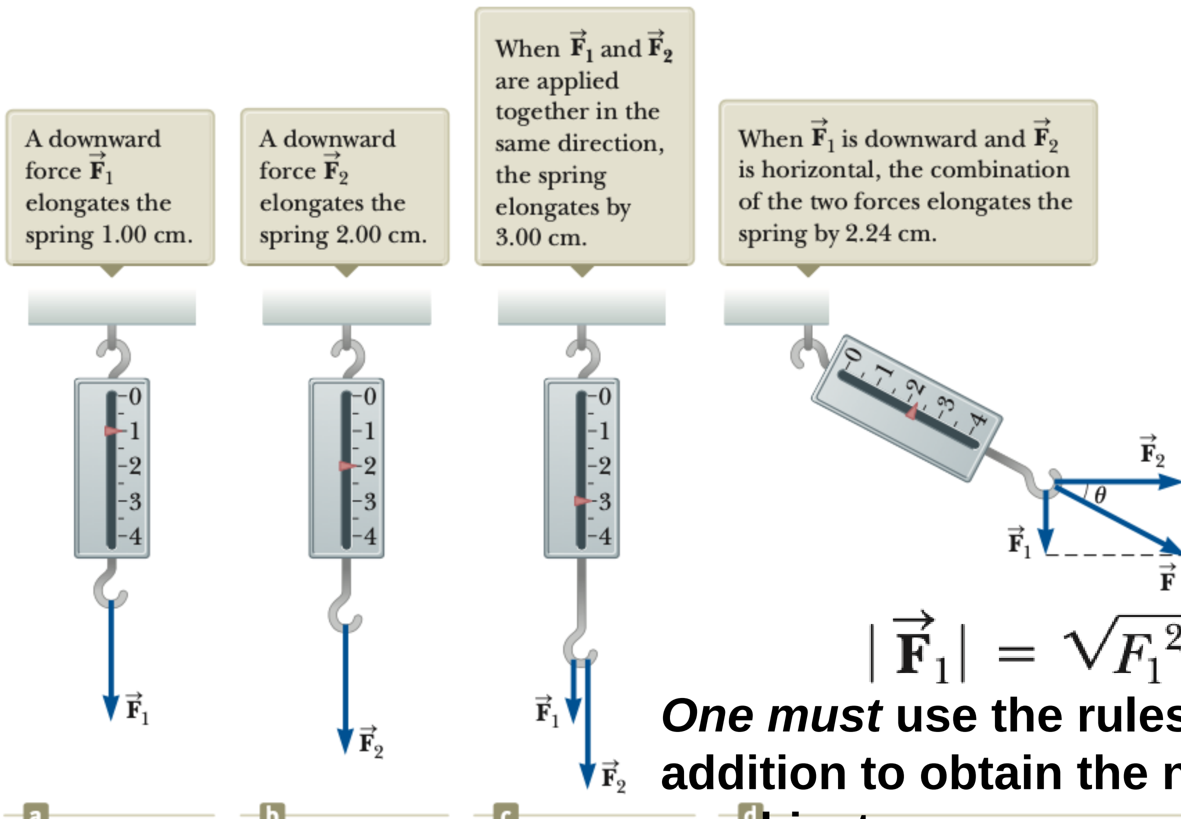
(2) *electromagnetic forces* between electric charges,
(3) *strong forces* between subatomic particles,

(4) *weak forces* that arise in certain radioactive decay processes.

In classical physics, we are concerned only with gravitational and electromagnetic forces.

The Vector Nature of Force

- We can use the deformation of a spring to measure force. A spring scale that has a fixed upper end.



$$|\vec{\mathbf{F}}_1| = \sqrt{F_1^{\ 2} + F_2^{\ 2}}$$

One must **use the rules of vector addition to obtain the net force on an object.**

5.2 Newton's First Law and Inertial Frames

Newton's first law of motion (the *law of inertia*):

If an object does not interact with other objects, it is possible to identify a reference frame in which the object has zero acceleration.

Such a reference frame is called an **inertial frame of reference.**

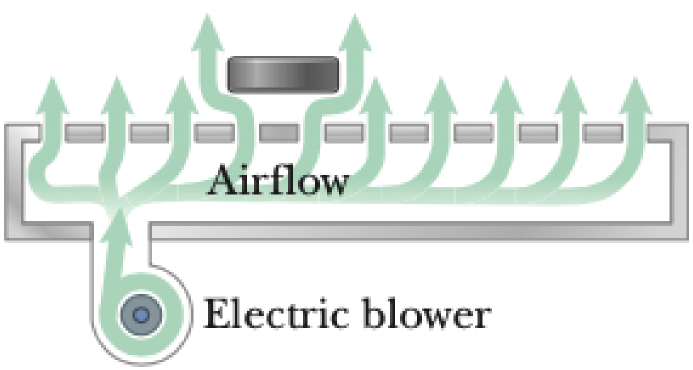


Figure : On an air hockey table, air blown through holes in the surface allows the puck to move almost without friction. If the table is not accelerating, a puck placed on the table will remain at rest.

Inertial frame of reference

- Any reference frame that moves with constant velocity relative to an inertial frame is itself an inertial frame
- A reference frame that moves with constant velocity relative to the distant stars is the best approximation of an inertial frame
- We model the Earth as an inertial frame, along with any other frame attached to it.

  -Centripetal accelerations of the Earth due to its orbital and rotational motion are small and can often be neglected.

Another statement of Newton's first law

In the absence of external forces and when viewed from an inertial reference frame, an object at rest remains at rest and an object in motion continues in motion with a constant velocity (that is, with a constant speed in a straight line).

That means when no force acts on an object, the acceleration of the object is zero.

The tendency of an object to resist any attempt to change its velocity is called **inertia.**

Definition of force: that which causes a change in motion of an object.

$$\Sigma F = 0 \,, \qquad a = 0$$

5.3 Mass

- **Mass** is an inherent property of an object that specifies how much resistance an object exhibits to changes in its velocity (*inertia).*
- The greater the mass of an object, the less that object accelerates under the action of a given applied force.

$$\frac{m_1}{m_2} = \frac{a_2}{a_1}$$

- Mass is a scalar quantity
- The SI unit of mass is *the kilogram* (kg).

Weight

- **The weight** of an object is equal to the magnitude of the gravitational force (vector) exerted on the object
  - Weight can varies with location

- The SI unit of force is the newton (N), 1 N = 1 $kg.m/s^2$
- Mass and weight are different quantities

5.4 Newton's Second Law

Newton's second law answers the question of what happens to an object when one or more forces act on it.

When viewed from an inertial reference frame, the acceleration of an object is directly proportional to the net force acting on it and inversely proportional to its mass:

$$\vec{\mathbf{a}} \propto \frac{\sum \vec{\mathbf{F}}}{m}$$

$$\boxed{\Sigma\vec{F} = m\vec{a}}$$

Newton's second law

The **net force** on an object is the vector sum of all forces acting on the object.

The **net force** = the *total force* = the *resultant force* = the *unbalanced force*

Many forces may be acting on an object, but there is only one acceleration.

Newton's second law: component form

$$\Sigma F_x = m\, a_x$$

$$\Sigma F_y = m\, a_y$$

$$\Sigma F_z = m\, a_z$$

Units of Force

The SI unit of force is the newton (N),

$1\ N = 1\ kg.m/s^2$

A force of 1 N is the force that, when acting on an object of mass 1 kg, produces an acceleration of 1 m/s^2

Table 5.1

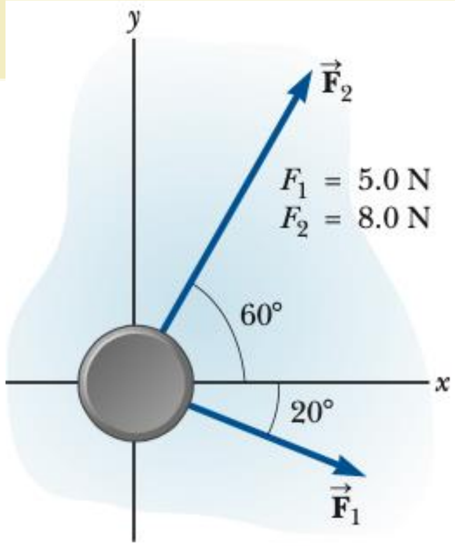
Units of Mass, Acceleration, and Force[a]			
System of Units	**Mass**	**Acceleration**	**Force**
SI	kg	m/s^2	$N = kg \cdot m/s^2$
U.S. customary	slug	ft/s^2	$lb = slug \cdot ft/s^2$

[a] 1 N = 0.225 lb.

