104 PHYS

Ch. 25

Electric Potential

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Contents

sec. 25.01 Potential Difference and Electric Potential

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- When a test charge q_0 is placed in an electric field $\vec{E}$ created by some source charge distribution, the electric force $\vec{F}_e$ acting on the test charge is $q_0\vec{E}$.
- The force $q_0\vec{E}$ is conservative because the force between charges described by Coulomb's law is conservative.
- For an infinitesimal displacement $d\vec{s}$ of a charge, the work done by the electric field on the charge is:

$$dW = \vec{F}_e \cdot d\vec{s} = q_0\, \vec{E} \cdot d\vec{s}$$

- As this amount of work is done by the field, the potential energy of the charge–field system is changed by an amount:

$$dU = -dW = -q_0\, \vec{E} \cdot d\vec{s}$$

- For a finite displacement of the charge from point A to point B, the change in potential energy of the system:

$$\Delta U = U_B - U_A = -q_0 \int_A^B \vec{E} \cdot d\vec{s}$$

- The integration is performed along the path that q_0 follows as it moves from A to B. Because the force $q_0\vec{E}$ is conservative, this line integral does not depend on the path taken from A to B.

Processes occurring during thunderstorms cause large differences in electric potential between a thundercloud and the ground. The result of this potential difference is an electrical discharge that we call lightning, such as this display over Tucson, Arizona.

Potential Difference and Electric Potential

- The electric potential V (or simply the potential) is the potential energy per unit charge.

$$V = \frac{U}{q_0}$$

- The electric potential is independent of the value of q_0 and has a value at every point in an electric field.
- The electric potential is a scalar quantity and has the units of $\mathrm{J/C}$, where $1\ \mathrm{J/C} = 1\ \mathrm{V}$.
- The potential difference $\Delta V = V_B - V_A$ between two points A and B in an electric field is defined as the change in potential energy of the system when a test charge is moved between the points divided by the test charge q_0:

$$\Delta V = \frac{\Delta U}{q_0} = -\int_A^B \vec{E} \cdot d\vec{s}$$

- The potential difference between A and B depends only on the source charge distribution (consider points A and B without the presence of the test charge), while the difference in potential energy exists only if a test charge is moved between the points.

Processes occurring during thunderstorms cause large differences in electric potential between a thundercloud and the ground. The result of this potential difference is an electrical discharge that we call lightning, such as this display over Tucson, Arizona.

Potential Difference and Electric Potential

- Because electric potential is a measure of potential energy per unit charge, the SI unit of both electric potential and potential difference (V) is joules per coulomb:

$$1\ \mathrm{V} = 1\ \frac{\mathrm{J}}{\mathrm{C}}$$

- The SI unit of electric field (N/C) can also be expressed in volts per meter:

$$1\ \frac{\mathrm{N}}{\mathrm{C}} = 1\ \frac{\mathrm{V}}{\mathrm{m}}$$

- A unit of energy commonly used in atomic and nuclear physics is the electron volt (eV), which is defined as the energy a charge–field system gains or loses when a charge of magnitude e (that is, an electron or a proton) is moved through a potential difference of 1 V.

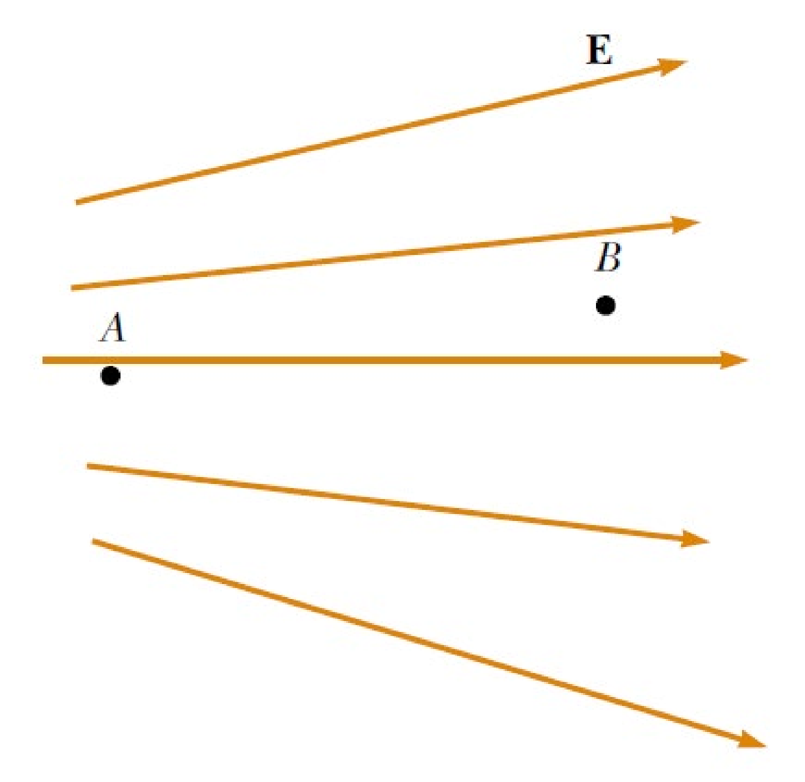
$$1\ \mathrm{eV} = 1.60 \times 10^{-19}\ \mathrm{C \cdot V} = 1.60 \times 10^{-19}\ \mathrm{J}$$

Processes occurring during thunderstorms cause large differences in electric potential between a thundercloud and the ground. The result of this potential difference is an electrical discharge that we call lightning, such as this display over Tucson, Arizona.

Quick Quiz 25.1 In the following figure, two points A and B are located within a region in which there is an electric field. The potential difference $\Delta V = V_B - V_A$ is:

(a) positive (b) negative (c) zero.

Answer: (b). When moving straight from A to B , $\vec{E}$ and $d\vec{s}$ both point toward the right. Thus, the dot product $\vec{E} \cdot d\vec{s}$ is positive and ΔV is negative.



Two points in an electric field.

Quick Quiz 25.2 In the following figure, a negative charge is placed at A and then moved to B. The change in potential energy of the charge–field system for this process is:

(a) positive (b) negative (c) zero.

Answer: (a). From the equation, $\Delta U = q_0 \Delta V$, so if a negative test charge is moved through a negative potential difference, the potential energy is positive. Work must be done to move the charge in the direction opposite to the electric force on it.

