Energy and Energy Transfer

7.1 Systems and Environments

7.2 Work Done by a Constant Force

7.3 The Scalar Product of Two Vectors

7.4 Work Done by a Varying Force

7.5 Kinetic Energy and the Work–Kinetic Energy Theorem

7.6 The Nonisolated System—Conservation of Energy

7.7 Situations Involving Kinetic Friction

7.8 Power

Introduction

- The definitions of quantities such as position, velocity, acceleration, and force and associated principles such as Newton's second law have allowed us to solve a variety of problems.
- Some problems that could theoretically be solved with Newton's laws, however, are very difficult in practice, but they can be made much simpler with a different approach.

- The concept of energy is one of the most important topics in science and engineering.
- In everyday life, we need fuels for transportation and heating, electricity for lights and appliances, and foods for consumption. The fuels are needed to do a job and that those fuels provide us with something we call energy.
- Energy is present in the Universe in various forms.
- Every physical process that occurs in the Universe involves energy and energy transfers or transformations.
- Energy cannot be easily defined.

A new simplification model

- The concept of energy can be applied to mechanical systems without resorting to Newton's laws.
- The energy approach to describing motion is particularly useful when the force is not constant
- We begin our new approach by focusing our attention on a new simplification model, a *system*, and analysis models based on the model of a system.

7.1 Systems and Environments

In the system model,

- we focus our attention on a small portion of the Universe – the system –
- and ignore details of the rest of the Universe outside of the system.

A critical skill in applying the system model to problems is *identifying the system.*

A valid system may

- be a single object or particle
- be a collection of objects or particles
- be a region of space (such as the interior of an automobile engine combustion cylinder)
- vary with time in size and shape (such as a rubber ball, which deforms upon striking a wall)

- Remember that:

Identifying the need for a system approach to solving a problem is part of the Categorize step in the General Problem-Solving Strategy.

Identifying the particular system is a second part of this step.

We identify a **system boundary,** an imaginary surface (not necessarily coinciding with a physical surface) that divides the Universe into the system and the **environment** surrounding the system.

Examples of systems

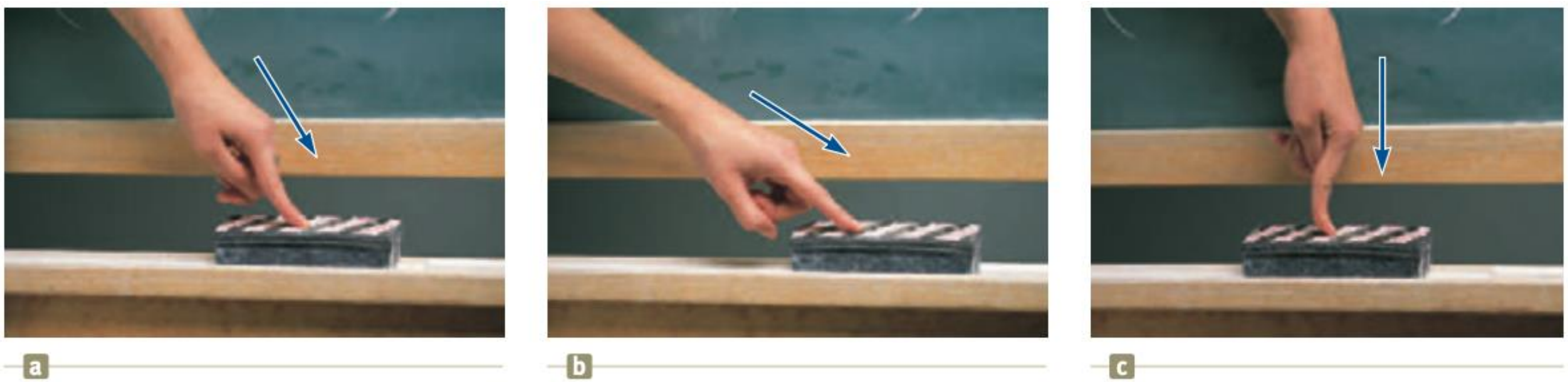
A force applied to an object in empty space.

- define the object as the system
- define its surface as the system boundary.
- the force applied to it is an influence on the system from the environment that acts across the system boundary.

We will see how to analyze this situation from a system approach in a subsequent section of this chapter.

7.2 Work Done by a Constant Force

We encounter a term whose meaning in physics is distinctly different from its every- day meaning: work.



An eraser being pushed along a chalkboard tray by a force acting at different angles with respect to the horizontal direction.

If we want to know how effective the force is in moving the eraser, we must consider not only the magnitude of the force but also its direction.

Work Is Done by...on...

Not only must you identify the system, you must also identify what agent in the environment is doing work on the system.

For example, “the work done by the hammer on the nail” identifies the nail as the system, and the force from the hammer represents the influence from the environment.

The **work** W done on a system by an agent exerting a constant force on the system is the product of the magnitude F of the force, the magnitude Δr of the displacement of the point of application of the force, and $\cos\theta$, where θ is the angle between the force and displacement vectors:

$$W \equiv F \Delta r \cos\theta \qquad (7.1)$$

A force does no work on an object if the force does not move through a displacement.
If $\Delta r = 0, \; W = 0$

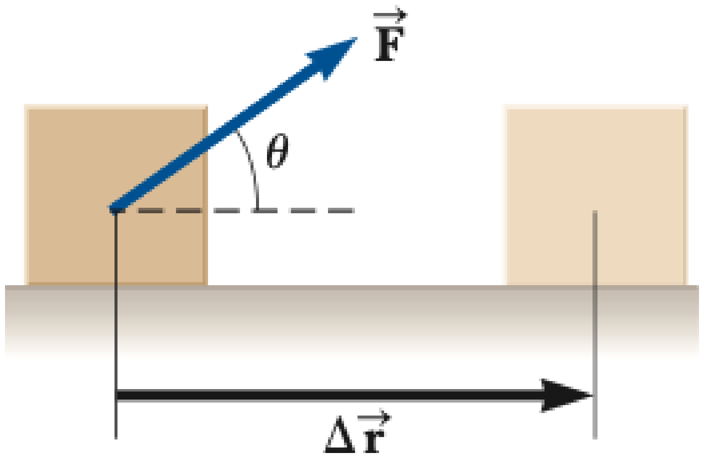


Figure 7.2 An object undergoes a displacement $\Delta\vec{r}$ under the action of a constant force $\vec{F}$.

The work done by a force on a moving object is zero when the force applied is perpendicular to the displacement of its point of application.
If $\theta = 90^o, \quad W = 0$

If an applied force **F** is in the same direction as the displacement $\Delta \boldsymbol{r}$, then $\theta = 0$ and $\cos 0 = 1$.
In this case,

$$W = F\Delta r$$

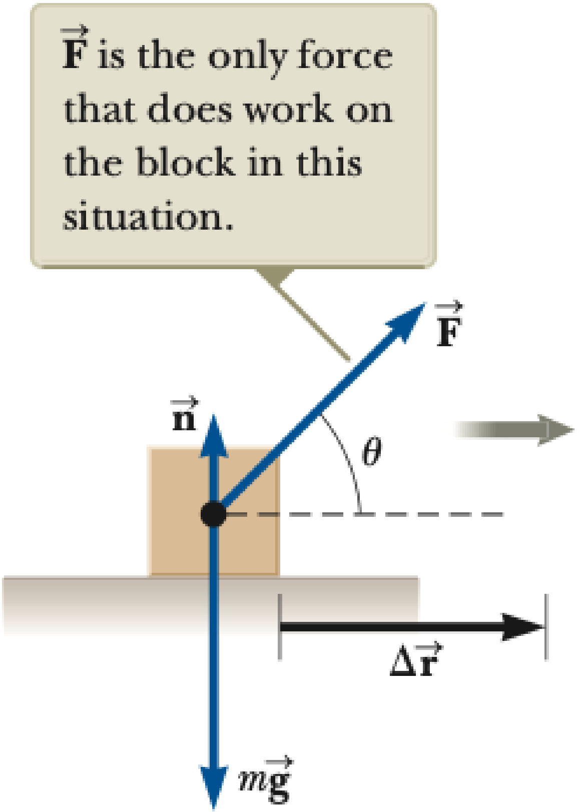


Figure 7.3 An object is displaced on a frictionless, horizontal surface. The normal force $\vec{n}$ and the gravitational force $m\vec{g}$ do no work on the object.

The sign of the work also depends on the direction of $\boldsymbol{F}$ relative to $\Delta\boldsymbol{r}$.

- The work done by the applied force on a system is positive when the projection of $\boldsymbol{F}$ onto $\Delta\boldsymbol{r}$ is in the same direction as the displacement.
- W is negative when the projection of $\boldsymbol{F}$ onto $\Delta\boldsymbol{r}$ is in the direction opposite the displacement.

Units of Work

- The units of work are those of force multiplied by those of length.
- The SI unit of work is the **newton · meter** ($N \cdot m = kg \cdot m^2/s^2$).
- This combination of units is used so frequently that it has been given a name of its own, the **joule** (J).

$$J = N \cdot m = kg \cdot m^2/s^2$$

Work is an energy transfer

If *W* is the work done on a system and *W* is positive, energy is transferred *to* the system.

if *W* is negative, energy is transferred *from* the system.

Therefore, if a system interacts with its environment, this interaction can be described as a transfer of energy across the system boundary.

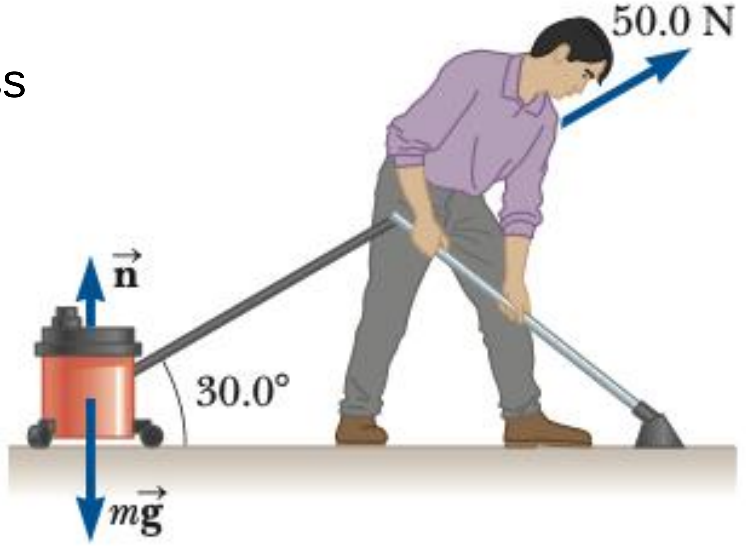
The result is a change in the energy stored in the system.