Example 5.1 An Accelerating Hockey Puck

AM

A hockey puck having a mass of 0.30 kg slides on the frictionless, horizontal surface of an ice rink. Two hockey sticks strike the puck simultaneously, exerting the forces on the puck shown in Figure 5.4. The force $\vec{\mathbf{F}}_1$ has a magnitude of 5.0 N, and is directed at $\theta = 20°$ below the x axis. The force $\vec{\mathbf{F}}_2$ has a magnitude of 8.0 N and its direction is $\phi = 60°$ above the x axis. Determine both the magnitude and the direction of the puck's acceleration.

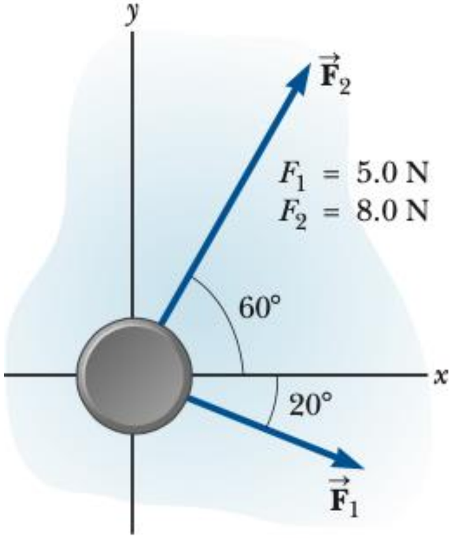


Solution

Find the component of the net force acting on the puck in the *x* direction:

$$\sum F_x = F_{1x} + F_{2x} = F_1 \cos\theta + F_2 \cos\phi$$

Find the component of the net force acting on the puck in the *y* direction:



$$\sum F_y = F_{1y} + F_{2y} = F_1 \sin\theta + F_2 \sin\phi$$

Use Newton’s second law in component form to find the *x* and *y* components of the puck’s acceleration:

$$a_x = \frac{\sum F_x}{m} = \frac{F_1 \cos\theta + F_2 \cos\phi}{m}$$

$$a_y = \frac{\sum F_y}{m} = \frac{F_1 \sin\theta + F_2 \sin\phi}{m}$$

Substitute numerical values:

$$a_x = \frac{(5.0\ \mathrm{N})\cos(-20^\circ) + (8.0\ \mathrm{N})\cos(60^\circ)}{0.30\ \mathrm{kg}} = 29\ \mathrm{m/s^2}$$

$$a_y = \frac{(5.0\ \mathrm{N})\sin(-20^\circ) + (8.0\ \mathrm{N})\sin(60^\circ)}{0.30\ \mathrm{kg}} = 17\ \mathrm{m/s^2}$$

Find the magnitude of the acceleration:

$$a = \sqrt{(29\ \mathrm{m/s^2})^2 + (17\ \mathrm{m/s^2})^2} = \boxed{34\ \mathrm{m/s^2}}$$

Find the direction of the acceleration relative to the positive *x* axis:

$$\theta = \tan^{-1}\left(\frac{a_y}{a_x}\right) = \tan^{-1}\left(\frac{17}{29}\right) = \boxed{31^\circ}$$

WHAT IF? Suppose three hockey sticks strike the puck simultaneously, with two of them exerting the forces shown in Figure 5.4. The result of the three forces is that the hockey puck shows *no* acceleration. What must be the components of the third force?

Answer If there is zero acceleration, the net force acting on the puck must be zero. Therefore, the three forces must cancel. The components of the third force must be of equal magnitude and opposite sign compared to the components of the net force applied by the first two forces so that all the components add to zero. Therefore, $F_{3x} = -\sum F_x = -(0.30\ \text{kg})(29\ \text{m/s}^2) = -8.7\ \text{N}$ and $F_{3y} = -\sum F_y = -(0.30\ \text{kg})(17\ \text{m/s}^2) = -5.2\ \text{N}$.

5.5 The Gravitational Force and Weight

All objects are attracted to the Earth. The attractive force exerted by the Earth on an object is called the **gravitational force $\mathbf{F}_g$** .

This force is directed toward the center of the Earth.

Its magnitude is called the **weight** of the object.

For a freely falling object of mass *m* experiences an acceleration $\boldsymbol{a} = \boldsymbol{g}$:

$$\vec{\mathbf{F}}_g = m\vec{\mathbf{g}}$$

The weight of an object

$$F_g = mg$$

Because it depends on *g*, weight varies with geographic location.

Because *g* decreases with increasing distance from the center of the Earth, objects weigh less at higher altitudes than at sea level.

Kilogram Is Not a Unit of Weight

You may have seen the "conversion" 1 kg = 2.2 lb. Despite popular statements of weights expressed in kilograms, the kilogram is not a unit of *weight*, it is a unit of *mass*. The conversion statement is not an equality; it is an *equivalence* that is valid only on the Earth's surface.

Gravitational Mass vs. Inertial Mass

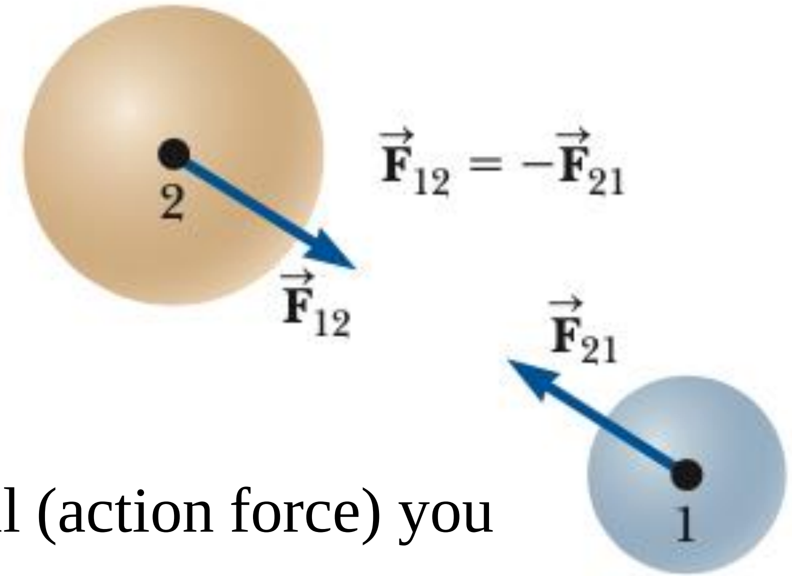
- In Newton's Laws, the mass is the inertial mass and measures the resistance to a change in the object's motion
- In the gravitational force, the mass is determining the gravitational attraction between the object and the Earth
- Experiments show that gravitational mass and inertial mass have the same value

5.6 Newton's Third Law

If two objects interact, the force $\vec{\mathbf{F}}_{12}$ exerted by object 1 on object 2 is equal in magnitude and opposite in direction to the force $\vec{\mathbf{F}}_{21}$ exerted by object 2 on object 1:

$$\vec{\mathbf{F}}_{12} = -\vec{\mathbf{F}}_{21} \tag{5.7}$$

$\mathbf{F}_{12}$ can be called *the* ***action force*** and $\mathbf{F}_{21}$ *the* ***reaction force***



For example, If you kick a football (action force) you can feel the force back (reaction force) on your foot.

Newton's Third Law, Alternative Statements

- Forces always occur in pairs
- A single isolated force cannot exist
- The action force is equal in magnitude to the reaction force and opposite in direction
  - One of the forces is the action force, the other is the reaction force
  - It doesn't matter which is considered the action and which the reaction
  - The action and reaction forces must act on different objects and be of the same type

Example, a computer monitor at rest on a table

The forces **n** and $m\mathbf{g}$ are equal in magnitude and opposite in direction.