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Example 25.01

How much work is done (by a battery, generator, or some other source of potential difference) in moving Avogadro's number of electrons from an initial point where the electric potential is 9.00 V to a point where the potential is −5.00 V? (The potential in each case is measured relative to a common reference point.)

$$\Delta V = -14.0 \text{ V}$$

$$Q = -N_A e = -\left(6.02 \times 10^{23} \times 1.60 \times 10^{-19}\right) = -9.63 \times 10^{4} \text{ C}$$

$$W = Q\Delta V = \left(-9.63 \times 10^{4}\right) \times (-14.0) = 1.35 \times 10^{6} \text{ J}$$

Problem 25.02

Additional problem

A positive ion accelerated through a potential difference of 115 V experiences an increase in kinetic energy of 7.37×10^{-17} J. Calculate the charge on the ion.

The ion–field system is isolated, so the mechanical energy of the system is conserved:

$$\Delta K + \Delta U = 0$$

$$\Delta K + q\Delta V = 0$$

$$q = -\frac{\Delta K}{\Delta V}$$

$$q = -\frac{7.37 \times 10^{-17}}{(-115)}$$

$$q = 6.41 \times 10^{-19}\ \text{C}$$

Problem 25.03

(a) Calculate the speed of a proton that is accelerated from rest through a potential difference of 120 V. (b) Calculate the speed of an electron that is accelerated through the same potential difference.

(a) Energy of the proton-field system is conserved as the proton moves from high to low potential, which can be defined for this problem as moving from 120 V down to 0 V.

$$\Delta K + \Delta U = 0$$

$$\left(\frac{1}{2}m{v_p}^2 - 0\right) + q_p\Delta V = 0$$

$$v_p = \sqrt{\frac{-(2q_p\Delta V)}{m_p}}$$

$$v_p = \sqrt{\frac{-2 \times 1.60 \times 10^{-19} \times (-120)}{1.67 \times 10^{-27}}}$$

$$v_p = 1.52 \times 10^5 \text{ m/s}$$

Problem 25.03

(a) Calculate the speed of a proton that is accelerated from rest through a potential difference of 120 V. (b) Calculate the speed of an electron that is accelerated through the same potential difference.

(b) The electron will gain speed in moving the other way, from $V_i = 0$ V to $V_f = 120$ V:

$$\Delta K + \Delta U = 0$$

$$\left(\frac{1}{2} m v_e{}^2 - 0\right) + q_e \Delta V = 0$$

$$v_e = \sqrt{\frac{-(2q_e\Delta V)}{e}}$$

$$v_e = \sqrt{\frac{-2 \times (-1.60 \times 10^{-19}) \times 120}{9.11 \times 10^{-31}}}$$

$$v_e = 6.49 \times 10^6 \text{ m/s}$$

Potential Differences in a Uniform Electric Field

- In a uniform electric field, the potential difference between two points A and B separated by a distance $|\vec{s}| = d$, where $\vec{s}$ is parallel to the field lines, is given by:

$$V_B - V_A = \Delta V = -\int_A^B \vec{E} \cdot d\vec{s} = -\int_A^B (E\cos 0°)\, ds = -\int_A^B E \cdot ds$$

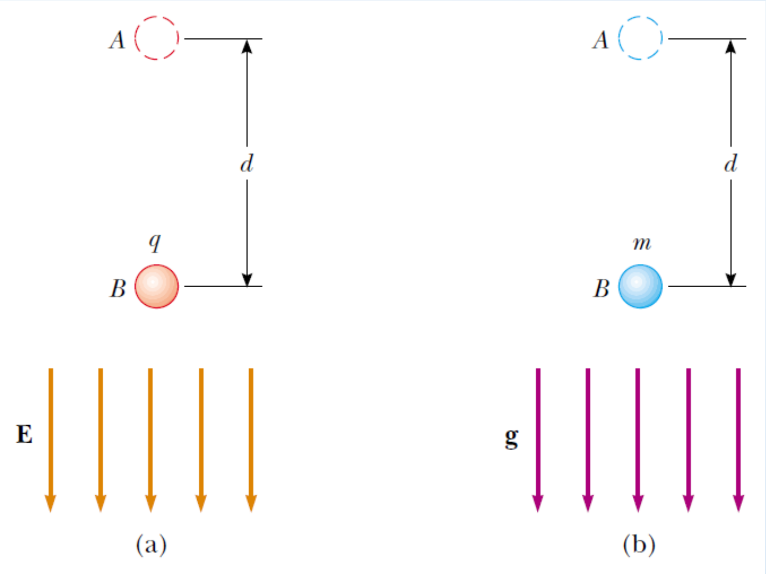
- Because E is constant, we can remove it from the integral sign; this gives:

$$\Delta V = -E\int_A^B ds = -Ed$$

- The negative sign indicates that the electric potential at point B is lower than at point A; that is, $V_B < V_A$. Electric field lines always point in the direction of decreasing electric potential.

- Now suppose that a test charge q_0 moves from A to B. We can calculate the change in the potential energy of the charge–field system:

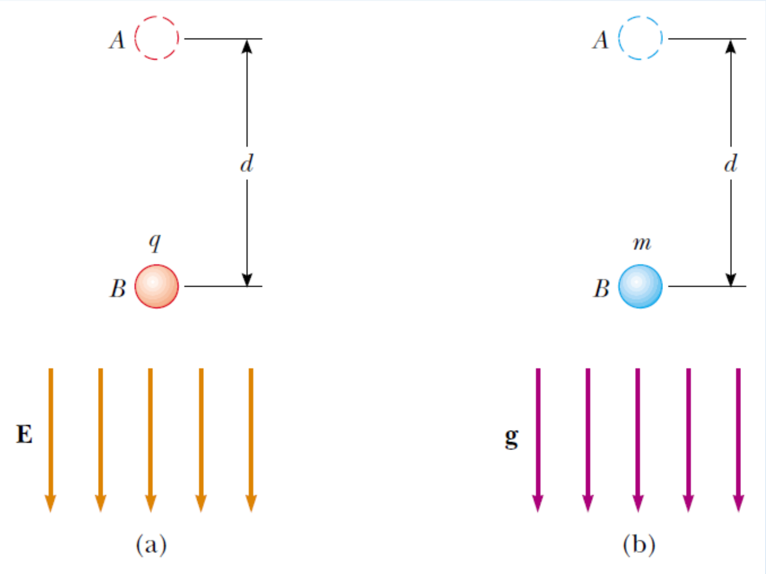
$$\Delta U = q_0 \Delta V = -q_0 Ed$$



(a) When the electric field $\vec{E}$ is directed downward, point B is at a lower electric potential than point A. When a positive test charge moves from point A to point B, the charge–field system loses electric potential energy. (b) When an object of mass m moves downward in the direction of the gravitational field g, the object–field system loses gravitational potential energy.