Example 7.1 Mr. Clean

A man cleaning a floor pulls a vacuum cleaner with a force of magnitude F = 50.0 N at an angle of 30.0^{o} with the horizontal. Calculate the work done by the force on the vacuum cleaner as the vacuum cleaner is displaced 3.00 m to the right.

Conceptualize
Think of it as as you pull an object across the floor with a rope or cord.

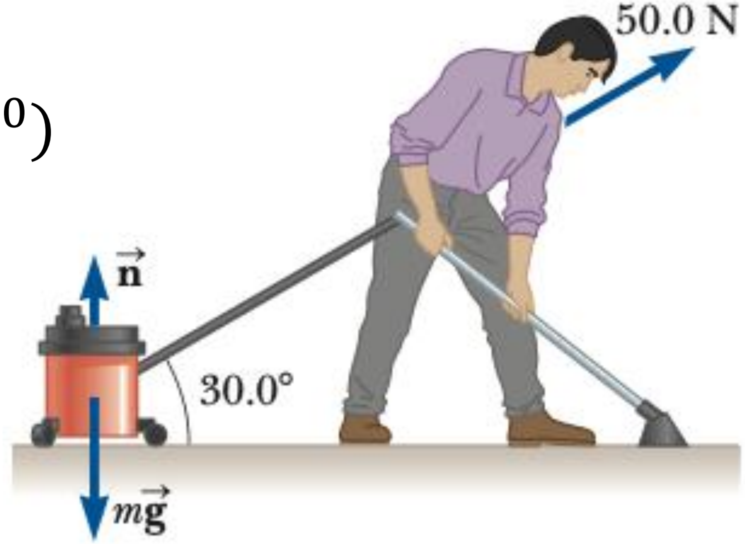
Categorize We are asked for W, the force on the object, the displacement of the object, and the angle between the two vectors are given.
We identify the vacuum cleaner as the system.



Example 7.1 Mr. Clean

A man cleaning a floor pulls a vacuum cleaner with a force of magnitude F = 50.0 N at an angle of 30.0° with the horizontal. Calculate the work done by the force on the vacuum cleaner as the vacuum cleaner is displaced 3.00 m to the right.

$$W = F\,\Delta r\,cos\theta$$
$$= (50.0\text{ N})\,(3.00\text{ m})(\cos 30.0^0)$$
$$= 130\text{ J}$$



7.3 The Scalar Product of Two Vectors

The scalar product of any two vectors $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ is defined as a scalar quantity equal to the product of the magnitudes of the two vectors and the cosine of the angle θ between them:

$$\vec{\mathbf{A}} \cdot \vec{\mathbf{B}} \equiv AB\ cos\theta$$

The scalar product or dot product

$$W = \vec{\mathbf{F}} \cdot \Delta\vec{\boldsymbol{r}} \equiv F\ \Delta r\ cos\theta$$

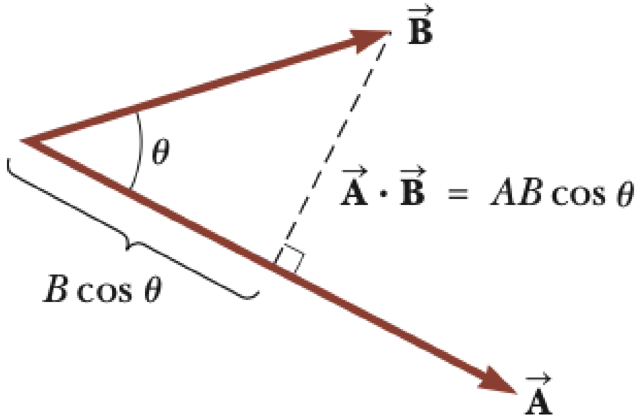


Figure 7.6 The scalar product $\vec{\mathbf{A}} \cdot \vec{\mathbf{B}}$ equals the magnitude of $\vec{\mathbf{A}}$ multiplied by $B\cos\theta$, which is the projection of $\vec{\mathbf{B}}$ onto $\vec{\mathbf{A}}$.

Some properties of the dot product:

-The scalar product is **commutative**

$$\vec{\mathbf{A}} \cdot \vec{\mathbf{B}} = \vec{\mathbf{B}} \cdot \vec{\mathbf{A}}$$

-The scalar product obeys the **distributive law of multiplication**

$$\vec{\mathbf{A}} \cdot (\vec{\mathbf{B}} + \vec{\mathbf{C}}) = \vec{\mathbf{A}} \cdot \vec{\mathbf{B}} + \vec{\mathbf{A}} \cdot \vec{\mathbf{C}}$$

If **A** is perpendicular to **B** ($\theta = 90^o$), then $\vec{\mathbf{A}} \cdot \vec{\mathbf{B}}$**=** 0.

If **A** is parallel to **B** ($\theta = 0^o$), then $\vec{\mathbf{A}} \cdot \vec{\mathbf{B}}$**=** AB.

If **A** is parallel to **B** ($\theta = 180^o$), then $\vec{\mathbf{A}} \cdot \vec{\mathbf{B}}$**= -**AB.

Using unit vectors i, j, and k, lie in the positive *x*, *y*, and *z* directions, respectively, of a right-handed coordinate system.

The scalar products of unit vectors

$$\hat{\mathbf{i}}\cdot\hat{\mathbf{i}} = \hat{\mathbf{j}}\cdot\hat{\mathbf{j}} = \hat{\mathbf{k}}\cdot\hat{\mathbf{k}} = 1$$

$$\hat{\mathbf{i}}\cdot\hat{\mathbf{j}} = \hat{\mathbf{i}}\cdot\hat{\mathbf{k}} = \hat{\mathbf{j}}\cdot\hat{\mathbf{k}} = 0$$

A and **B** vectors can be expressed in unit-vector from

$$\vec{\mathbf{A}} = A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}} + A_z\hat{\mathbf{k}}$$

$$\vec{\mathbf{B}} = B_x\hat{\mathbf{i}} + B_y\hat{\mathbf{j}} + B_z\hat{\mathbf{k}}$$

The scalar product of **A** and **B** reduces to

$$\vec{\mathbf{A}}\cdot\vec{\mathbf{B}} = A_xB_x + A_yB_y + A_zB_z$$

Example 7.2 The Scalar Product

The vectors $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ are given by $\vec{\mathbf{A}} = 2\,\hat{\mathbf{i}} + 3\,\hat{\mathbf{j}}$ and $\vec{\mathbf{B}} = -\,\hat{\mathbf{i}} + 2\,\hat{\mathbf{j}}$.

(A) Determine the scalar product $\vec{\mathbf{A}} \cdot \vec{\mathbf{B}}$

Conceptualize There is no physical system to imagine here. Rather, it is purely a mathematical exercise involving two vectors.

Categorize Because we have a definition for the scalar product, we categorize this example as a substitution problem.

$$\begin{aligned}\vec{\mathbf{A}} \cdot \vec{\mathbf{B}} &= (2\hat{\mathbf{i}} + 3\hat{\mathbf{j}}) \cdot (-\hat{\mathbf{i}} + 2\hat{\mathbf{j}}) \\ &= -2\hat{\mathbf{i}} \cdot \hat{\mathbf{i}} + 2\hat{\mathbf{i}} \cdot 2\hat{\mathbf{j}} - 3\hat{\mathbf{j}} \cdot \hat{\mathbf{i}} + 3\hat{\mathbf{j}} \cdot 2\hat{\mathbf{j}} \\ &= -2(1) + 4(0) - 3(0) + 6(1) = -2 + 6 = \boxed{4}\end{aligned}$$

The same result is obtained when we use $\vec{\mathbf{A}} \cdot \vec{\mathbf{B}} = A_x B_x + A_y B_y$ directly

(B) Find the angle θ between **A** and **B** .

$$A = \sqrt{A_x^2 + A_y^2} = \sqrt{(2)^2 + (3)^2} = \sqrt{13}$$

$$B = \sqrt{B_x^2 + B_y^2} = \sqrt{(-1)^2 + (2)^2} = \sqrt{5}$$

$$\cos\theta = \frac{\vec{\mathbf{A}} \cdot \vec{\mathbf{B}}}{AB} = \frac{4}{\sqrt{13}\sqrt{5}} = \frac{4}{\sqrt{65}}$$

$$\theta = \cos^{-1}\frac{4}{\sqrt{65}} = 60.3°$$

Example 7.3 Work Done by a Constant Force

A particle moving in the *xy* plane undergoes a displacement given by $\Delta\vec{\mathbf{r}} = (2.0\,\hat{\mathbf{i}} + 3.0\,\hat{\mathbf{j}})$ m as a constant force $\vec{\mathbf{F}} = (5.0\,\hat{\mathbf{i}} + 2.0\,\hat{\mathbf{j}})$ N acts on the particle. Calculate the work done by $\vec{\mathbf{F}}$ on the particle.

$$W = \vec{\mathbf{F}} \cdot \Delta\vec{\mathbf{r}} = [(5.0\hat{\mathbf{i}} + 2.0\hat{\mathbf{j}})\text{ N}] \cdot [(2.0\hat{\mathbf{i}} + 3.0\hat{\mathbf{j}})\text{ m}]$$

$$= (5.0\hat{\mathbf{i}} \cdot 2.0\hat{\mathbf{i}} + 5.0\hat{\mathbf{i}} \cdot 3.0\hat{\mathbf{j}} + 2.0\hat{\mathbf{j}} \cdot 2.0\hat{\mathbf{i}} + 2.0\hat{\mathbf{j}} \cdot 3.0\hat{\mathbf{j}})\text{ N} \cdot \text{m}$$

$$= [10 + 0 + 0 + 6]\text{ N} \cdot \text{m} = 16\text{ J}$$

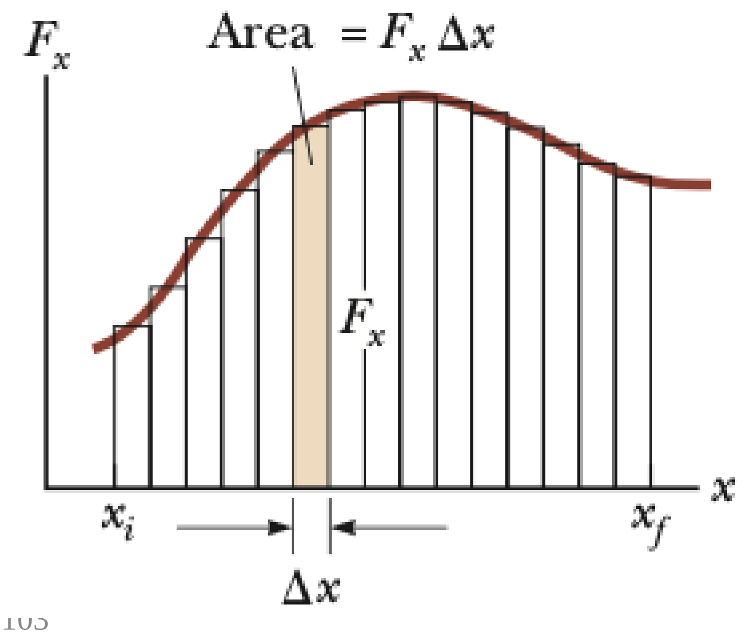
7.4 Work Done by a Varying Force

Consider a particle being displaced along the x axis under the action of a force that varies with position.