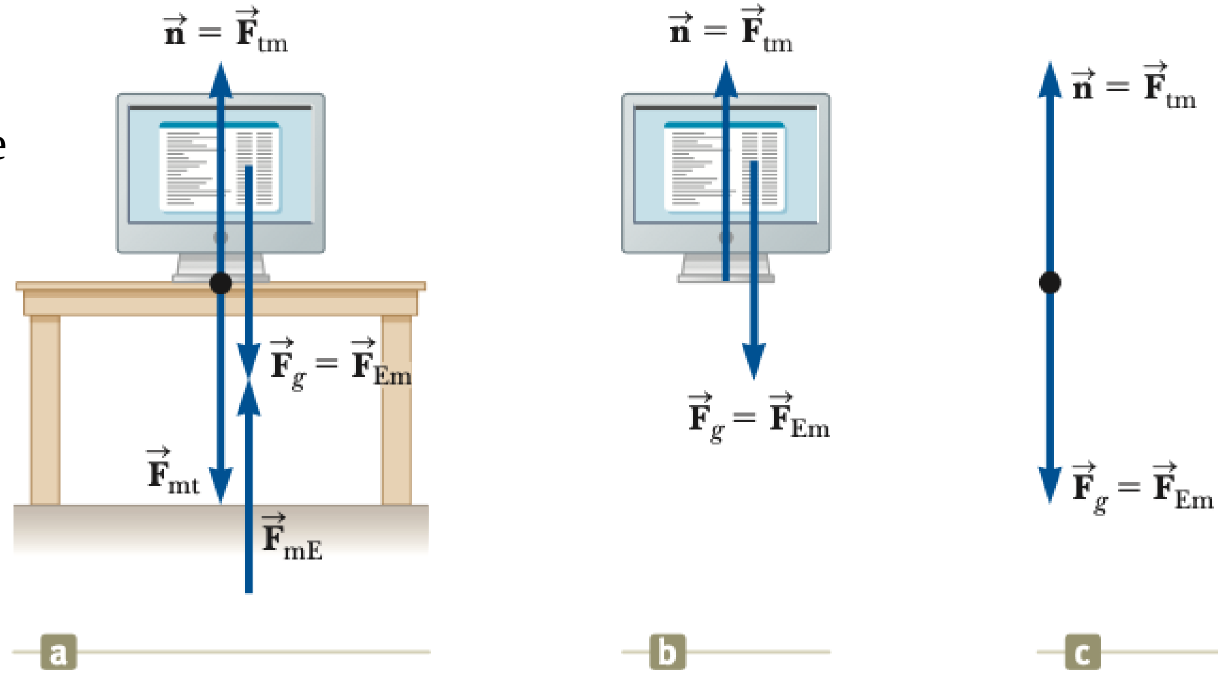


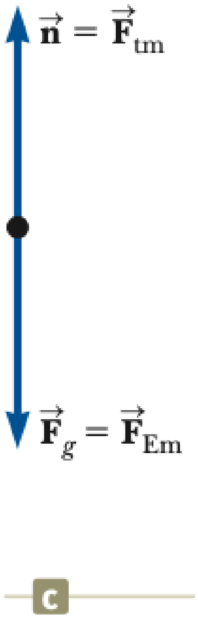
Figure 5.6 (a) When a computer monitor is at rest on a table, the forces acting on the monitor are the normal force $\vec{\mathbf{n}}$ and the gravitational force $\vec{\mathbf{F}}_g$. The reaction to $\vec{\mathbf{n}}$ is the force $\vec{\mathbf{F}}_{\text{mt}}$ exerted by the monitor on the table. The reaction to $\vec{\mathbf{F}}_g$ is the force $\vec{\mathbf{F}}_{\text{mE}}$ exerted by the monitor on the Earth. (b) A *force diagram* shows the forces on the monitor. (c) A *free-body diagram* shows the monitor as a black dot with the forces acting on it.

Free-Body Diagrams

The *most important* step in solving a problem using Newton's laws is to draw a proper sketch, the free-body diagram.

Be sure to draw *only* those forces that act on the object you are isolating.

Be sure to draw *all* forces acting on the object, including any field forces, such as the gravitational force.



Conceptual Example 5.3

A large man and a small boy stand facing each other on frictionless ice. They put their hands together and push against each other so that they move apart.

A) Who moves away with the higher speed?

B) Who moves farther while their hands are in contact?

(A) Who moves away with the higher speed?

SOLUTION

This situation is similar to what we saw in Quick Quiz 5.5. According to Newton's third law, the force exerted by the man on the boy and the force exerted by the boy on the man are a third-law pair of forces, so they must be equal in magnitude. (A bathroom scale placed between their hands would read the same, regardless of which way it faced.) Therefore, the boy, having the smaller mass, experiences the greater acceleration. Both individuals accelerate for the same amount of time, but the greater acceleration of the boy over this time interval results in his moving away from the interaction with the higher speed.

(B) Who moves farther while their hands are in contact?

SOLUTION

Because the boy has the greater acceleration and therefore the greater average velocity, he moves farther than the man during the time interval during which their hands are in contact.

5.7 Applications of Newton's Second Law

We discuss two analysis models for solving problems:

a) The Particle in Equilibrium

b) The Particle Under a Net Force

-Interested only in external forces that act on the object

-The objects are modeled as particles

-Neglect the mass of any ropes, strings, or cables involved (*light* and *of negligible mass*)

-Initially Neglecting the effects of friction

Problem-Solving Hints

Conceptualize Inspect the drawing, draw a diagram, choose a convenient coordinate system for each object.

Categorize A *particle in equilibrium*. *Or* a particle under a net force.

Analyze Construct a diagram of the forces acting on the object. Draw free-body. Include only forces acting on the object. Establish coordinate system. Find components. Apply the appropriate equations. Solve for the unknown.

Finalize Check the results. Check extreme values.

Analysis Model: The Particle in Equilibrium

If the acceleration of an object is zero.

Then, the net force on the object is zero:

$$\Sigma F = 0, \qquad a = 0$$

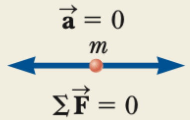
The net force on the objects in the *x* and *y* directions,

$$\Sigma F_x = 0 \qquad \text{and} \qquad \Sigma F_y = 0$$

Analysis Model **Particle in Equilibrium**

Imagine an object that can be modeled as a particle. If it has several forces acting on it so that the forces all cancel, giving a net force of zero, the object will have an acceleration of zero. This condition is mathematically described as

$$\sum \vec{\mathbf{F}} = 0 \qquad \textbf{(5.8)}$$



Examples

- a chandelier hanging over a dining room table
- an object moving at terminal speed through a viscous medium (Chapter 6)
- a steel beam in the frame of a building (Chapter 12)
- a boat floating on a body of water (Chapter 14)

Examples of Particles in Equilibrium

Example: a lamp suspended from a light chain fastened to the ceiling

The forces acting on the lamp are:
-the downward gravitational force **F***g*
-the upward force **T** exerted by the chain.

There are no forces in the *x* direction,

$$\Sigma F_x = 0$$

The condition $\Sigma F_y = 0$

$$\Sigma F_y = T - F_g = 0$$

$$T = F_g$$

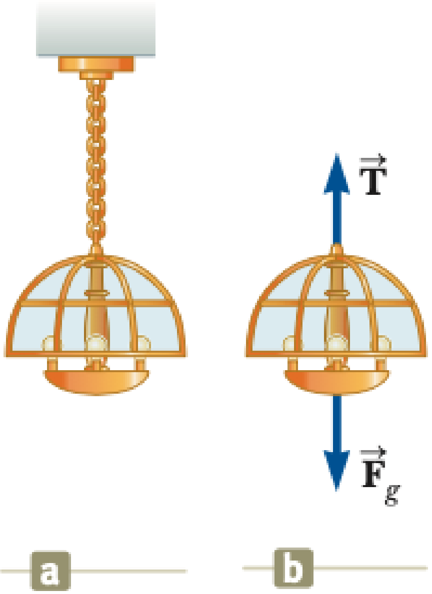
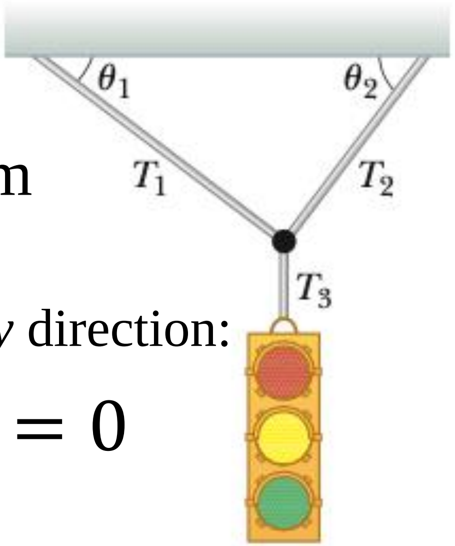


Figure 5.7 (a) A lamp suspended from a ceiling by a chain of negligible mass. (b) The forces acting on the lamp are the gravitational force $\vec{\mathbf{F}}_g$ and the force $\vec{\mathbf{T}}$ exerted by the chain.

The reaction force to **T** is a downward force exerted by the lamp on the chain.