Potential Differences in a Uniform Electric Field

- If q_0 is positive, then ΔU is negative. We conclude that a system consisting of a positive charge and an electric field loses electric potential energy when the charge moves in the direction of the field.
- This means that an electric field does work on a positive charge when the charge moves in the direction of the electric field.
- If a positive test charge is released from rest in this electric field, it experiences an electric force $q_0\vec{E}$ in the direction of $\vec{E}$(downward in Fig. (a)).
- Therefore, it accelerates downward, gaining kinetic energy. As the charged particle gains kinetic energy, the charge–field system loses an equal amount of potential energy.
- If q_0 is negative, then ΔU is positive and the situation is reversed: A system consisting of a negative charge and an electric field gains electric potential energy when the charge moves in the direction of the field.
- If a negative charge is released from rest in an electric field, it accelerates in a direction opposite the direction of the field. In order for the negative charge to move in the direction of the field, an external agent must apply a force and do positive work on the charge.



(a) When the electric field $\vec{E}$ is directed downward, point B is at a lower electric potential than point A. When a positive test charge moves from point A to point B, the charge–field system loses electric potential energy. (b) When an object of mass m moves downward in the direction of the gravitational field g, the object–field system loses gravitational potential energy.

Potential Differences in a Uniform Electric Field

- Now consider the more general case of a charged particle that moves between A and B in a uniform electric field such that the vector $\vec{s}$ is not parallel to the field lines,

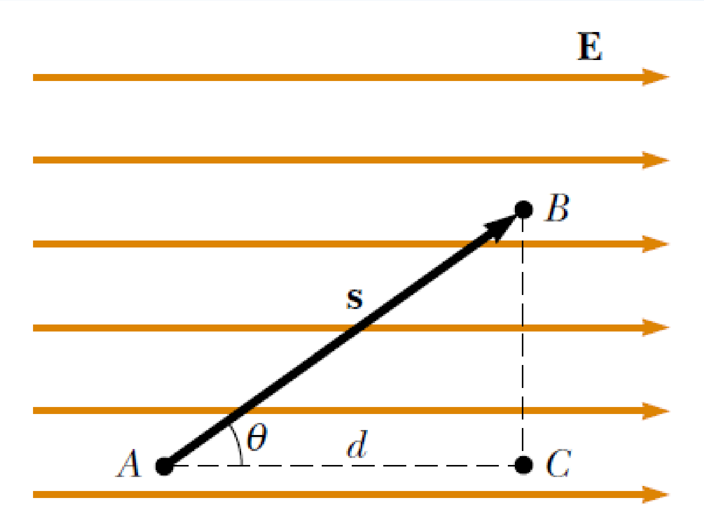
$$\Delta V = -\int_A^B \vec{E}\cdot d\vec{s} = -\vec{E}\int_A^B d\vec{s} = -\vec{E}\cdot\vec{s}$$

- We are able to remove $\vec{E}$ from the integral because it is constant. The change in potential energy of the charge–field system is:

$$\Delta U = q_0\Delta V = -q_0\,\vec{E}\cdot\vec{s}$$

- All points in a plane perpendicular to a uniform electric field are at the same electric potential (the potential difference $V_B - V_A$ is equal to the potential difference $V_c - V_A \Rightarrow V_B = V_c$).

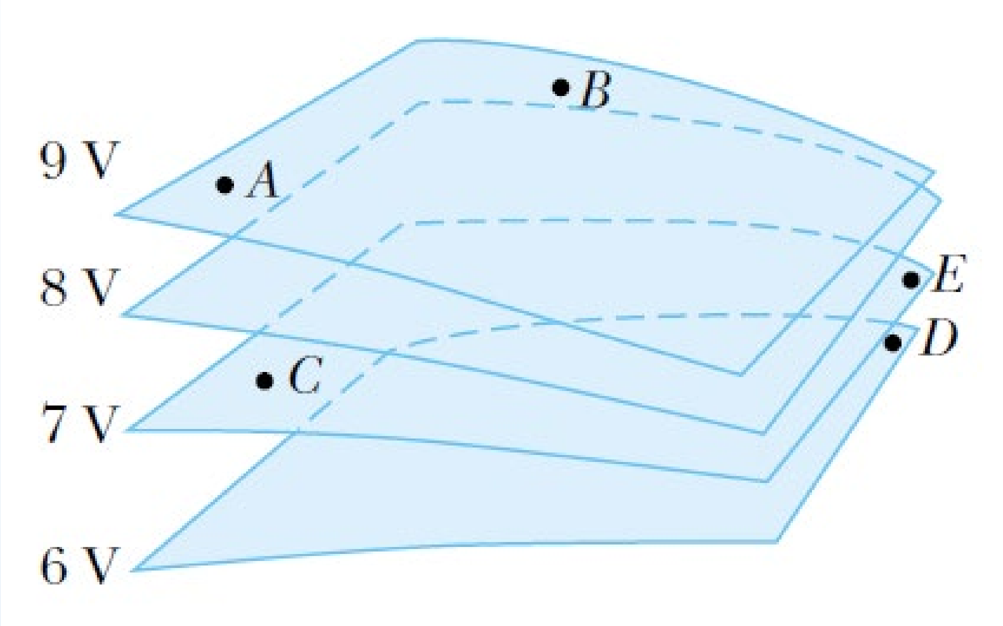
- <u>The name equipotential surface is given to any surface consisting of a continuous distribution of points having the same electric potential</u>.



A uniform electric field directed along the positive x axis. Point B is at a lower electric potential than point A. Points B and C are at the same electric potential.

Quick Quiz 25.3 The labeled points in the following figure are on a series of equipotential surfaces associated with an electric field. Rank (from greatest to least) the work done by the electric field on a positively charged particle that moves from A to B; from B to C; from C to D; from D to E.

Answer: $B \rightarrow C$, $C \rightarrow D$, $A \rightarrow B$, $D \rightarrow E$. Moving from B to C decreases the electric potential by 2 V, so the electric field performs 2 J of work on each coulomb of positive charge that moves. Moving from C to D decreases the electric potential by 1 V, so 1 J of work is done by the field. It takes no work to move the charge from A to B because the electric potential does not change. Moving from D to E increases the electric potential by 1 V, and thus the field does -1 J of work per unit of positive charge that moves.