For a particle undergoing a very small displacement Δx, the x component F_x of the force is approximately constant over this small interval, we can approximate the work done on the particle by using

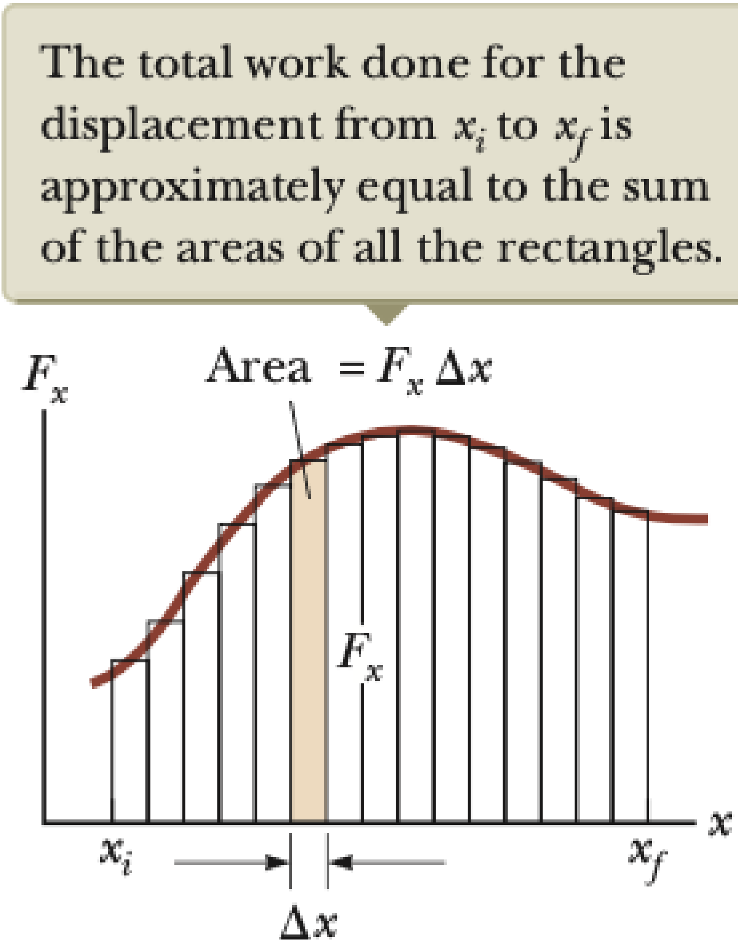
$$W \approx F_x \, \Delta x$$

which is the area of the shaded rectan



The total work done for the displacement from x_i to x_f is approximately equal to the sum of a large number of such terms:

$$W \approx \sum_{x_i}^{x_f} F_x \, \Delta x$$



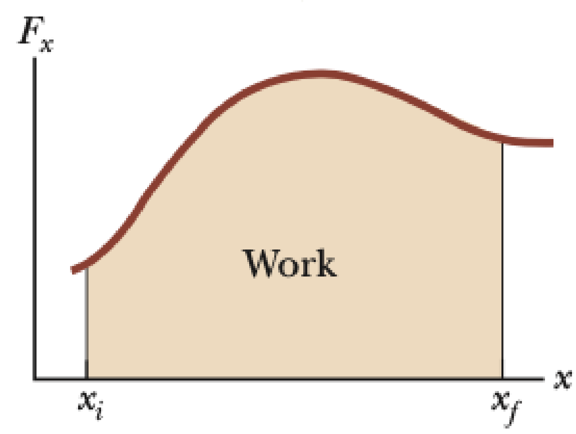
If the size of the small displacements is allowed to approach zero, then the value of the sum approaches:

$$\lim_{\Delta x \to 0} \sum_{x_i}^{x_f} F_x \, \Delta x = \int_{x_i}^{x_f} F_x \, dx$$

The work done by F_x on the system of the particle as it moves from x_i to x_f as

$$W = \int_{x_i}^{x_f} F_x \, dx$$

The work done by the component F_x of the varying force as the particle moves from x_i to x_f is *exactly* equal to the area under the curve.



If more than one force acts on a system

If more than one force acts on a system *and the system can be modeled as a particle*,

the total work done on the system is just the work done by the net force.

$$\sum W = W_{ext} = \int_{x_i}^{x_f} \left(\sum F_x \right) dx$$

For the general case of the net force (magnitude and direction may both vary):

$$\sum W = W_{ext} = \int_{x_i}^{x_f} \left(\sum \vec{\mathbf{F}} \right) \cdot d\vec{\boldsymbol{r}}$$

Example 7.4 Calculating Total Work Done from a Graph

A force acting on a particle varies with *x*, as shown in Figure 7.8. Calculate the work done by the force as the particle moves from $x = 0$ to $x = 6.0$ m.

SOLUTION

Conceptualize Imagine a particle subject to the force in Figure 7.8. The force remains constant as the particle moves through the first 4.0 m and then decreases linearly to zero at 6.0 m. In terms of earlier discussions of motion, the particle could be modeled as a particle under constant acceleration for the first 4.0 m because the force is constant. Between 4.0 m and 6.0 m, however, the motion does not fit into one of our earlier analysis models because the acceleration of the particle is changing. If the particle starts from rest, its speed increases throughout the motion, and the particle is always moving in the positive *x* direction. These details about its speed and direction are not necessary for the calculation of the work done, however.

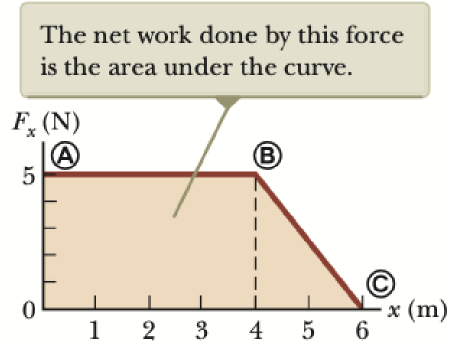
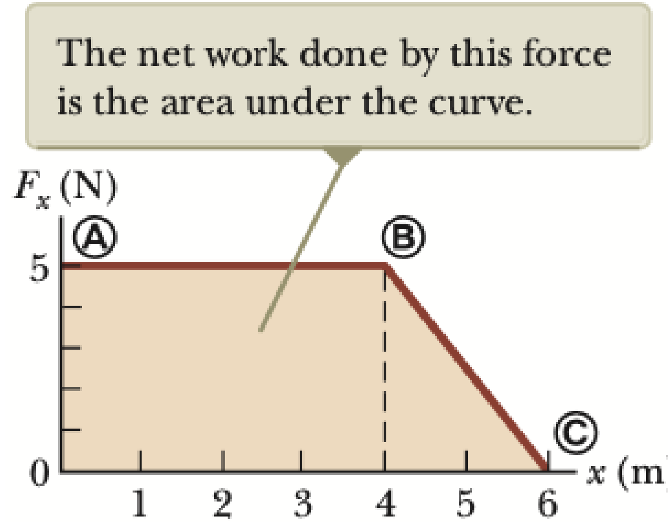


Figure 7.8 (Example 7.4) The force acting on a particle is constant for the first 4.0 m of motion and then decreases linearly with x from $x_{Ⓑ} = 4.0$ m to $x_{Ⓒ} = 6.0$ m.

Categorize Because the force varies during the motion of the particle, we must use the techniques for work done by varying forces. In this case, the graphical representation in Figure 7.8 can be used to evaluate the work done.



Analyze The work done by the force is equal to the area under the curve from $x_Ⓐ = 0$ to $x_Ⓒ = 6.0$ m. This area is equal to the area of the rectangular section from Ⓐ to Ⓑ plus the area of the triangular section from Ⓑ to Ⓒ.