Example: A traffic light suspended by cables (at Rest)

If the acceleration of a particle is zero, the particle is in equilibrium



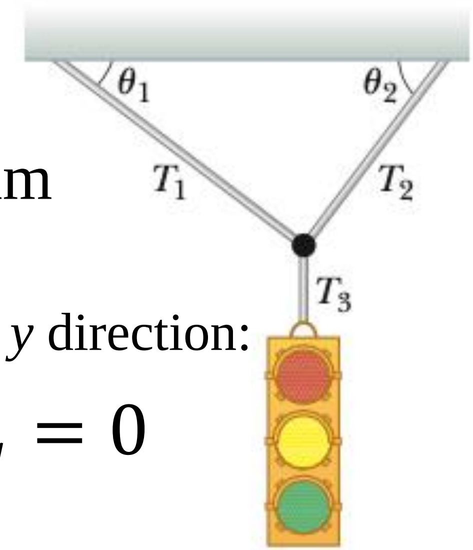
1) At the traffic light in the *y* direction:

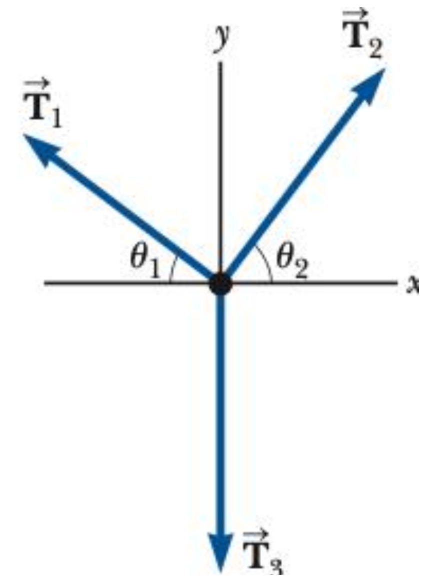
$$\Sigma F_y = 0 \rightarrow T_3 - F_g = 0$$

$$T_3 = F_g$$

Example: A traffic light suspended by cables (at Rest)

If the acceleration of a particle is zero, the particle is in equilibrium





1) At the traffic light in the *y* direction:

$$\Sigma F_y = 0 \rightarrow T_3 - F_g = 0$$

$$T_3 = F_g$$

2) Resolve the forces acting on the knot into their components:

Force	*x* Component	*y* Component
$\vec{\mathbf{T}}_1$	$-T_1 \cos\theta_1$	$T_1 \sin\theta_1$
$\vec{\mathbf{T}}_2$	$T_2 \cos\theta_2$	$T_2 \sin\theta_2$
$\vec{\mathbf{T}}_3$	0	$-F_g$

$$\Sigma F_x = T_2 \cos\theta_2 - T_1 \cos\theta_1 = 0 \quad (1)$$

$$\Sigma F_y = T_1 \sin\theta_1 + T_2 \sin\theta_2 - T_3 = 0 \quad (2)$$

Solve Equation (1) for T_2 in terms of T_1 :

$$\boxed{T_2 = T_1\left(\frac{\cos\theta_1}{\cos\theta_2}\right)} \qquad (3)$$

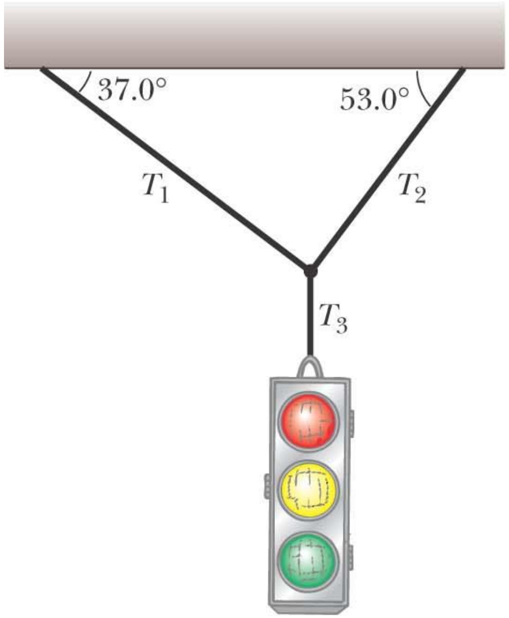
Substitute this value for T_2 into Equation (2):

$$T_1 \sin\theta_1 + T_1\left(\frac{\cos\theta_1}{\cos\theta_2}\right)(\sin\theta_2) - F_g = 0$$

Solve for T_1:

$$\boxed{T_1 = \frac{F_g}{\sin\theta_1 + \cos\theta_1 \tan\theta_2}} \qquad (4)$$

Example 5.4: A traffic light weighing 122 N hangs from a cable tied to two other cables fastened to a support, as in the figure below. These upper cables are not as strong as the vertical cable, and will break if the tension in them exceeds 100 N. Does the traffic light remain hanging in this situation, or will one of the cables break?



$$T_1 = \frac{F_g}{\sin\theta_1 + \cos\theta_1 \tan\theta_2}$$

$$T_1 = \frac{122\text{ N}}{\sin 37.0^\circ + \cos 37.0^\circ \tan 53.0^\circ} = 73.4\text{ N}$$

$$T_2 = T_1\left(\frac{\cos\theta_1}{\cos\theta_2}\right)$$

$$T_2 = (73.4\text{ N})\left(\frac{\cos 37.0^\circ}{\cos 53.0^\circ}\right) = 97.4\text{ N}$$

Both values are less than 100 N (just barely for T_2), so the cables will not break.

WHAT IF? Suppose the two angles in Figure 5.10a are equal. What would be the relationship between T_1 and T_2?

Answer We can argue from the symmetry of the problem that the two tensions T_1 and T_2 would be equal to each other. Mathematically, if the equal angles are called θ, Equation (3) becomes

$$T_2 = T_1\left(\frac{\cos\theta}{\cos\theta}\right) = T_1$$

which also tells us that the tensions are equal. Without knowing the specific value of θ, we cannot find the values of T_1 and T_2. The tensions will be equal to each other, however, regardless of the value of θ.

Analysis Model: The Particle Under a Net Force

If an object experiences an acceleration, its motion can be analyzed with: the **particle under a net force** model.

Draw a free-body diagram,

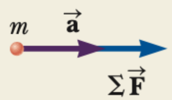
Apply Newton's second law, $\boxed{\Sigma \vec{F} = m\vec{a}}$

In component form in 2D: $\boxed{\Sigma F_x = m\, a_x}$ $\boxed{\Sigma F_y = m\, a_y}$

Analysis Model **Particle Under a Net Force**

Imagine an object that can be modeled as a particle. If it has one or more forces acting on it so that there is a net force on the object, it will accelerate in the direction of the net force. The relationship between the net force and the acceleration is

$$\sum \vec{\mathbf{F}} = m\vec{\mathbf{a}} \qquad (5.2)$$



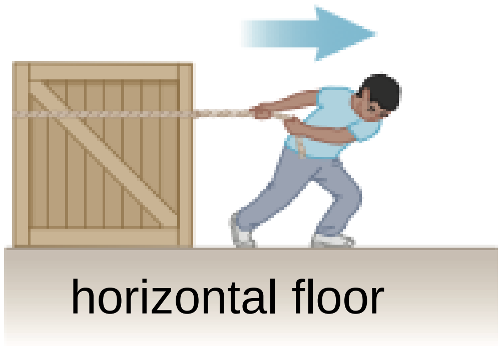
Examples

- a crate pushed across a factory floor
- a falling object acted upon by a gravitational force
- a piston in an automobile engine pushed by hot gases (Chapter 22)
- a charged particle in an electric field (Chapter 23)

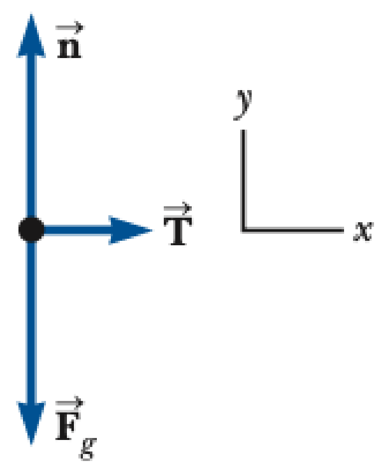
Examples on Particles Under a Net Force

Example: a rope is attached to a crate, and the crate is **pulled** to the right. The rope exerts a force **T** on the crate, and its magnitude (T) is called the tension.

One can find the acceleration of the crate and the force the floor exerts on it.



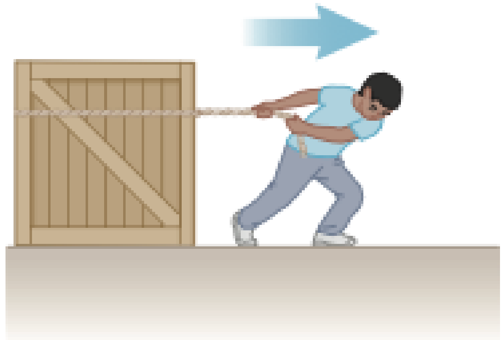
The forces acting on the crate are:
- The horizontal force **T** (The magnitude of **T** is equal to the tension in the rope)
- The gravitational force Fg
- The normal force **n** exerted by the floor on the crate.



The free-body diagram representing external forces acting on the crate.