Four equipotential surfaces.

Quick Quiz 25.4 For the equipotential surfaces in the following figure, what is the approximate direction of the electric field? (a) Out of the page (b) Into the page (c) Toward the right edge of the page (d) Toward the left edge of the page (e) Toward the top of the page (f) Toward the bottom of the page.

Answer: (f). The electric field points in the direction of decreasing electric potential.

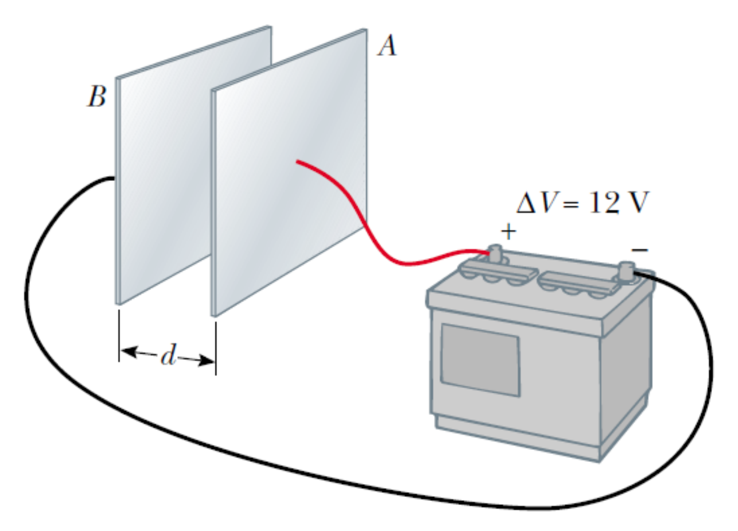
Example 25.01

A battery produces a specified potential difference ΔV between conductors attached to the battery terminals. A 12 V battery is connected between two parallel plates, as shown in the following figure. The separation between the plates is $d = 0.30$ cm, and we assume the electric field between the plates to be uniform. (This assumption is reasonable if the plate separation is small relative to the plate dimensions and if we do not consider locations near the plate edges.) Find the magnitude of the electric field between the plates.

The electric field is directed from the positive plate (A) to the negative one (B), and the positive plate is at a higher electric potential than the negative plate is. The potential difference between the plates must equal the potential difference between the battery terminals. We can understand this by noting that all points on a conductor in equilibrium are at the same electric potential (the electric field vanishes within a conductor in electrostatic equilibrium; thus, the path integral between any two points in the conductor must be zero); no potential difference exists between a terminal and any portion of the plate to which it is connected. Therefore, the magnitude of the electric field between the plates is:

$$E = -\frac{V_B - V_A}{d} = -\frac{-12}{0.30 \times 10^{-2}} = 4.0 \times 10^3 \text{ V/m}$$

The configuration of plates in the following figure is called a parallel-plate capacitor, and is examined in greater detail in Chapter 26.



A 12 V battery connected to two parallel plates. The electric field between the plates has a magnitude given by the potential difference ΔV divided by the plate separation d.

Example 25.02

A proton is released from rest in a uniform electric field that has a magnitude of 8.0×10^4 V/m (see the figure below). The proton undergoes a displacement of 0.50 m in the direction of $\vec{E}$. (a) Find the change in electric potential between points A and B. (b) Find the change in potential energy of the proton–field system for this displacement. (c) Find the speed of the proton after completing the 0.50 m displacement in the electric field.

(a) Because the positively charged proton moves in the direction of the field, we expect it to move to a position of lower electric potential.

$$\Delta V = -Ed = -8.0 \times 10^4 \times 0.50 = -4.0 \times 10^4 \text{ V}$$

(b)

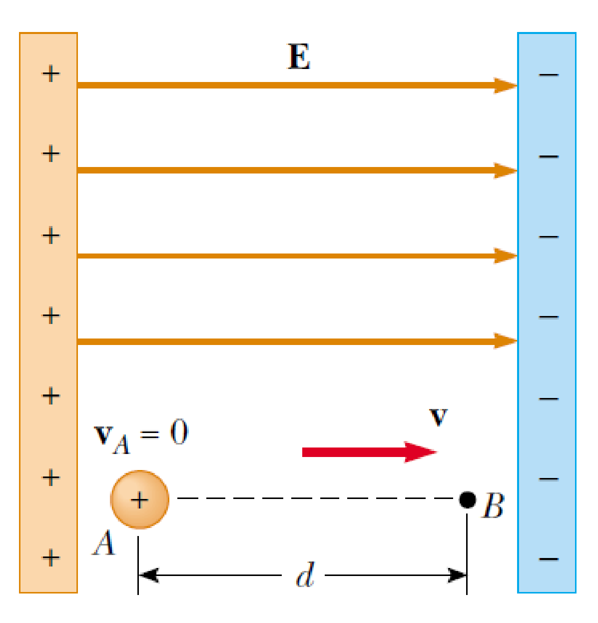
$$\Delta U = q_0 \Delta V$$

$$\Delta U = e\Delta V$$

$$\Delta U = 1.6 \times 10^{-19} \times \left(-4.0 \times 10^4\right)$$

$$\Delta U = -6.4 \times 10^{-15} \text{ J}$$

The negative sign means the potential energy of the system decreases as the proton moves in the direction of the electric field. As the proton accelerates in the direction of the field, it gains kinetic energy and at the same time the system loses electric potential energy.



A proton accelerates from A to B in the direction of the electric field.

Example 25.02

A proton is released from rest in a uniform electric field that has a magnitude of 8.0×10^4 V/m (see the figure below). The proton undergoes a displacement of 0.50 m in the direction of $\vec{E}$. (a) Find the change in electric potential between points A and B. (b) Find the change in potential energy of the proton–field system for this displacement. (c) Find the speed of the proton after completing the 0.50 m displacement in the electric field.