Evaluate the area of the rectangle: $W_{Ⓐ \text{ to } Ⓑ} = (5.0 \text{ N})(4.0 \text{ m}) = 20 \text{ J}$

Evaluate the area of the triangle: $W_{Ⓑ \text{ to } Ⓒ} = \frac{1}{2}(5.0 \text{ N})(2.0 \text{ m}) = 5.0 \text{ J}$

Find the total work done by the force on the particle: $W_{Ⓐ \text{ to } Ⓒ} = W_{Ⓐ \text{ to } Ⓑ} + W_{Ⓑ \text{ to } Ⓒ} = 20 \text{ J} + 5.0 \text{ J} = 25 \text{ J}$

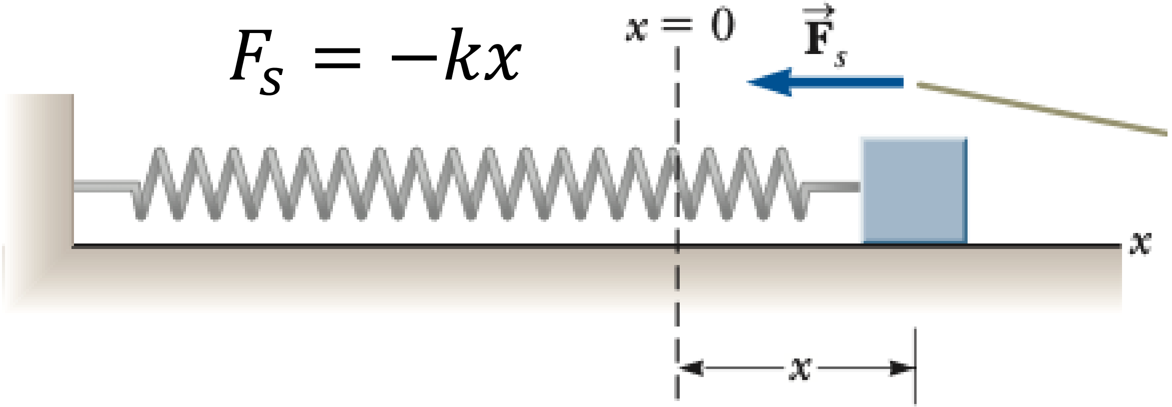
Finalize Because the graph of the force consists of straight lines, we can use rules for finding the areas of simple geometric models to evaluate the total work done in this example. If a force does not vary linearly as in Figure 7.7, such rules cannot be used and the force function must be integrated as in Equation 7.7 or 7.8.

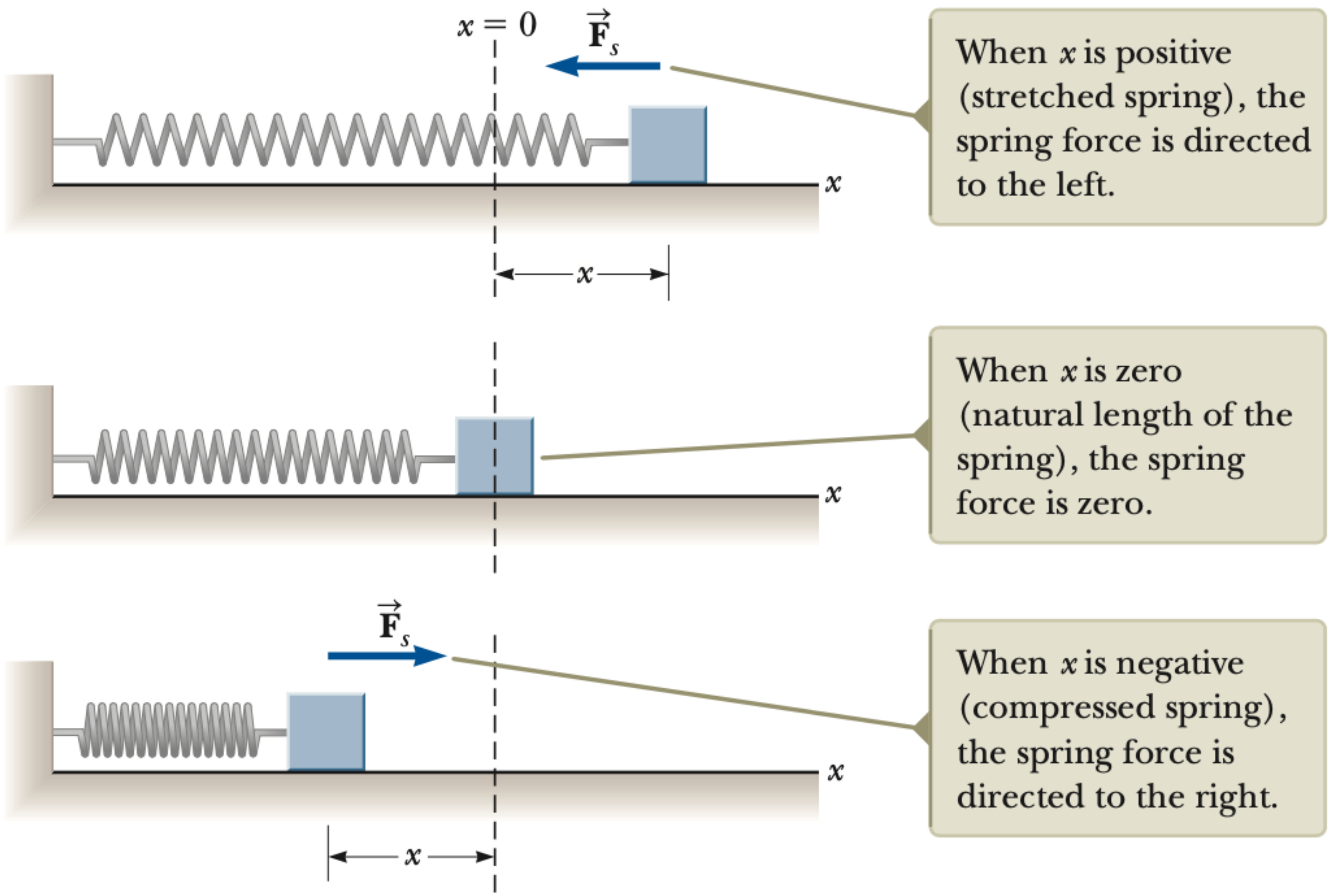
Work Done by a Spring

- A model of a common physical system on which the force varies with position.
- The system is a block on a frictionless, horizontal surface and connected to a spring.
- If the spring is either stretched or compressed a small distance from its unstretched (equilibrium) configuration, it exerts on the block a force that can be mathematically modeled as

$$F_s = -kx$$

k: force constant
or spring constant





x = 0
$\vec{F}_s$
x
x
When x is positive (stretched spring), the spring force is directed to the left.
x
When x is zero (natural length of the spring), the spring force is zero.
$\vec{F}_s$
x
x
When x is negative (compressed spring), the spring force is directed to the right.

This force law for springs is known as **Hooke's law.**

$$F_s = -kx$$

The vector form

$$F_s\hat{\mathbf{i}} = -kx\hat{\mathbf{i}}$$

The value of k is a measure of the *stiffness* of the spring.

The units of k are N/m.

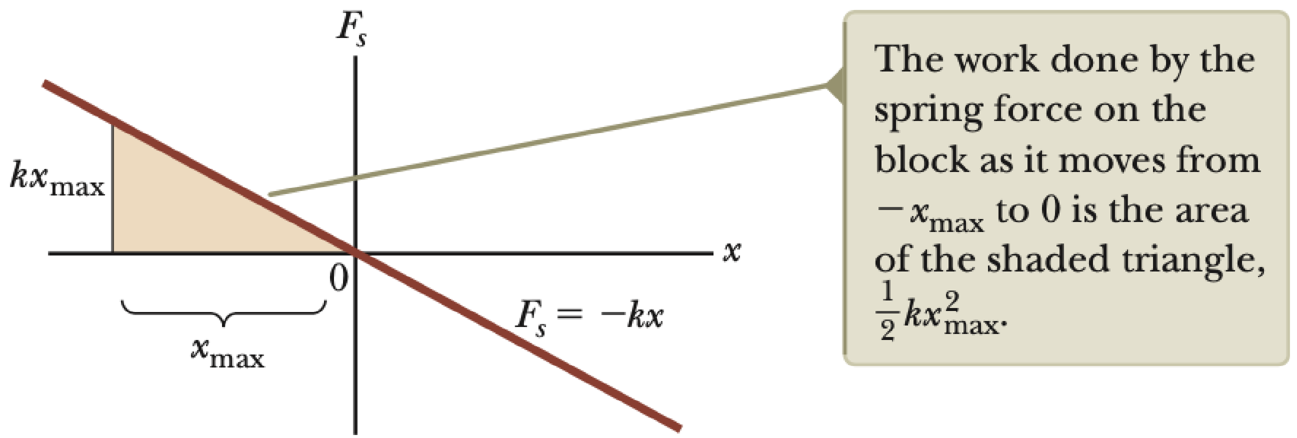
The negative sign means that the force exerted by the spring is always directed *opposite* the displacement from equilibrium.

If the spring is compressed until the block is at the point $–x_{\mathrm{max}}$ and is then released,

the block moves from $–x_{max}$ through zero to $+x_{max}$.

It then reverses direction, returns to $-x_{max}$, and continues oscillating back and forth.

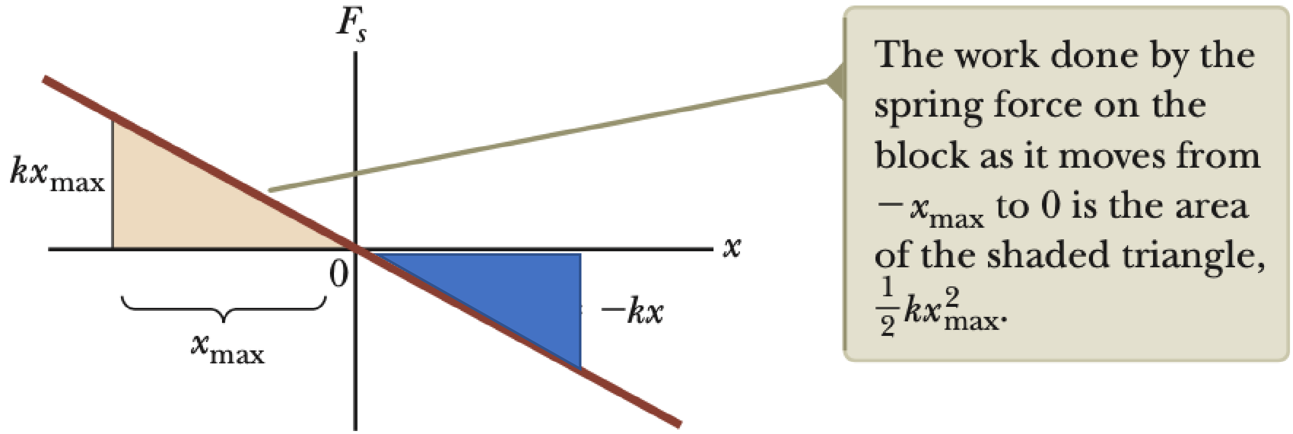
To calculate the work W_s done by the spring force on the block as the block moves from x_i = -x_{max} to x_f = 0.



$$W_s = \int_{x_i}^{x_f} \overrightarrow{\mathbf{F}_s} \cdot \Delta\vec{\boldsymbol{r}} = \int_{x_i}^{x_f} (F_s\hat{\mathbf{i}}) \cdot (dx\hat{\mathbf{i}}) = \int_{x_i}^{x_f} F_s\, dx = \int_{-x_{\max}}^{0} (-kx)\, dx = \frac{1}{2}kx_{max}^2$$

The work done by the spring force is positive because the force is in the same direction as its displacement (both are to the right).