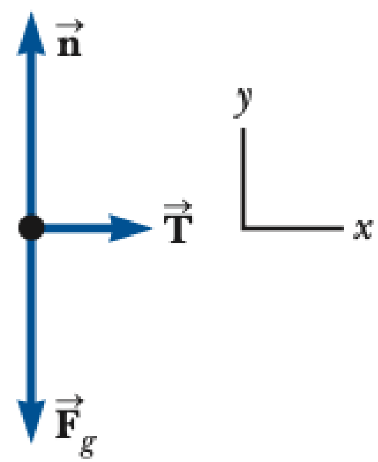
In the x direction (horizontal motion)

$$\Sigma F_x = m\, a_x$$
$$\mathrm{T} = m\, a_x$$
$$a_x = \frac{T}{m}$$



In the *y* direction:

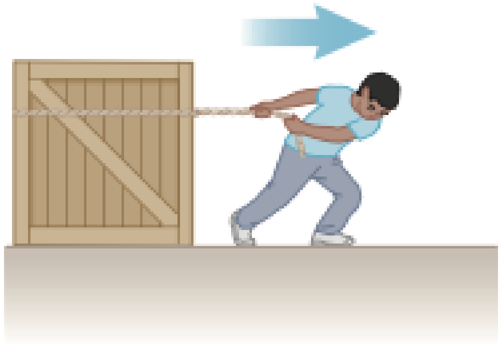
$$\Sigma F_y = n - F_g = 0$$
$$n = F_g$$



If the tension is constant, then a is constant and the kinematic equations can be used to more fully describe the motion of the crate.

If **T** is a constant force, a_x is also constant.
One can use kinematics equations to obtain the crate's position x and velocity v_x as functions of time.

$$\boxed{a_x = \frac{T}{m}}$$



$$v_{xf} = v_{xi} + \left(\frac{T}{m}\right)t$$

$$x_f = x_i + v_{xi}t + \frac{1}{2}\left(\frac{T}{m}\right)t^2$$

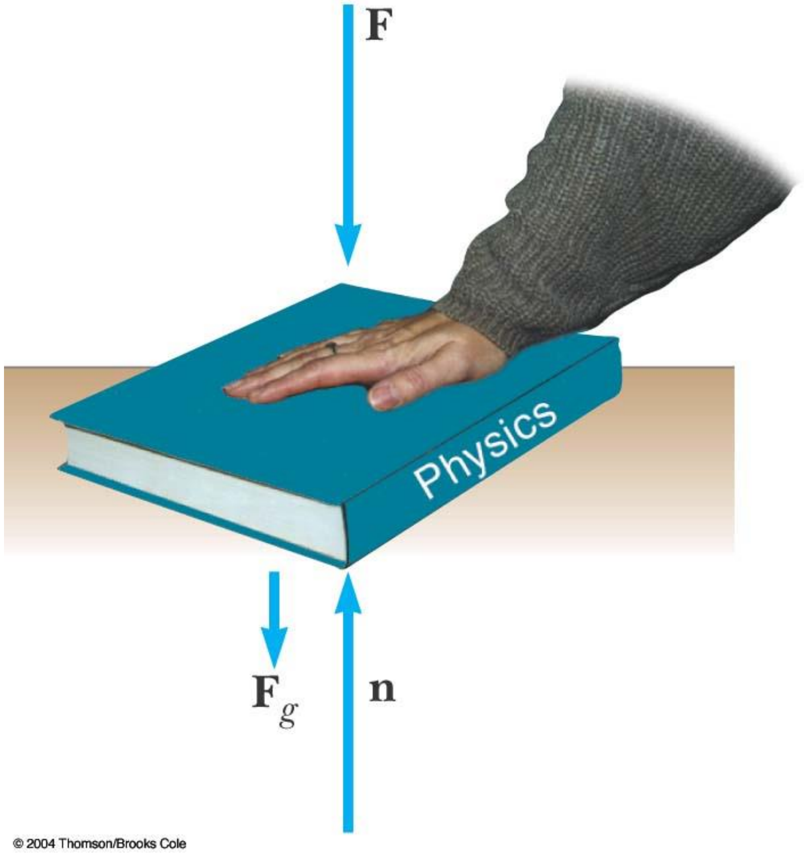
Note About the Normal Force

- The normal force is **not** always equal to the gravitational force of the object
- For example, in this case

$$\sum F_y = n - F_g - F = 0$$

and $n = F_g + F$

- **n** may also be less than $\mathbf{F}_g$

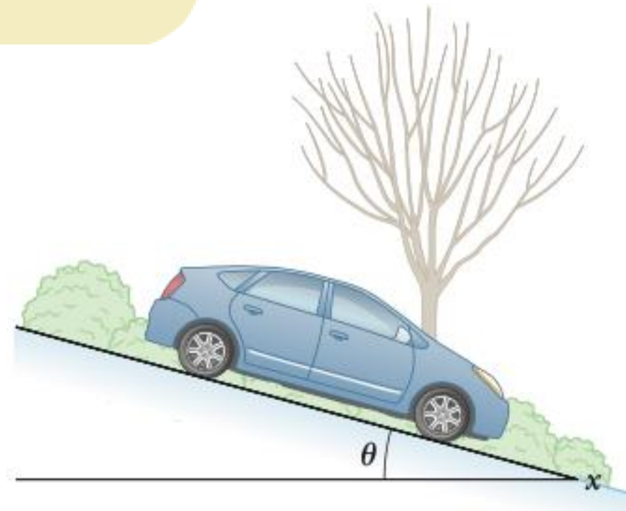


Example 5.6 The Runaway Car AM

A car of mass m is on an icy driveway inclined at an angle θ as in Figure 5.11a.

(A) Find the acceleration of the car, assuming the driveway is frictionless.

Conceptualize Use Figure 5.11a to conceptualize the situation. From everyday experience, we know that a car on an icy incline will accelerate down the incline. (The same thing happens to a car on a hill with its brakes not set.)



Categorize

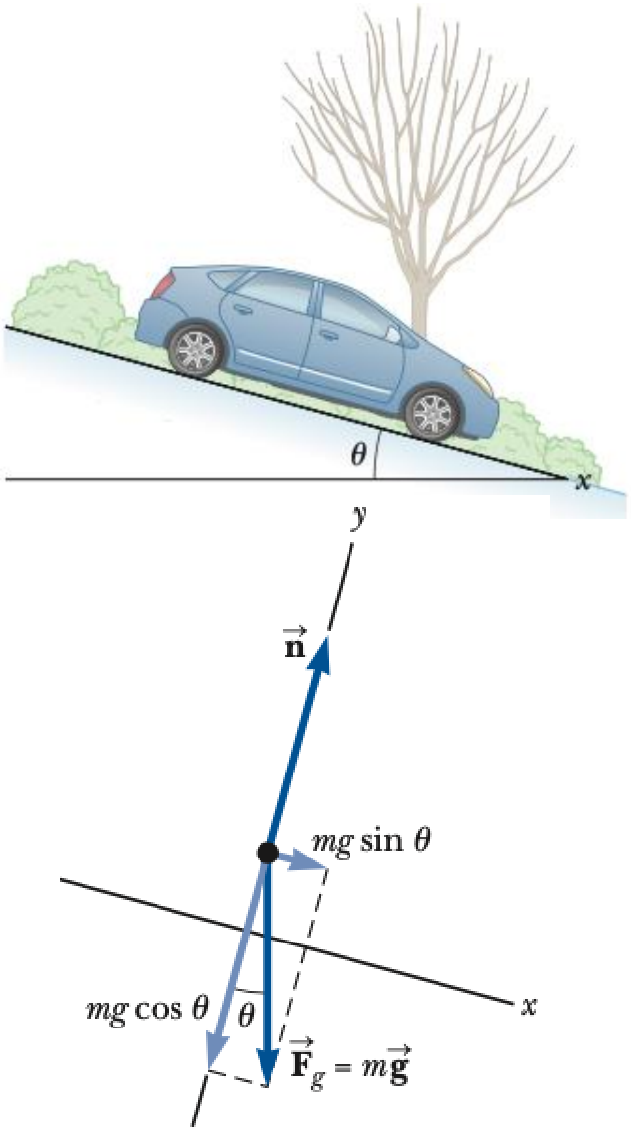
a particle under a net force

Analyze The only forces acting on the car are the normal force and gravitational force

$$\boxed{\Sigma F_x = mg\sin\theta = ma_x}$$

$$\boxed{\Sigma F_y = n - mg\cos\theta = 0}$$

$$\boxed{a_x = g\sin\theta}$$



Finalize Note that the acceleration component a_x is independent of the mass of the car

(B) Suppose the car is released from rest at the top of the incline and the distance from the car's front bumper to the bottom of the incline is d. How long does it take the front bumper to reach the bottom of the hill, and what is the car's speed as it arrives there?