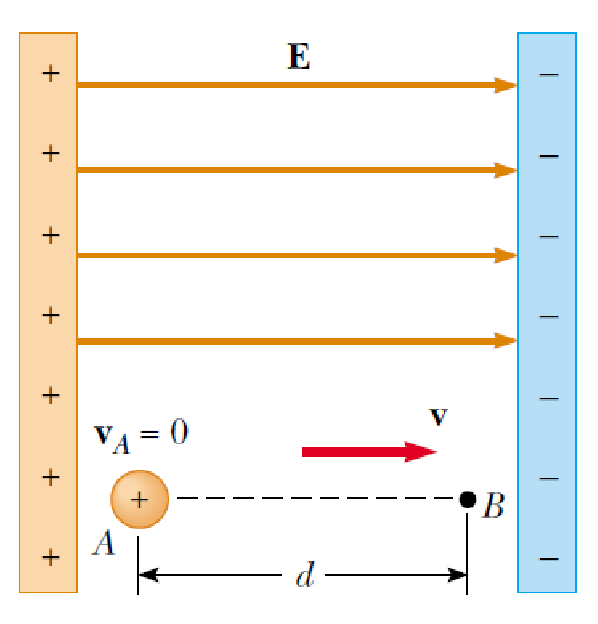
(c) The charge–field system is isolated, so the mechanical energy of the system is conserved:

$$\Delta K + \Delta U = 0$$

$$\left(\frac{1}{2}mv^2 - 0\right) + e\Delta V = 0$$

$$v = \sqrt{\frac{-(2e\Delta V)}{m}}$$

$$v = \sqrt{\frac{-2 \times 1.6 \times 10^{-19} \times (-4.0 \times 10^4)}{1.67 \times 10^{-27}}}$$



A proton accelerates from A to B in the direction of the electric field.

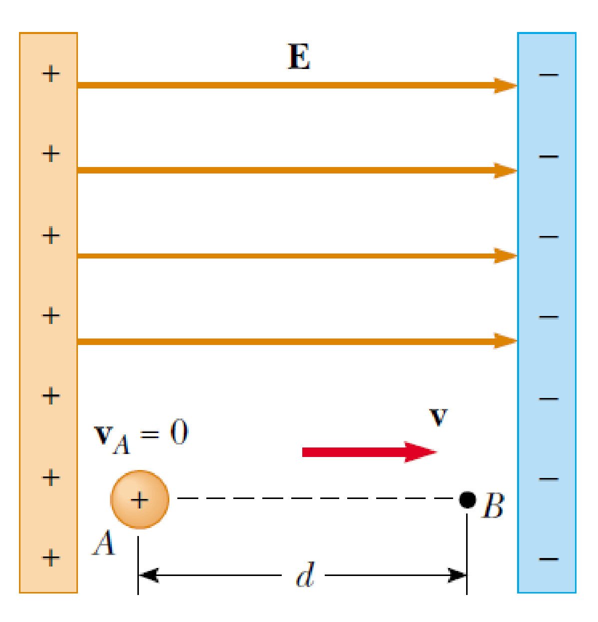
Example 25.02

A proton is released from rest in a uniform electric field that has a magnitude of 8.0×10^4 V/m (see the figure below). The proton undergoes a displacement of 0.50 m in the direction of $\vec{E}$. (a) Find the change in electric potential between points A and B. (b) Find the change in potential energy of the proton–field system for this displacement. (c) Find the speed of the proton after completing the 0.50 m displacement in the electric field.

$$v = 2.8 \times 10^6 \text{ m/s}$$

What If? What if the situation is exactly the same as that shown in the following figure, but no proton is present? Could both parts (a) and (b) of this example still be answered?

Part (a) of the example would remain exactly the same because the potential difference between points A and B is established by the source charges in the parallel plates. The potential difference does not depend on the presence of the proton, which plays the role of a test charge. Part (b) of the example would be meaningless if the proton is not present. A change in potential energy is related to a change in the charge–field system. In the absence of the proton, the system of the electric field alone does not change.



A proton accelerates from A to B in the direction of the electric field.

Problem 25.05

A uniform electric field of magnitude 250 V/m is directed in the positive x direction. A $+12.0\ \mu\text{C}$ charge moves from the origin to the point $(x, y) = (20.0\text{ cm}, 50.0\text{ cm})$. (a) What is the change in the potential energy of the charge–field system? (b) Through what potential difference does the charge move?

(a) We follow the path from $(0, 0)$ to $(20.0\text{ cm}, 0)$ to $(20.0\text{ cm}, 50.0\text{ cm})$.

$$\Delta U = -(\text{work done})$$

$$\Delta U = -\left(\text{work from origin to } (20.0\text{ cm}, 0)\right) - \left(\text{work from } (20.0\text{ cm}, 0) \text{ to } (20.0\text{ cm}, 50.0\text{ cm})\right)$$

Note that the last term is equal to 0 because the force is perpendicular to the displacement.

$$\Delta U = -(qE_x)\Delta x = -\left(12.0 \times 10^{-6} \times 250 \times 0.200\right) = -6.00 \times 10^{-4}\text{ J}$$

(b) $$\Delta V = \frac{\Delta U}{q} = \frac{-6.00 \times 10^{-4}}{12.0 \times 10^{-6}} = -50.0\text{ V}$$

Problem 25.06

Additional problem

The difference in potential between the accelerating plates in the electron gun of a TV picture tube is about 25000 V. If the distance between these plates is 1.50 cm, what is the magnitude of the uniform electric field in this region?

$$\Delta V = -Ed$$

$$E = -\frac{\Delta V}{d}$$

$$E = -\frac{25.0 \times 10^{3}}{1.50 \times 10^{-2}}$$

$$E = -1.67 \times 10^{6}\ \mathrm{N/C}$$

Electric Potential and Potential Energy Due to Point Charges

- An isolated positive point charge q produces an electric field that is directed radially outward from the charge.
- To find the electric potential at a point located a distance r from the charge, we begin with the general expression for potential difference between two arbitrary points A and B:

$$V_B - V_A = -\int_A^B \vec{E} \cdot d\vec{s}$$

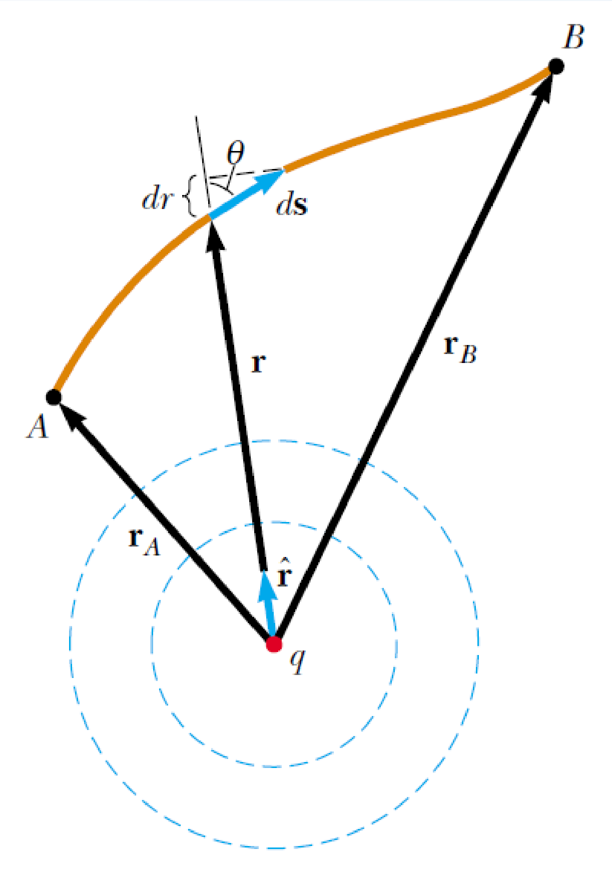
- At any point in space, the electric field due to the charge is $\vec{E} = k_e\, q\hat{r}/r^2$. The quantity $\vec{E} \cdot d\vec{s}$ can be expressed as:

$$\vec{E} \cdot d\vec{s} = k_e \frac{q}{r^2} \hat{r} \cdot d\vec{s} = k_e \frac{q}{r^2} \cos\theta\, ds = k_e \frac{q}{r^2} dr$$

$$V_B - V_A = -k_e q \int_{r_A}^{r_B} \frac{dr}{r^2} = \left. \frac{k_e q}{r} \right]_{r_A}^{r_B}$$

$$V_B - V_A = k_e q \left[\frac{1}{r_B} - \frac{1}{r_A} \right]$$

- The potential difference between any two points A point and B in a field created by a point charge depends only on the radial coordinates r_A and r_B.



The potential difference between points A and B due to a point charge q depends <u>only</u> on the initial and final radial coordinates r_A and r_B . The two dashed circles represent intersections of spherical equipotential surfaces with the page.

Electric Potential and Potential Energy Due to Point Charges

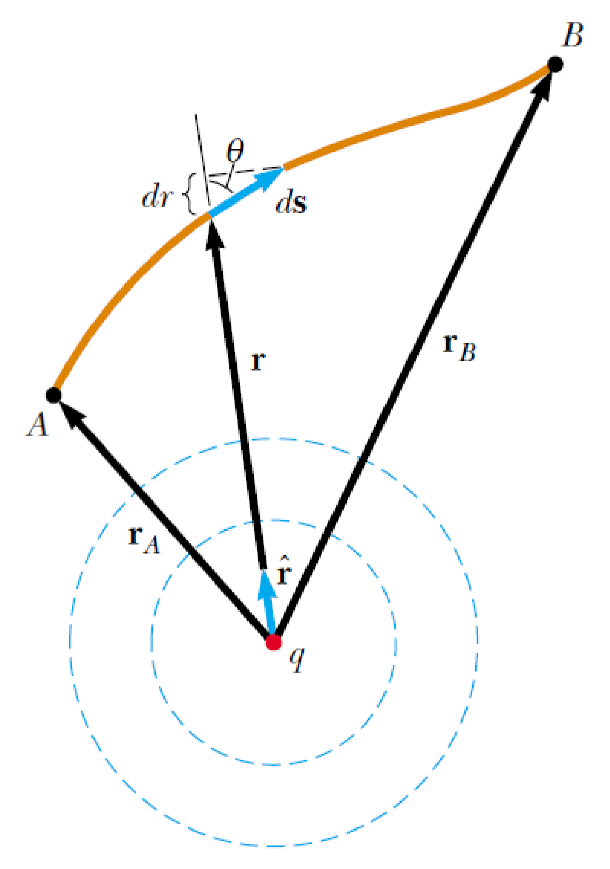
- It is customary to choose the reference of electric potential for a point charge to be $V_A = 0$ at $r_A = \infty$. With this reference choice, the electric potential created by a point charge at any distance r from the charge is:

$$V = k_e \frac{q}{r}$$

- We obtain the electric potential resulting from two or more point charges by applying the superposition principle.
- For a group of point charges, we can write the total electric potential at P in the form:

$$V = k_e \sum_i \frac{q_i}{r_i}$$

- The potential is taken to be zero at infinity ($V_\infty = 0$).
- Note that the sum in the previous Eq. is an algebraic sum of scalars rather than a vector sum. Thus, it is often much easier to evaluate V than to evaluate $\vec{E}$.



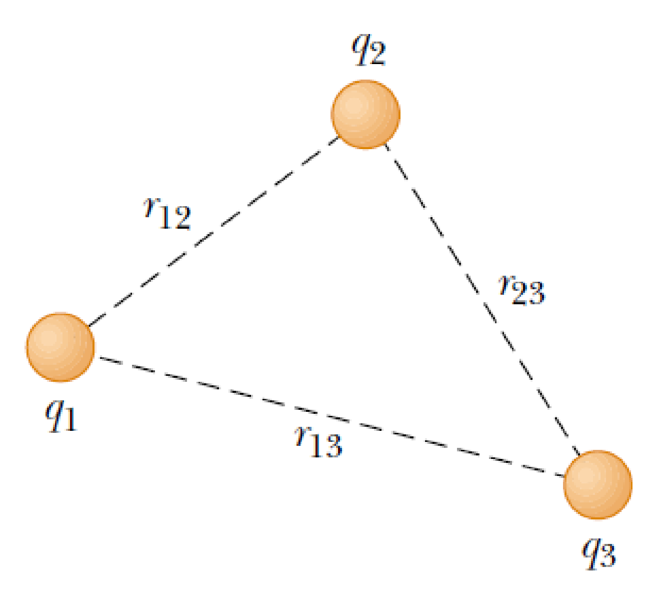
The potential difference between points A and B due to a point charge q depends <u>only</u> on the initial and final radial coordinates r_A and r_B. The two dashed circles represent intersections of spherical equipotential surfaces with the page.