The block arrives at $x = 0$ with some speed, and continue moving until it reaches $x = x_{max}$



$$W_s = \int_0^{x_{max}} (-kx)\,dx = -\frac{1}{2}kx_{max}^2$$

The work is negative because for this part of the motion the spring force is to the left and its displacement is to the right.

If the block undergoes an arbitrary displacement from $x = x_i$ to $x = x_f$, the work done by the spring force on the block is

$$W_s = \int_{x_i}^{x_f} (-kx)\, dx = \frac{1}{2} k x_i^2 - \frac{1}{2} k x_f^2$$

The work done by the spring on the block.

The work done by the spring force is zero for any motion that ends where it began ($x_i = x_f$).

The work done on the system by the external agent:

The *applied force* $\mathbf{F}_{app}$ is equal in magnitude and opposite in direction to the spring force $\mathbf{F}_s$,
The work done by this applied force on the system of the block is

$$W_{ext} = \int_{x_i}^{x_f} kx\,dx = \frac{1}{2}kx_f^2 - \frac{1}{2}kx_i^2$$

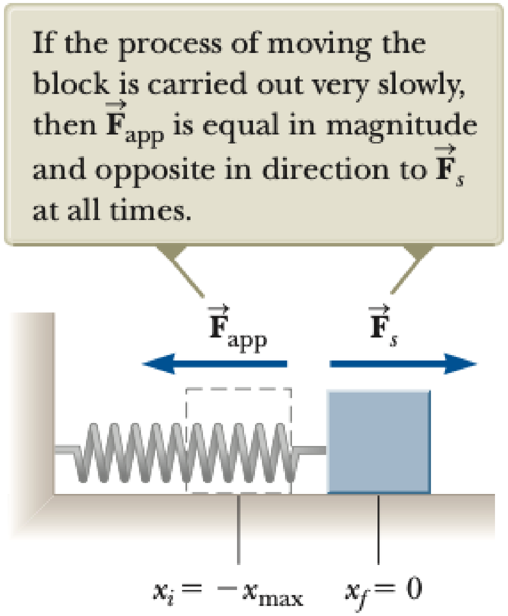


Figure 7.10 A block moves from $x_i = -x_{max}$ to $x_f = 0$ on a frictionless surface as a force $\vec{\mathbf{F}}_{app}$ is applied to the block.

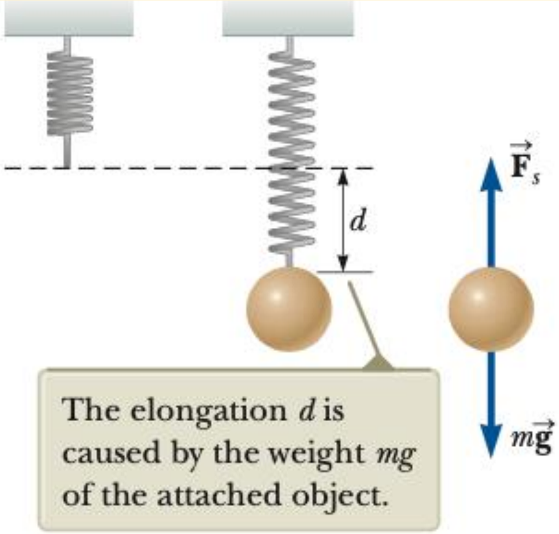
Example 7.6 Measuring k for a Spring

A common technique used to measure the force constant of a spring is demonstrated by the setup in Figure 7.12. The spring is hung vertically, and an object of mass *m* is attached to its lower end. Under the action of the "load" *mg*, the spring stretches a distance *d* from its equilibrium position.

(A) If a spring is stretched 2.0 cm by a suspended object having a mass of 0.55 kg, what is the force constant of the spring?

Conceptualize Figure shows what happens to the spring when the object is attached to it. Simulate this situation by hanging an object on a rubber band.

Categorize The object in Figure (b) is at rest and not accelerating, so it is modeled as a *particle in equilibrium.*



Analyze Because the object (the system) is in equilibrium, the net force on it is zero and the upward spring force balances the downward gravitational force $m\mathbf{g}$ (Fig. 7.11c).

Apply the particle in equilibrium model to the object:

$$\overrightarrow{\mathbf{F}}_s + m\vec{\mathbf{g}} = 0 \rightarrow \mathrm{F}_s - mg = 0 \rightarrow \mathrm{F}_s = mg$$

Apply Hooke's law to give $|\mathrm{F}_s| = k\,d$ and solve for k:

$$k = \frac{mg}{d} = \frac{(0.55\ \mathrm{kg})(9.80\ \mathrm{m/s^2})}{2.0 \times 10^{-2}\ \mathrm{m}} = \boxed{2.7 \times 10^{2}\ \mathrm{N/m}}$$

(B) How much work is done by the spring on the object as it stretches through this distance?

To find the work done by the spring on the object:

$$W_s = 0 - \frac{1}{2}kd^2 = -\frac{1}{2}(2.7 \times 10^2\ \text{N/m})(2.0 \times 10^{-2}\ \text{m})^2$$

$$= -5.4 \times 10^{-2}\ \text{J}$$

Finalize This work is negative because the spring force acts upward on the object, but its point of application (where the spring attaches to the object) moves downward. As the object moves through the 2.0-cm distance, the gravitational force also does work on it. This work is positive because the gravitational force is downward and so is the displacement of the point of application of this force. Would we expect the work done by the gravitational force, as the applied force in a direction opposite to the spring force, to be the negative of the answer above? Let's find out.

Evaluate the work done by the gravitational force on the object:

$$W = \vec{\mathbf{F}} \cdot \Delta\vec{\mathbf{r}} = (mg)(d)\cos 0 = mgd$$

$$= (0.55\ \text{kg})(9.80\ \text{m/s}^2)(2.0 \times 10^{-2}\ \text{m}) = 1.1 \times 10^{-1}\ \text{J}$$

7.5 Kinetic Energy and the Work–Kinetic Energy Theorem

Kinetic Energy

We have stated that **work** is an influence on a system from the environment, but we have not yet discussed the *result* of this influence on the system.

One possible result of doing work on a system is that the system changes its **speed**. This results an energy called ***kinetic energy***.

The net work done on the block by the external net force

$$W_{ext} = \int_{x_i}^{x_f} \sum F\, dx$$

$$\sum F = ma$$

$$W_{ext} = \int_{x_i}^{x_f} ma\, dx = \int_{x_i}^{x_f} m \frac{dv}{dt} dx = \int_{x_i}^{x_f} m \frac{dv}{dx}\frac{dx}{dt} dx = \int_{x_i}^{x_f} m \frac{dv}{\cancel{dx}} v\, \cancel{dx}$$

$$= \int_{x_i}^{x_f} mv\, dv$$

$$W_{ext} = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

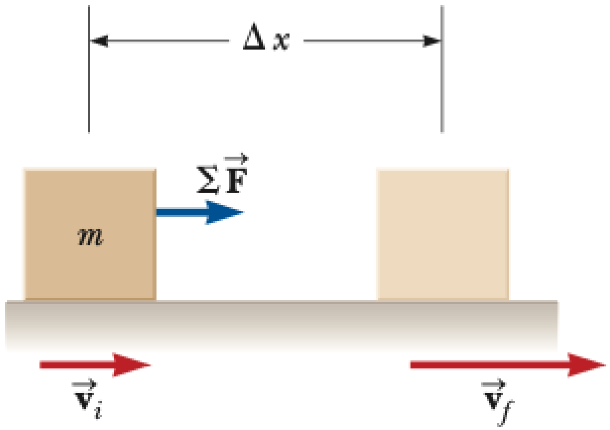


Figure 7.12 An object undergoing a displacement $\Delta\vec{r} = \Delta x\hat{i}$ and a change in velocity under the action of a constant net force $\sum \vec{F}$.

Kinetic Energy:

$$K = \frac{1}{2}mv^2$$

Kinetic energy represents the energy associated with the motion of the particle.

It is a scalar quantity and has the same units as work.

The work done on a particle by a net force acting on it equals the change in kinetic energy of the particle.

$$W_{ext} = K_f - K_i = \Delta K$$

The final kinetic energy of an object is equal to its initial kinetic energy plus the change in energy due to the net work done on it.

$$K_f = K_i + W_{ext}$$

$$W_{ext} = K_f - K_i = \Delta K$$

$$K = \frac{1}{2}mv^2$$

$$W_{ext} = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

Table 7.1

Kinetic Energies for Various Objects

Object	Mass (kg)	Speed (m/s)	Kinetic Energy (J)
Earth orbiting the Sun	5.98×10^{24}	2.98×10^{4}	2.66×10^{33}
Moon orbiting the Earth	7.35×10^{22}	1.02×10^{3}	3.82×10^{28}
Rocket moving at escape speed[a]	500	1.12×10^{4}	3.14×10^{10}
Automobile at 65 mi/h	2 000	29	8.4×10^{5}
Running athlete	70	10	3 500
Stone dropped from 10 m	1.0	14	98
Golf ball at terminal speed	0.046	44	45
Raindrop at terminal speed	3.5×10^{-5}	9.0	1.4×10^{-3}
Oxygen molecule in air	5.3×10^{-26}	500	6.6×10^{-21}

[a] Escape speed is the minimum speed an object must reach near the Earth's surface in order to move infinitely far away from the Earth.