SOLUTION

Conceptualize Imagine the car is sliding down the hill and you use a stopwatch to measure the entire time interval until it reaches the bottom.

Categorize This part of the problem belongs to kinematics rather than to dynamics, and Equation (3) shows that the acceleration a_x is constant. Therefore, you should categorize the car in this part of the problem as a particle under constant acceleration.

Analyze Defining the initial position of the front bumper as $x_i = 0$ and its final position as $x_f = d$, and recognizing that $v_{xi} = 0$, choose Equation 2.16 from the particle under constant acceleration model, $x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2$:

$$d = \tfrac{1}{2}a_x t^2$$

Solve for t:

$$(4) \quad t = \sqrt{\frac{2d}{a_x}} = \boxed{\sqrt{\frac{2d}{g\sin\theta}}}$$

Use Equation 2.17, with $v_{xi} = 0$, to find the final velocity of the car:

$$v_{xf}^{\,2} = 2a_x d$$

$$(5) \quad v_{xf} = \sqrt{2a_x d} = \boxed{\sqrt{2gd\sin\theta}}$$

Finalize We see from Equations (4) and (5) that the time t at which the car reaches the bottom and its final speed v_{xf} are independent of the car's mass, as was its acceleration. Notice that we have combined techniques from Chapter 2 with new techniques from this chapter in this example. As we learn more techniques in later chapters, this process of combining analysis models and information from several parts of the book will occur more often. In these cases, use the General Problem-Solving Strategy to help you identify what analysis models you will need.

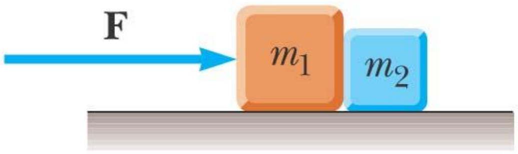
WHAT IF? What previously solved problem does this situation become if $\theta = 90°$?

Answer Imagine θ going to 90° in Figure 5.11. The inclined plane becomes vertical, and the car is an object in free fall! Equation (3) becomes

$$a_x = g \sin \theta = g \sin 90° = g$$

which is indeed the free-fall acceleration. (We find $a_x = g$ rather than $a_x = -g$ because we have chosen positive x to be downward in Fig. 5.11.) Notice also that the condition $n = mg \cos \theta$ gives us $n = mg \cos 90° = 0$. That is consistent with the car falling downward *next to* the vertical plane, in which case there is no contact force between the car and the plane.

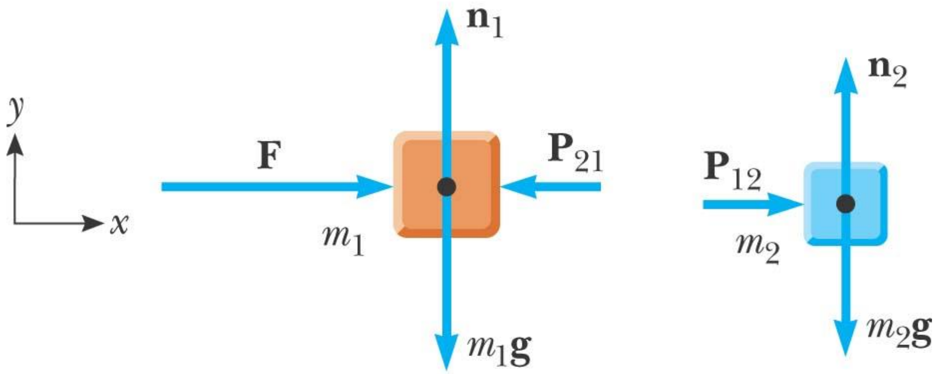
Example 5.7: Two blocks of masses m_1 and m_2, with m_1 + m_2, are placed in contact with each other on a frictionless, horizontal surface. A constant horizontal force **F** is applied to m_1.



$$\sum F_x = F = (m_1 + m_2)a$$

The magnitude of the acceleration of the system

$$a = \frac{F}{(m_1 + m_2)}$$



$$\sum F_x = P = m_2 a$$

P is the contact force between the two blocks

$$P = \left(\frac{m_2}{m_1 + m_2}\right)F$$

Example 5.8: A person weighs a fish of mass m on a spring scale attached to the ceiling of an elevator.

The weight of the fish when elevator accelerates **upward**:

$$\sum F_y = T - w = m.a$$

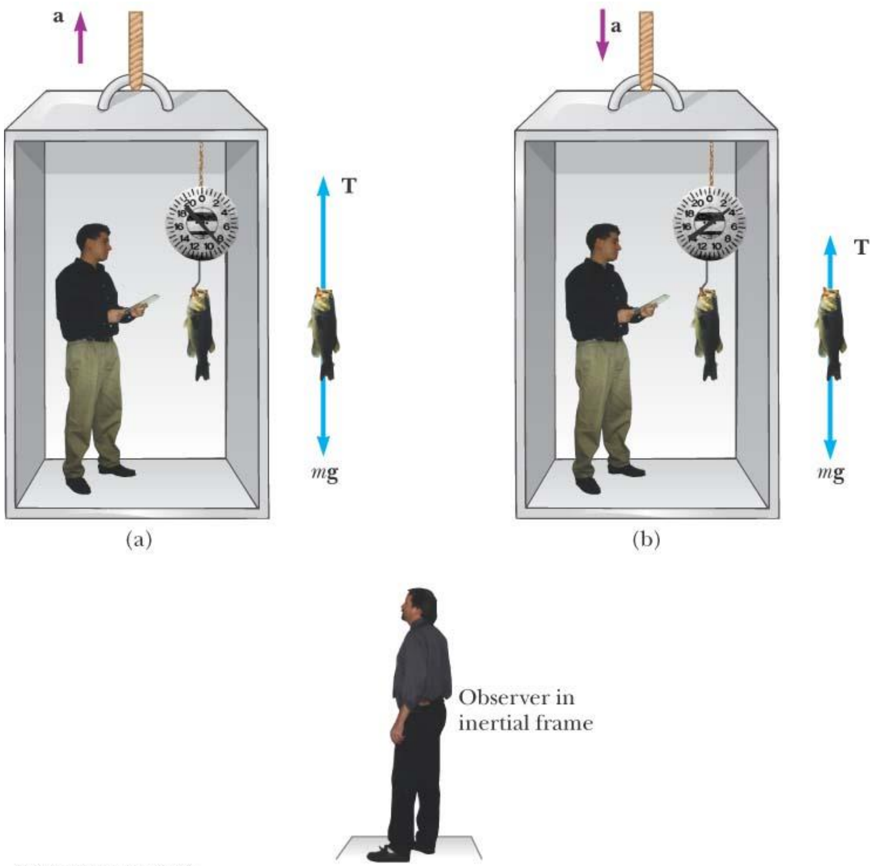
The weight of the fish when elevator accelerates **downward**:

$$\sum F_y = T - w = -m.a$$

$$T = w \pm m.a = mg \pm ma$$

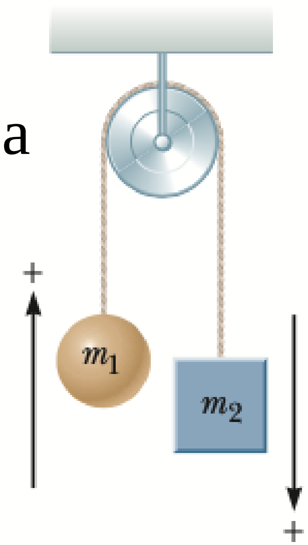
$$T = mg\left(1 \pm \frac{a}{g}\right)$$

$$T = w\left(1 \pm \frac{a}{g}\right)$$



Example 5.9: Atwood Machine

Two objects of unequal mass are hung vertically over a frictionless pulley of negligible mass.

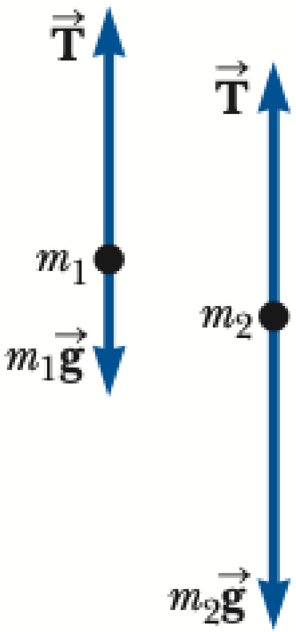


$$\sum F_y = T - m_1 g = m_1 a$$

$$\sum F_y = T - m_2 g = -m_2 a$$

$$\sum F_x = 0$$

$$a = \left(\frac{m_2 - m_1}{m_2 + m_1}\right) g \qquad , T = \left(\frac{2m_1 m_2}{m_1 + m_2}\right) g$$



The magnitude of the acceleration of the two objects.	The tension in the lightweight cord.