- We can express the potential energy of the system of two charged particles as:

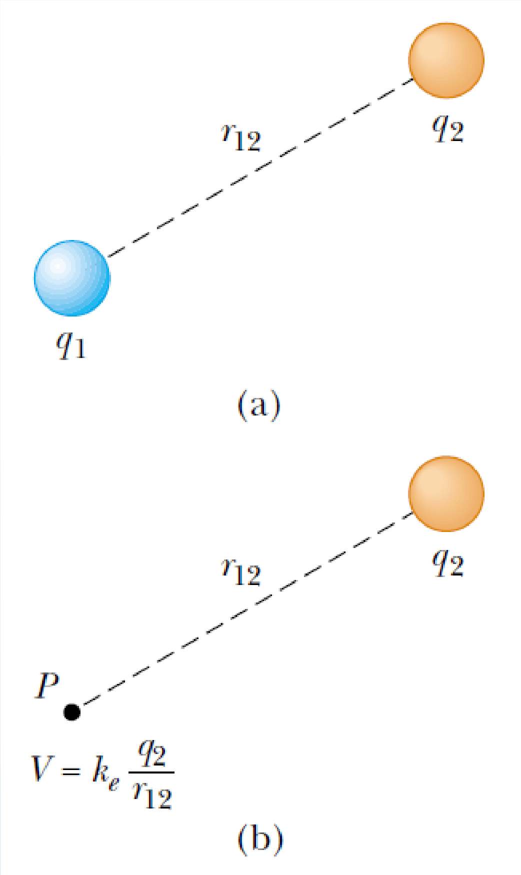
$$U = k_e \frac{q_1 q_2}{r_{12}}$$

- The total potential energy of the system of three charges:

$$U = k_e \left(\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}} \right)$$



Three point charges are fixed at the positions shown. The potential energy of this system of charges is given by the previous Eq.



(a) If two point charges are separated by a distance r_{12}, the potential energy of the pair of charges is given by $k_e q_1 q_2 / r_{12}$. (b) If charge q_1 is removed, a potential $k_e q_2 / r_{12}$ exists at point P due to charge q_2.

Example 25.03

A charge $q_1 = 2.00$ μC is located at the origin, and a charge $q_2 = -6.00$ μC is located at $(0, 3.00)$ m , as shown in Fig. (a). (a) Find the total electric potential due to these charges at the point P, whose coordinates are $(4.00, 0)$ m . (b) Find the change in potential energy of the system of two charges plus a charge $q_3 = 3.00$ μC as the latter charge moves from infinity to point P (Fig. (b)).

(a) For two charges, the total electric potential at the point P:

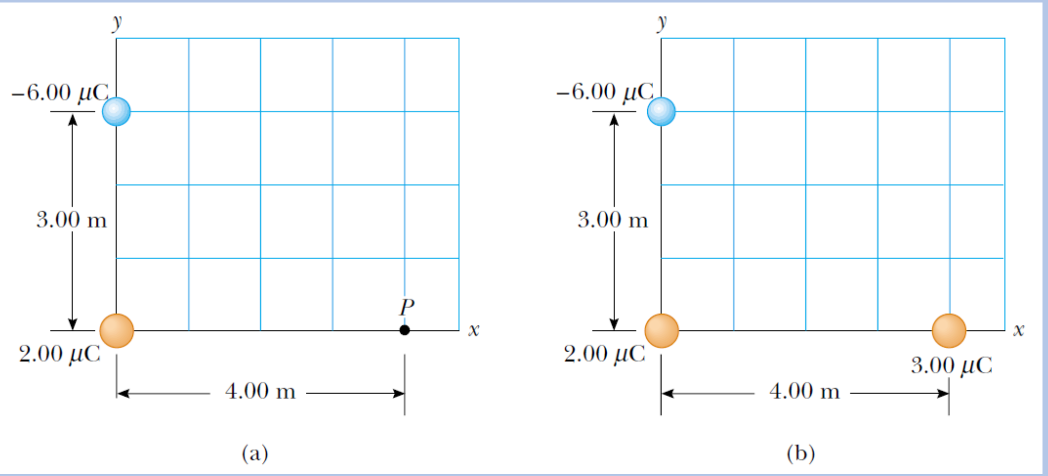
$$V_P = k_e\left(\frac{q_1}{r_1} + \frac{q_2}{r_2}\right)$$

$$V_P = 9 \times 10^9 \times \left(\frac{2.00 \times 10^{-6}}{4.00} - \frac{6.00 \times 10^{-6}}{5.00}\right)$$

$$V_P = -6.29 \times 10^3 \text{ V}$$

(b) When the charge q_3 is at infinity, let us define $U_i = 0$ for the system, and when the charge is at P , $U_f = q_3 V_P$; therefore:

$$\Delta U = q_3 V_P - 0 = 3.00 \times 10^{-6} \times (-6.29 \times 10^3) = -1.89 \times 10^{-2} \text{ J}$$



(a) The electric potential at P due to the two charges q_1 and q_2 is the algebraic sum of the potentials due to the individual charges. (b) A third charge $q_3 = 3.00$ μC is brought from infinity to a position near the other charges.

Example 25.03

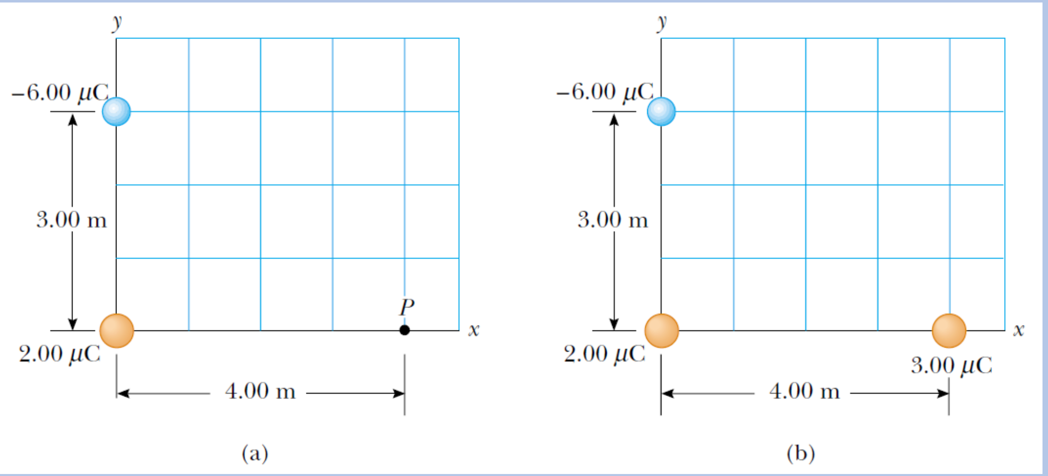
A charge $q_1 = 2.00\ \mu C$ is located at the origin, and a charge $q_2 = -6.00\ \mu C$ is located at $(0, 3.00)$ m , as shown in Fig. (a). (a) Find the total electric potential due to these charges at the point P, whose coordinates are $(4.00, 0)$ m . (b) Find the change in potential energy of the system of two charges plus a charge $q_3 = 3.00\ \mu C$ as the latter charge moves from infinity to point P (Fig. (b)).