The **work–kinetic energy theorem:**

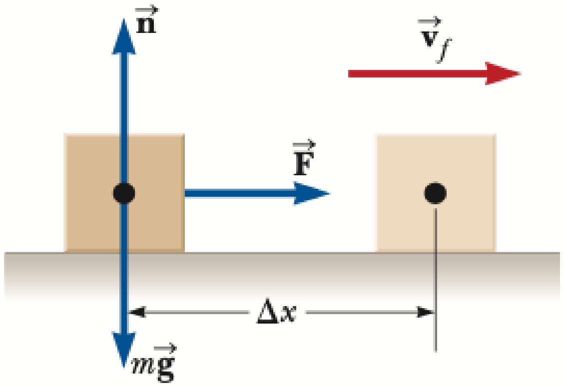
When work is done on a system and the only change in the system is in its speed, the net work done on the system equals the change in kinetic energy of the system, as expressed by Equation

$$W_{ext} = K_f - K_i = \Delta K$$

The speed of a system *increases* if the net work done on it is *positive* because the final kinetic energy is greater than the initial kinetic energy.

Example 7.7 A Block Pulled on a Frictionless Surface

A 6.0-kg block initially at rest is pulled to the right along a horizontal, frictionless surface by a constant horizontal force of 12 N. Find the speed of the block after it has moved 3.0 m.



The work done by this force is

$$W = F\Delta x = (12\,\mathrm{N})(3.0\,\mathrm{m}) = 36\,\mathrm{J}$$

Using the work–kinetic energy theorem and noting that the initial kinetic energy is zero, we obtain

$$W = K_f - K_i = \tfrac{1}{2}m{v_f}^2 - 0$$

$$v_f = \sqrt{\frac{2W}{m}} = \sqrt{\frac{2(36\,\mathrm{J})}{6.0\,\mathrm{kg}}} = 3.5\,\mathrm{m/s}$$

What if? Suppose the magnitude of the force in this example is doubled to $F' = 2F$. The 6.0-kg block accelerates to 3.5 m/s due to this applied force while moving through a displacement $\Delta x'$. How does the displacement $\Delta x'$ compare with the original displacement Δx?

Answer If we pull harder, the block should accelerate to a given speed in a shorter distance, so we expect that $\Delta x' < \Delta x$. In both cases, the block experiences the same change in kinetic energy ΔK. Mathematically, from the work– kinetic energy theorem, we find that

$$W_{ext} = F'\Delta x' = \Delta K = F\Delta x$$

$$\Delta x' = \frac{F}{F'}\Delta x = \frac{F}{2F}\Delta x = \frac{1}{2}\Delta x$$

and the distance is shorter as suggested by our conceptual argument.

Example 7.8 Does the Ramp Lessen the Work Required?

A man wishes to load a refrigerator onto a truck using a ramp at angle θ as shown in Figure 7.14. He claims that less work would be required to load the truck if the length L of the ramp were increased. Is his claim valid?

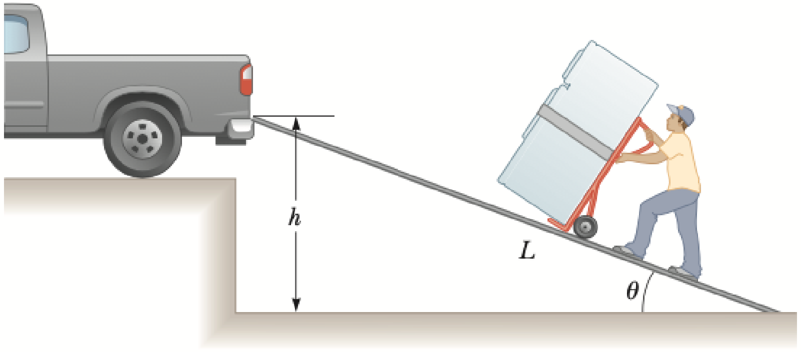


Figure 7.14 (Conceptual Example 7.7) A refrigerator attached to a frictionless, wheeled hand truck is moved up a ramp at constant speed.

SOLUTION

No. Suppose the refrigerator is wheeled on a hand truck up the ramp at constant speed. In this case, for the system of the refrigerator and the hand truck, $\Delta K = 0$. The normal force exerted by the ramp on the system is directed at 90° to the displacement of its point of application and so does no work on the system. Because $\Delta K = 0$, the work–kinetic energy theorem gives

$$W_{\text{ext}} = W_{\text{by man}} + W_{\text{by gravity}} = 0$$

The work done by the gravitational force equals the product of the weight mg of the system, the distance L through which the refrigerator is displaced, and $\cos(\theta + 90°)$. Therefore,

$$W_{\text{by man}} = -W_{\text{by gravity}} = -(mg)(L)[\cos(\theta + 90°)]$$
$$= mgL \sin\theta = mgh$$

where $h = L \sin\theta$ is the height of the ramp. Therefore, the man must do the same amount of work mgh on the system *regardless* of the length of the ramp. The work depends only on the height of the ramp. Although less force is required with a longer ramp, the point of application of that force moves through a greater displacement.

7.6 The Nonisolated System—Conservation of Energy

- If we choose the object as the system,
- for a *nonisolated system*, energy crosses the boundary of the system during some time interval due to an interaction with the environment.

Example, the work done by the external force (the interaction of the system with its environment), and the quantity in the system that changes is the kinetic energy.

Work, is a method of transferring energy to a system by applying a force to the system such that the point of application of the force undergoes a displacement.

Energy transfer mechanisms

Ways to transfer energy into/from a system:

Work,

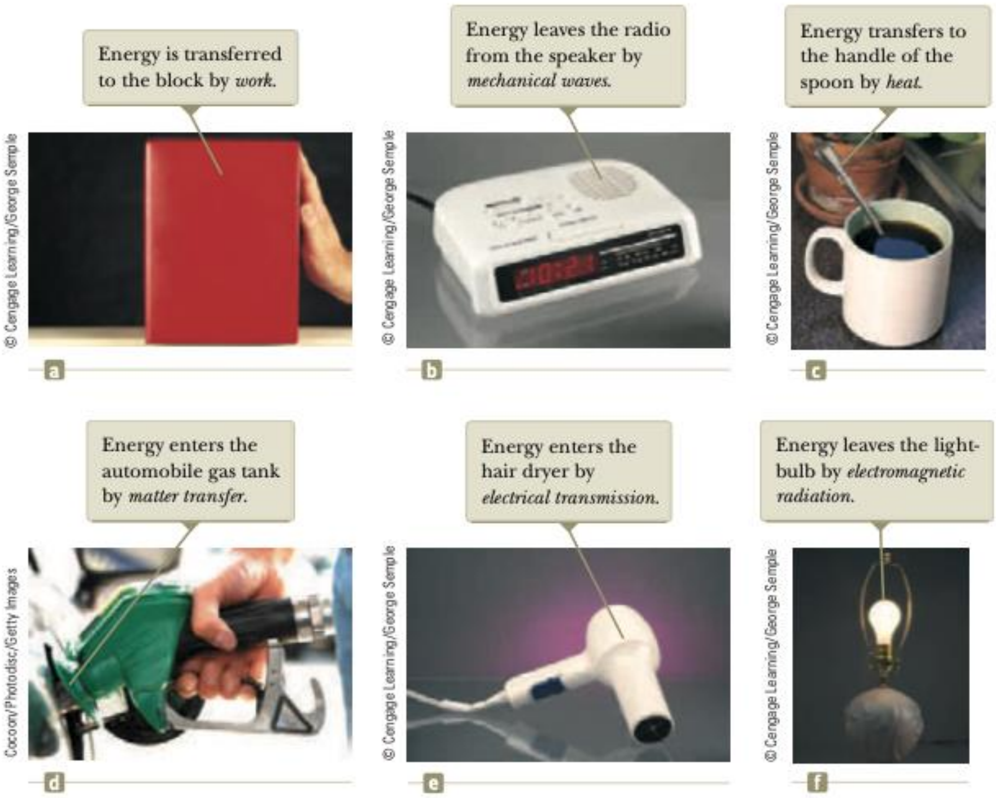
Mechanical waves,

Heat,

Matter transfer,

Electrical transmission,

Electromagnetic radiation.



Conservation of energy

We can neither create nor destroy energy, that energy is always ***conserved***.

If the total amount of energy in a system **changes**, it can *only* be **because energy has crossed the boundary** of the system by a transfer mechanism such as one of the methods listed above.

- Energy is one of several quantities in physics that are conserved.
- We will see other conserved quantities in subsequent chapters.
- There are many physical quantities that do not obey a conservation principle.

Conservation of energy

The general statement of the principle of **conservation of energy** can be described as

$$\Delta E_{system} = \Sigma\, T$$

The total energy of the system, including all methods of energy storage (kinetic, potential, and internal)

The amount of energy transferred across the system boundary by some mechanism

$$\Delta K + \Delta U + \Delta E_{int} = W + Q + T_{MW} + T_{MT} + T_{ET} + T_{ER}$$

If, for a given system, all terms on the right side of the conservation of energy equation are zero, the system is an *isolated system,* which we study in the next section.

Example, suppose a force is applied to a nonisolated system and the point of application of the force moves through a displacement. Then suppose the only effect on the system is to change its speed.

Then, **conservation of energy** Equation becomes

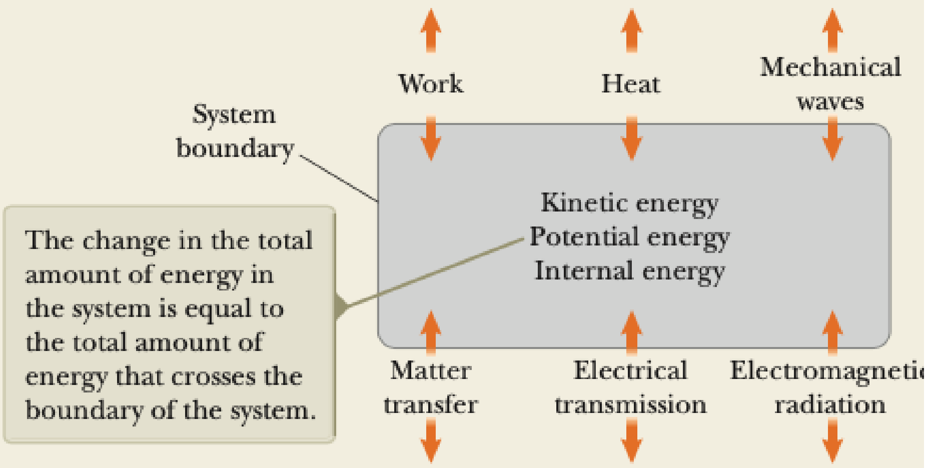
$$\Delta K = W$$

which is the work–kinetic energy theorem. This theorem is a special case of the more general principle of conservation of energy.