Describe the motion of the system if

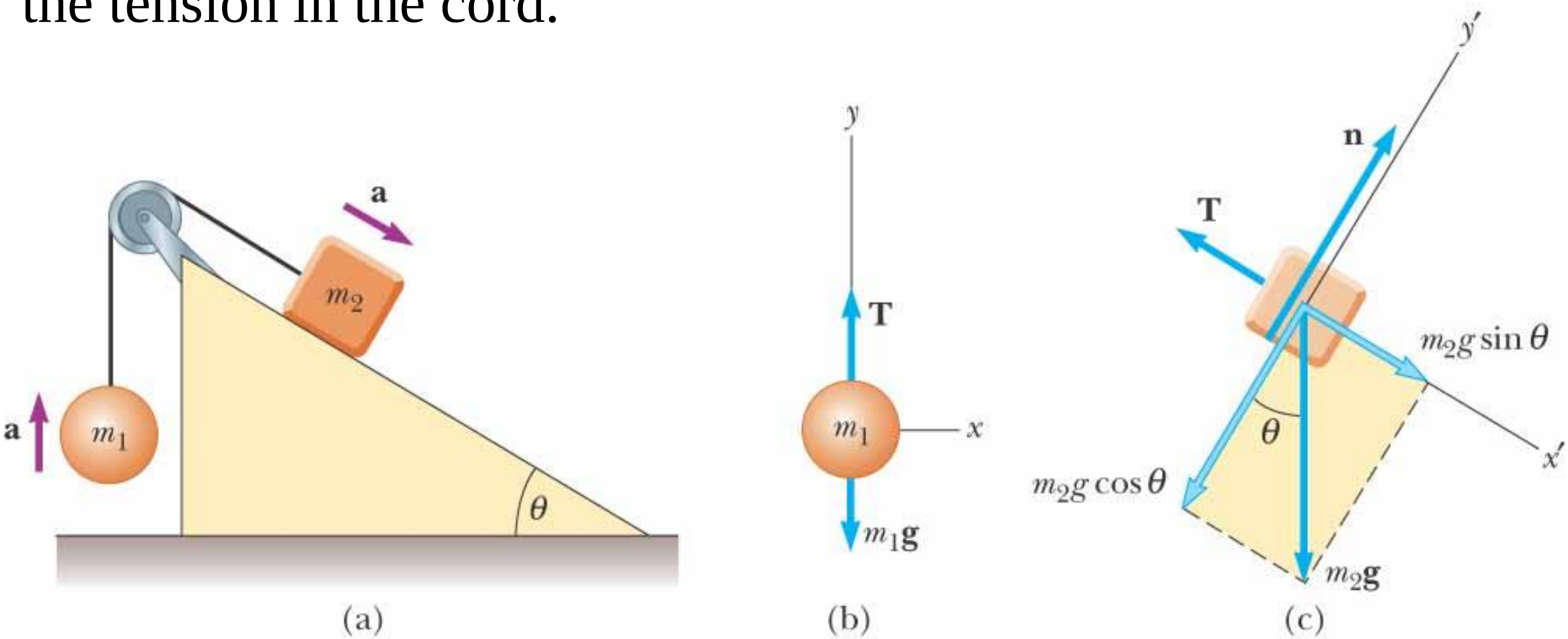
a) the objects have equal masses, $m_1 = m_2$.

b) if one of the masses is much larger than the other, $m_1 >> m_2$.

Example 5.10: Two Objects Connected by a Cord

A ball of mass m_1 and a block of mass m_2 are attached by a lightweight cord that passes over a frictionless pulley of negligible mass. The block lies on a frictionless incline of angle θ.

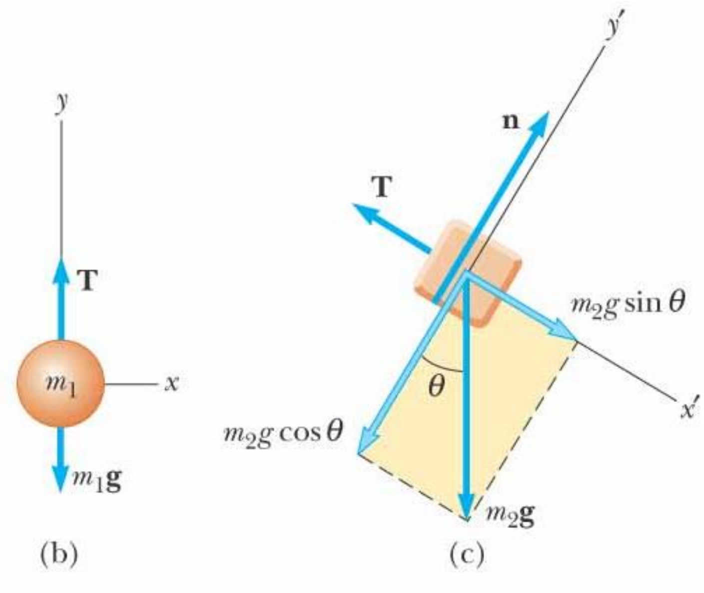
Find the magnitude of the acceleration of the two objects and the tension in the cord.



For the mass m_1:

$$\sum F_x = 0 \; , \; \sum F_y = T - m_1 g = \mathrm{m_1 a}$$

$$T > m_1 g$$



For the mass m_2:

$$\sum F_x = m_2 g \sin\theta - T = \mathrm{m_2 \, .a}$$

$$\sum F_y = N - m_2 g \cos\theta = 0$$

$$a = \frac{m_2 g \sin\theta - m_1 g}{m_1 + m_2}$$

$$T = \frac{m_1 m_2 g (1 + \sin\theta)}{m_1 + m_2}$$

What happens in these situations

a) if the angle θ= 90°?

b) b) if the mass m_1 = 0?

5.8 Forces of Friction

- When an object is in motion on a surface or through a viscous medium, there will be a resistance to the motion
  - This is due to the interactions between the object and its environment
- This resistance is called the *force of friction*

Forces of Friction, cont.

- Friction is proportional to the normal force
  - $f_s \leq \mu_s\, n$ and $f_k = \mu_k\, n$
  - These equations relate the magnitudes of the forces, they are not vector equations
- The force of static friction is generally greater than the force of kinetic friction
- The coefficient of friction (μ) depends on the surfaces in contact

Forces of Friction, final

- The direction of the frictional force is opposite the direction of motion and parallel to the surfaces in contact
- The coefficients of friction are nearly independent of the area of contact

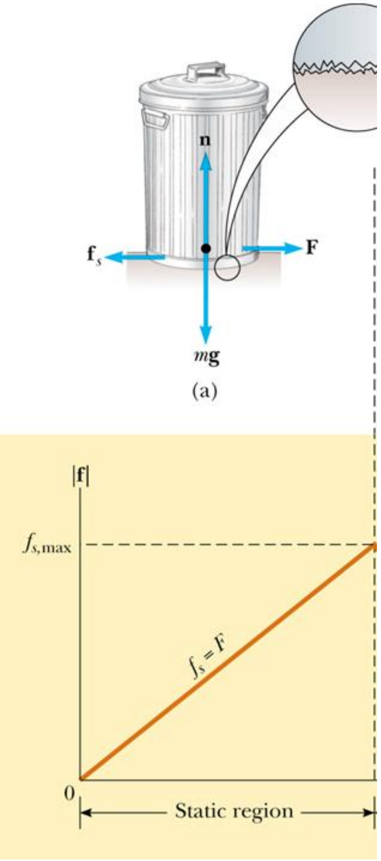
Static Friction

- Static friction acts to keep the object from moving
- If **F** increases, so does $\boldsymbol{f}_{\text{s}}$
- If **F** decreases, so does $\boldsymbol{f}_{\text{s}}$

$$f_s \leq \mu_s n$$

where the equality holds when the surfaces are on the verge of slipping

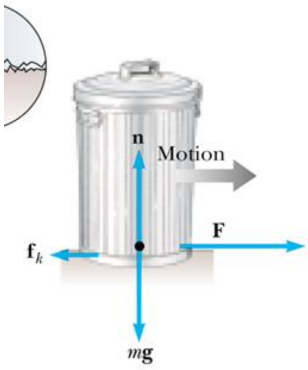
  - Called impending motion



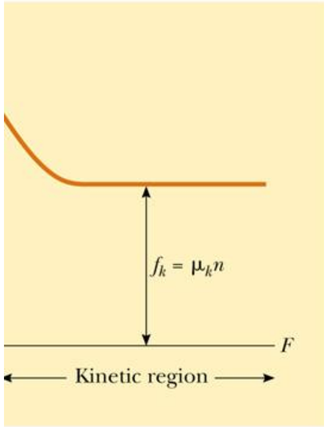
Kinetic Friction

- The force of kinetic friction acts when the object is in motion
- Although μ_k can vary with speed, we shall neglect any such variations

$$f_k = \mu_k n$$



(b)