Analysis Model Nonisolated System (Energy)

Imagine you have identified a system to be analyzed and have defined a system boundary. Energy can exist in the system in three forms: kinetic, potential, and internal. The total of that energy can be changed when energy crosses the system boundary by any of six transfer methods shown in the diagram here. The total change in the energy in the system is equal to the total amount of energy that has crossed the system boundary. The mathematical statement of that concept is expressed in the **conservation of energy equation**:

$$\Delta E_{\text{system}} = \Sigma\, T \qquad \textbf{(8.1)}$$



The full expansion of Equation 8.1 shows the specific types of energy storage and transfer:

$$\Delta K + \Delta U + \Delta E_{\text{int}} = W + Q + T_{\text{MW}} + T_{\text{MT}} + T_{\text{ET}} + T_{\text{ER}} \qquad \textbf{(8.2)}$$

For a specific problem, this equation is generally reduced to a smaller number of terms by eliminating the terms that are equal to zero because they are not appropriate to the situation.

Examples:

- a force does work on a system of a single object, changing its speed: the work–kinetic energy theorem, $W = \Delta K$
- a gas contained in a vessel has work done on it and experiences a transfer of energy by heat, resulting in a change in its temperature: the first law of thermodynamics, $\Delta E_{\text{int}} = W + Q$ (Chapter 20)
- an incandescent light bulb is turned on, with energy entering the filament by electricity, causing its temperature to increase, and leaving by light: $\Delta E_{\text{int}} = T_{\text{ET}} + T_{\text{ER}}$ (Chapter 27)
- a photon enters a metal, causing an electron to be ejected from the metal: the photoelectric effect, $\Delta K + \Delta U = T_{\text{ER}}$ (Chapter 40)

7.7 Situations Involving Kinetic Friction

- Consider the book sliding across the surface in the preceding section. As the book moves through a distance d, the only force in the horizontal direction is the force of kinetic friction. This force causes a change $-f_k d$ in the kinetic energy of the book.
- The change in kinetic energy is equal to the work done by all forces other than friction minus a term $f_k d$ associated with the friction force.

$$\Sigma W_{\text{other forces}} - f_k d = \Delta K$$

$$\Delta E_{\text{int}} = f_k d$$

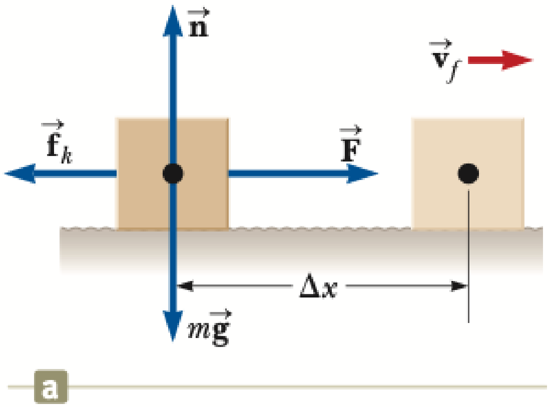
$$\Sigma W_{\text{other forces}} = W = \Delta K + \Delta E_{\text{int}}$$

Example 7.9 A Block Pulled on a Rough Surface

A 6.0-kg block initially at rest is pulled to the right along a horizontal surface by a constant horizontal force of 12 N.

(A) Find the speed of the block after it has moved 3.0 m if the surfaces in contact have a coefficient of kinetic friction of 0.15.

Conceptualize The rough surface applies a friction force on the block opposite to the applied force.



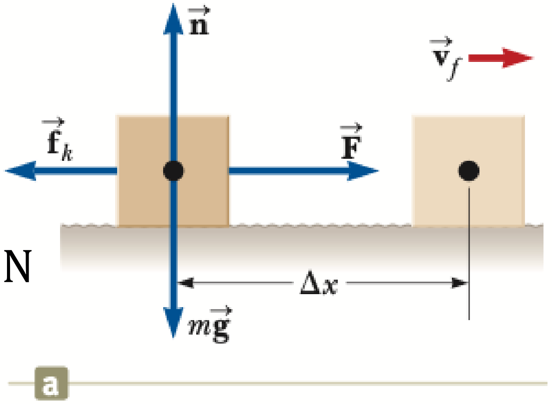
Categorize The block is pulled by a force and the surface is rough, so the block and the surface are modeled as a *nonisolated system* with a nonconservative force acting.

Analyze Neither the normal force nor the gravitational force does work on the system because their points of application are displaced horizontally.

$$W = \Delta K + \Delta E_{\text{int}}$$

Find the work done on the system by the applied force

$$W = F\Delta x = (12\text{ N})(3.0\text{ m}) = 36\text{J}$$



The magnitude of the friction force:

$$f_k = \mu_k n = \mu_k mg = (0.15)(6.0\text{ kg})(9.80\text{ m/s}^2) = 8.82\text{ N}$$

The change in kinetic energy of the block due to friction is

$$\Delta K = -f_k d = -(8.82\text{ N})(3.0\text{ m}) = -26.5\text{ J}$$

The final speed of the block follows from Equation $\Sigma W_{\text{other forces}} - f_k d = \Delta K$

$$\left(\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2\right) = -f_k d + \Sigma\, W_{\text{other force}}$$

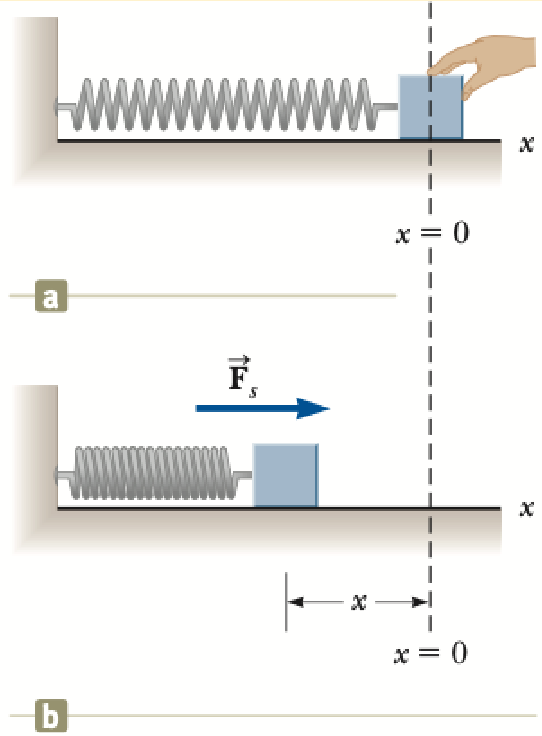
$$v_f = \sqrt{\frac{2}{m}(-f_k \text{d} + F\Delta x)} = 1.8\text{ m/s}$$

Finalize As expected, this value is less than the 3.5 m/s found in the case of the block sliding on a frictionless surface (Example 7.7).

Example 7.11 A Block—Spring System

A block of mass 1.6 kg is attached to a horizontal spring that has a force constant of 1 000 N/m, as shown in the Figure. The spring is compressed 2.0 cm and is then released from rest.

(A) Calculate the speed of the block as it passes through the equilibrium position $x = 0$ if the surface is frictionless.



Conceptualize it is easy to visualize the block being pushed to the right by the spring and moving with some speed at $x = 0$.

Categorize We identify the system as the block and model the block as a *nonisolated system*.

Analyze In this situation, the block starts with $v_i = 0$ at $x_i = -2.0$ cm, and we want to find v_f at $x_f = 0$.

The work done by the spring on the system with $x_{max} = x_i$

$$W_s = \frac{1}{2} k x_{max}^2$$

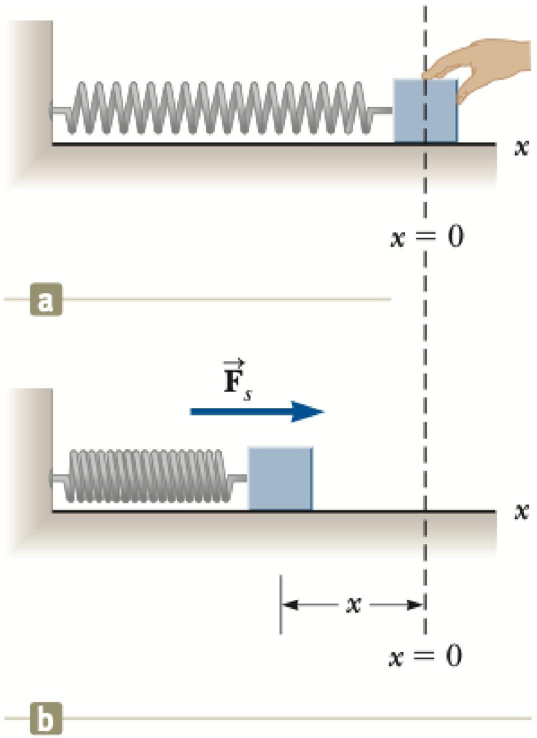
The change in kinetic energy of the block equal to the work done on it by the spring

$$W_s = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2$$

$$v_f = \sqrt{v_i^2 + \frac{2}{m} W_s} = \sqrt{v_i^2 + \frac{2}{m}\left(\frac{1}{2} k x_{max}^2\right)}$$

$$v_f = \sqrt{0 + \frac{2}{1.6\ \mathrm{kg}}\left[\tfrac{1}{2}(1\,000\ \mathrm{N/m})(0.020\ \mathrm{m})^2\right]}$$

$$= 0.50\ \mathrm{m/s}$$



(B) Calculate the speed of the block as it passes through the equilibrium position if a constant friction force of 4.0 N retards its motion from the moment it is released.

Conceptualize The correct answer must be less than that found in part (A) because the friction force retards the motion.