Some Coefficients of Friction

Table 5.2

Coefficients of Friction[a]

	μ_s	μ_k
Steel on steel	0.74	0.57
Aluminum on steel	0.61	0.47
Copper on steel	0.53	0.36
Rubber on concrete	1.0	0.8
Wood on wood	0.25–0.5	0.2
Glass on glass	0.94	0.4
Waxed wood on wet snow	0.14	0.1
Waxed wood on dry snow	—	0.04
Metal on metal (lubricated)	0.15	0.06
Ice on ice	0.1	0.03
Teflon on Teflon	0.04	0.04
Synovial joints in humans	0.01	0.003

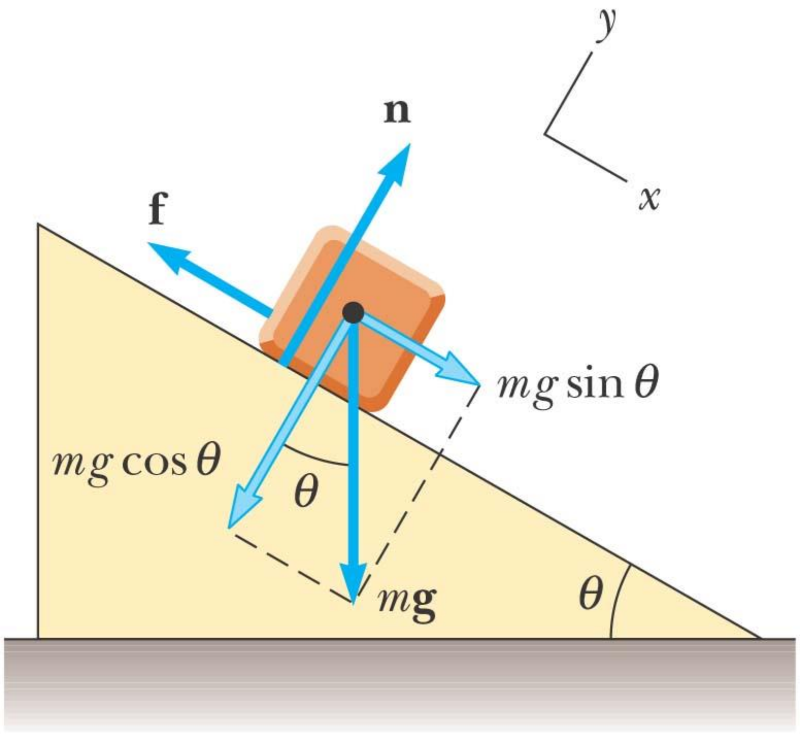
[a] All values are approximate. In some cases, the coefficient of friction can exceed 1.0.

Friction in Newton's Laws Problems

- Friction is a force, so it simply is included in the $\Sigma\mathbf{F}$ in Newton's Laws
- The rules of friction allow you to determine the direction and magnitude of the force of friction

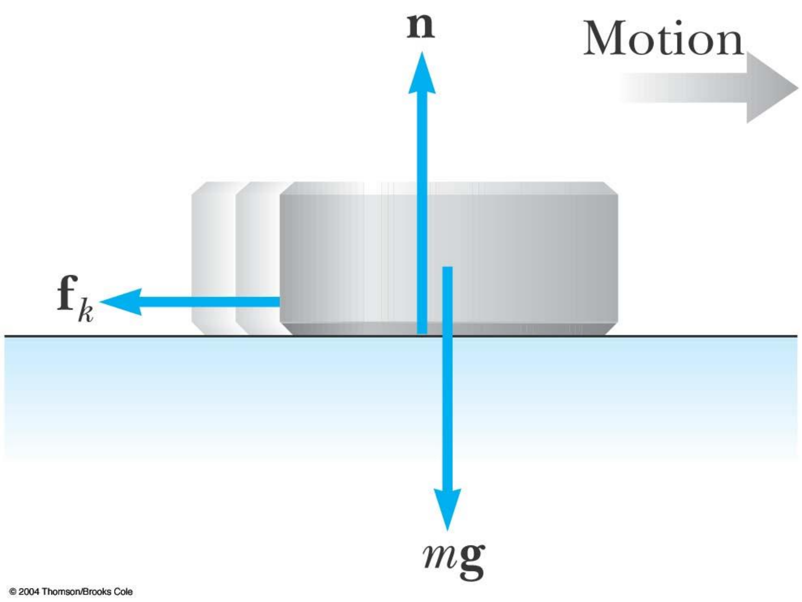
Friction Example, 1

- The block is sliding down the plane, so friction acts up the plane
- This setup can be used to experimentally determine the coefficient of friction
- $\mu = \tan\ \theta$
  - For μ_s, use the angle where the block just slips
  - For μ_k, use the angle where the block slides down at a constant speed

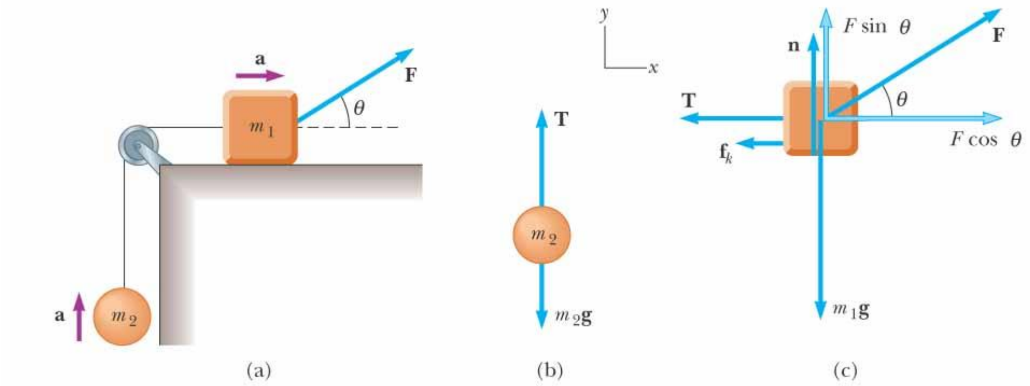


Friction, Example 2

- Draw the free-body diagram, including the force of kinetic friction
  - Opposes the motion
  - Is parallel to the surfaces in contact
- Continue with the solution as with any Newton's Law problem



Friction, Example 3



- Friction acts only on the object in contact with another surface
- Draw the free-body diagrams
- Apply Newton's Laws as in any other multiple object system problem

Summary

Definitions

An **inertial frame of reference** is a frame in which an object that does not interact with other objects experiences zero acceleration. Any frame moving with constant velocity relative to an inertial frame is also an inertial frame.

We define **force as that which causes a change in motion of an object.**

Concepts and Principles

Newton's first law states that it is possible to find an inertial frame in which an object that does not interact with other objects experiences zero acceleration, or, equivalently, in the absence of an external force, when viewed from an inertial frame, an object at rest remains at rest and an object in uniform motion in a straight line maintains that motion.

Newton's second law states that the acceleration of an object is directly proportional to the net force acting on it and inversely proportional to its mass.

Newton's third law states that if two objects interact, the force exerted by object 1 on object 2 is equal in magnitude and opposite in direction to the force exerted by object 2 on object 1.

The **gravitational force** exerted on an object is equal to the product of its mass (a scalar quantity) and the free-fall acceleration:

$$\vec{\mathbf{F}}_g = m\vec{\mathbf{g}} \qquad (5.5)$$

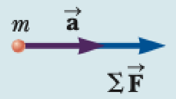
The **weight** of an object is the magnitude of the gravitational force acting on the object:

$$F_g = mg \qquad (5.6)$$

Analysis Models for Problem Solving

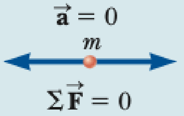
Particle Under a Net Force If a particle of mass m experiences a nonzero net force, its acceleration is related to the net force by Newton's second law:

$$\sum \vec{\mathbf{F}} = m\vec{\mathbf{a}} \tag{5.2}$$



Particle in Equilibrium If a particle maintains a constant velocity (so that $\vec{\mathbf{a}} = 0$), which could include a velocity of zero, the forces on the particle balance and Newton's second law reduces to

$$\sum \vec{\mathbf{F}} = 0 \tag{5.8}$$



Suggested Problems from Chapter 5

Problems: 3, 7, 11, 16, 18, 24, 25, 26, 28, 30, 31, 37, 41, 44, 45, 46, 68