Categorize We identify the system as the block and the surface, a *nonisolated system* because of the work done by the spring. There is a nonconservative force acting within the system: the friction between the block and the surface.

$$W_s = \Delta K + \Delta E_{\text{int}} = \left(\frac{1}{2}mv_f^2 - 0\right) + f_k d$$

$$v_f = \sqrt{\frac{2}{m}(W_s - f_k \mathrm{d})}$$

$$v_f = \sqrt{\frac{2}{m}\left(\frac{1}{2}kx_{max}^2 - f_k \mathrm{d}\right)}$$

Finalize this value is less than the 0.50 m/s found in part (A).

$$v_f = \sqrt{\frac{2}{1.6\text{ kg}}\left[\tfrac{1}{2}(1\,000\text{ N/m})(0.020\text{ m})^2 - (4.0\text{ N})(0.020\text{ m})\right]} = 0.39\text{ m/s}$$

7.8 Power

The time rate of energy transfer is called the **instantaneous power** P and is defined as

$$P \equiv \frac{dE}{dt}$$

The notion of power is valid for *any* means of energy transfer.

- We will focus on work as the energy transfer method.
- If an external force is applied to an object and if the work done by this force on the object in the time interval Δt is W, the **average power** during this interval is

$$P_{\text{avg}} = \frac{W}{\Delta t}$$

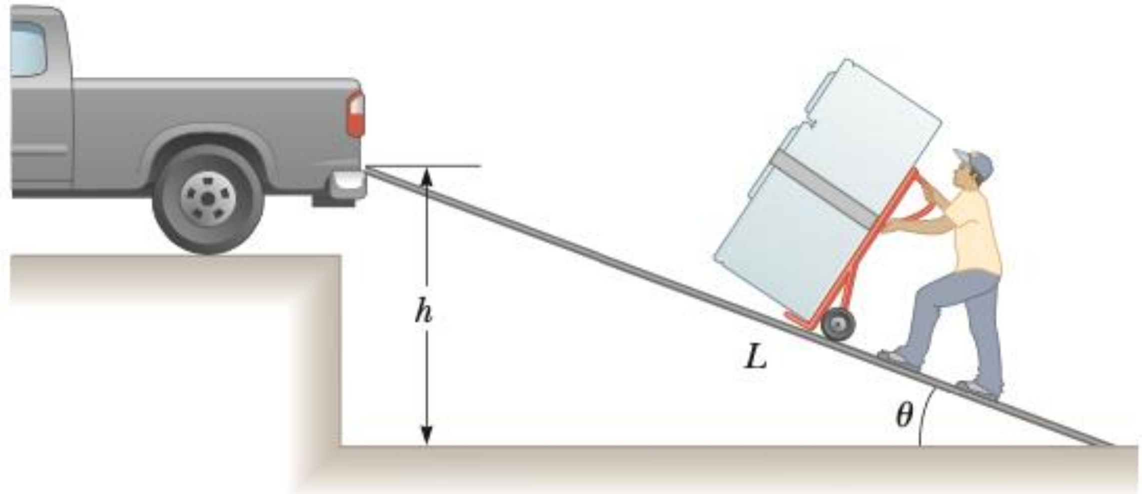


Figure 7.14 (Conceptual Example 7.7) A refrigerator attached to a frictionless, wheeled hand truck is moved up a ramp at constant speed.

The man must do the same amount of work *mgh* on the system *regardless* of the length of the ramp. The work depends only on the height of the ramp. Although less force is required with a longer ramp, the point of application of that force moves through a greater displacement.

Although the same work is done in rolling the refrigerator up both ramps, less power is required for the longer ramp.

The instantaneous power is the limiting value of the average power as Δt approaches zero:

$$P = \lim_{\Delta t \to 0} \frac{W}{\Delta t} = \frac{dW}{dt}$$

$$dW = \vec{\boldsymbol{F}} \cdot d\vec{\boldsymbol{r}}$$

$$\mathrm{P} = \frac{dW}{dt} = \vec{\boldsymbol{F}} \cdot \frac{d\vec{\boldsymbol{r}}}{dt} = \vec{\boldsymbol{F}} \cdot \vec{\mathrm{v}}$$

The SI unit of power is joules per second (J/s), also called the **watt** (W) after James Watt:

$$1\ \text{W} = 1\ \text{J/s} = 1\ \text{kg} \cdot \text{m}^2/\text{s}^3$$

A unit of power in the U.S. customary system is the **horsepower** (hp):

1 hp = 746 W

A unit of energy (or work) can be defined in terms of the unit of power.

One **kilowatt-hour** (kWh) is the energy transferred in 1 h at the constant rate of 1 kW = 1 000 J/s.

The amount of energy represented by 1 kWh is

1 kWh = $(10^3$ W)(3 600 s) = 3.60×10^6 J

A kilowatt-hour is a unit of energy, not power. When you pay your electric bill, you are buying energy, and the amount of energy transferred by electrical transmission into a home during the period represented by the electric bill is usually expressed in kilowatt-hours.

For example, your bill may state that you used 900 kWh of energy during a month and that you are being charged at the rate of 10¢ per kilowatt-hour. Your obligation is then $90 for this amount of energy.

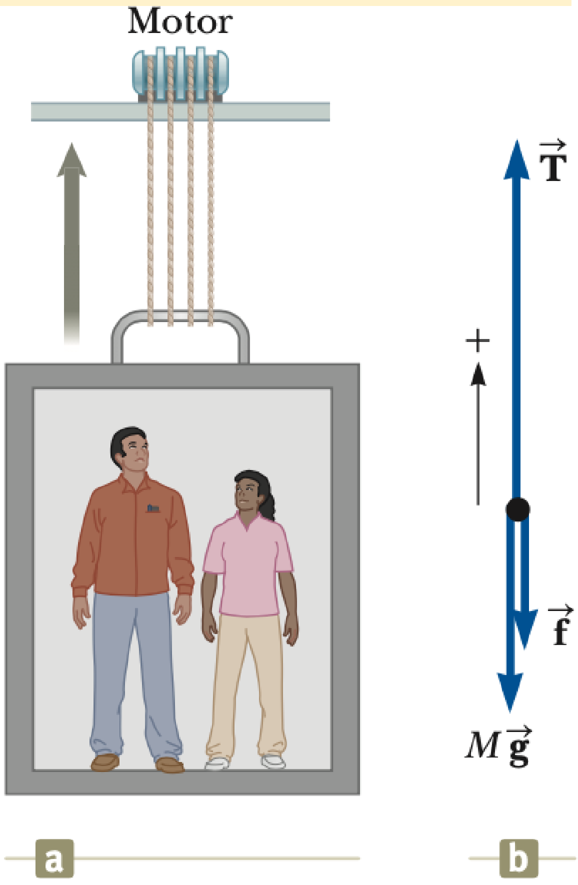
As another example, suppose an electric bulb is rated at 100 W. In 1.00 h of operation, it would have energy transferred to it by electrical transmission in the amount of

$$(0.100 \text{ kW})(1.00 \text{ h}) = 0.100 \text{ kWh} = 3.60 \times 10^5 \text{ J}.$$

Example 7.12 Power Delivered by an Elevator Motor

An elevator car has a mass of 1 600 kg and is carrying passengers having a combined mass of 200 kg. A constant friction force of 4 000 N retards its motion upward, as shown in the Figure.

(A) How much power must a motor deliver to lift the elevator car and its passengers at a constant speed of 3.00 m/s?



Conceptualize The motor must supply the force of magnitude *T* that pulls the elevator car upward.

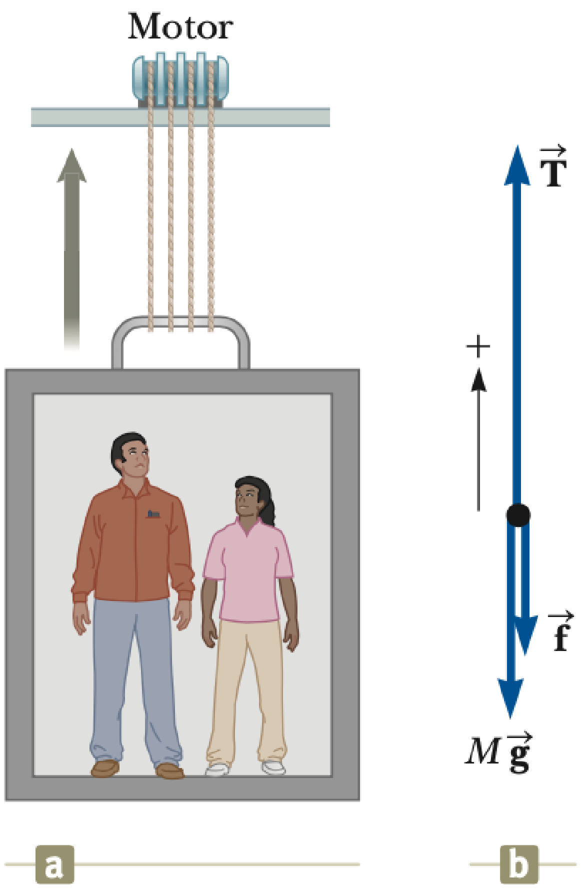
Categorize The friction force increases the power necessary to lift the elevator. The problem states that the speed of the elevator is constant, which tells us that $a = 0$. We model the elevator as a *particle in equilibrium*.

$$\sum F_y = T - f - Mg = 0$$

The *total* mass *M* of the system is equal to 1 800 kg.

$$T = Mg + f$$

$$P = \vec{\mathbf{T}} \cdot \vec{\mathbf{v}} = Tv = (Mg + f)v$$



$$P = [(1\ 800\ \text{kg})(9.80\ \text{m/s}^2) + (4\ 000\ \text{N})](3.00\ \text{m/s})$$

$$= 6.49 \times 10^4\ \text{W}$$

(B) What power must the motor deliver at the instant the speed of the elevator is v if the motor is designed to provide the elevator car with an upward acceleration of 1.00 m/s^2?

$$\sum F_y = T - f - Mg = Ma$$

$$T = M(a + g) + f$$

$$P = Tv = [M(a + g) + f]v$$

$$P = [(1\,800 \text{ kg})(1.00 \text{ m/s}^2 + 9.80 \text{ m/s}^2) + 4\,000 \text{ N}]v$$

$$= (2.34 \times 10^4)v$$

Finalize To compare with part (A), let $v = 3.00$ m/s, giving a power of

$$P = (2.34 \times 10^4 \text{ N})(3.00 \text{ m/s}) = 7.02 \times 10^4 \text{ W}$$

which is larger than the power found in part (A), as expected.

Suggested Problems from Chapter 7

Problems: 1, 4, 7, 13, 14, 15, 16, 19, 21, 24, 25, 26, 28, 31, 32, 33, 35, 37, 40