Therefore, because the potential energy of the system has decreased, positive work would have to be done by an external agent to remove the charge from point P back to infinity.

What If? You are working through this example with a classmate and he says, "Wait a minute! In part (b), we ignored the potential energy associated with the pair of charges q_1 and q_2!" How would you respond?

Given the statement of the problem, it is not necessary to include this potential energy, because part (b) asks for the change in potential energy of the system as q_3 is brought in from infinity. Because the configuration of charges q_1 and q_2 does not change in the process, there is no ΔU associated with these charges. However, if part (b) had asked to find the change in potential energy when all three charges start out infinitely far apart and are then brought to the positions in Fig. (b), we would need to calculate the change as follows:

$$U = k_e\left(\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}}\right)$$



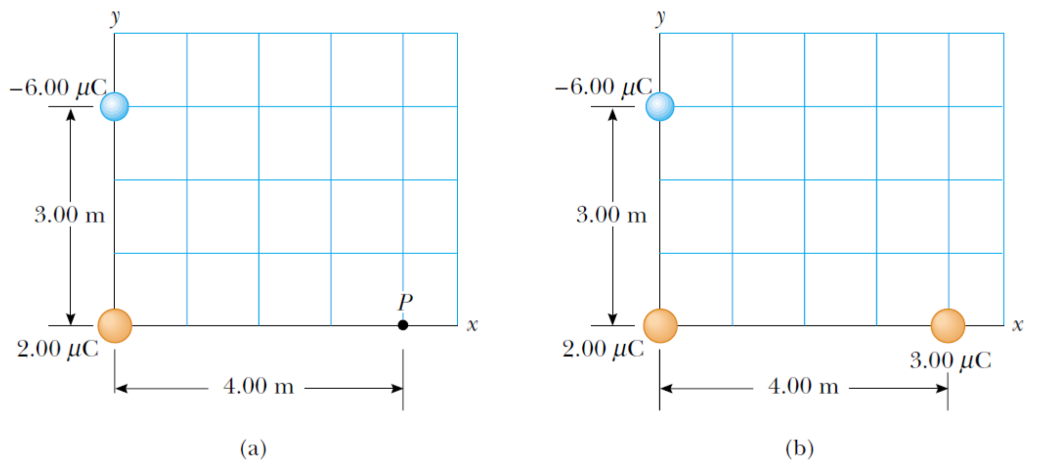
(a) The electric potential at P due to the two charges q_1 and q_2 is the algebraic sum of the potentials due to the individual charges. (b) A third charge $q_3 = 3.00\ \mu C$ is brought from infinity to a position near the other charges.

Example 25.03

A charge $q_1 = 2.00\ \mu\text{C}$ is located at the origin, and a charge $q_2 = -6.00\ \mu\text{C}$ is located at $(0, 3.00)$ m , as shown in Fig. (a). (a) Find the total electric potential due to these charges at the point P, whose coordinates are $(4.00, 0)$ m . (b) Find the change in potential energy of the system of two charges plus a charge $q_3 = 3.00\ \mu\text{C}$ as the latter charge moves from infinity to point P (Fig. (b)).

$$U = 9 \times 10^9 \times \left(\frac{2.00 \times 10^{-6} \times (-6.00 \times 10^{-6})}{3.00} + \frac{2.00 \times 10^{-6} \times 3.00 \times 10^{-6}}{4.00} + \frac{(-6.00 \times 10^{-6}) \times 3.00 \times 10^{-6}}{5.00} \right)$$

$$U = -5.48 \times 10^{-2}\ \text{J}$$



(a) The electric potential at P due to the two charges q_1 and q_2 is the algebraic sum of the potentials due to the individual charges. (b) A third charge $q_3 = 3.00\ \mu\text{C}$ is brought from infinity to a position near the other charges.

Problem 25.16

Additional problem

Given two 2.00 μC charges, as shown in the following figure, and a positive test charge $q = 1.28 \times 10^{-18}$ C at the origin, (a) what is the net force exerted by the two 2.00 μC charges on the test charge q? (b) What is the electric field at the origin due to the two 2.00 μC charges? (c) What is the electric potential at the origin due to the two 2.00 μC charges?

(a) Since the charges are equal and placed symmetrically, $F = 0$

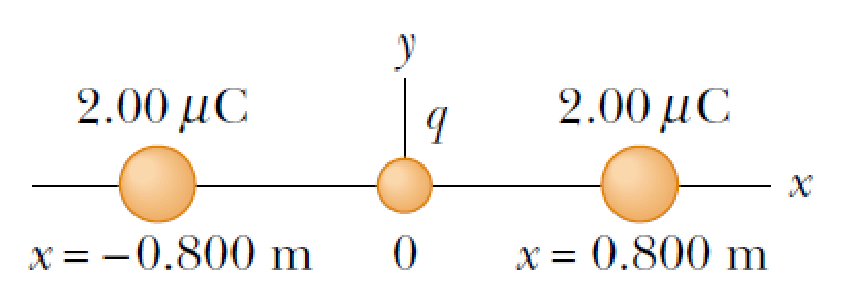
(b) Since $F = qE$, $E = 0$

(c) $V = 2k_e \frac{Q}{r}$

$$V = 2 \times 8.99 \times 10^9 \times \frac{2.00 \times 10^{-6}}{0.800}$$

$$V = 4.50 \times 10^4 \text{ V}$$

$$V = 45.0 \text{ kV}$$



Problem 25.17

At a certain distance from a point charge, the magnitude of the electric field is 500 V/m and the electric potential is −3.00 kV. (a) What is the distance to the charge? (b) What is the magnitude of the charge?

(a) $E = k_e \frac{|Q|}{r^2}$

$V = k_e \frac{Q}{r}$

$r = \frac{|V|}{|E|} = \frac{3000}{500} = \boxed{6.00\ \text{m}}$

(b) $Q = \frac{rV}{k_e} = \frac{6.00 \times (-3000)}{8.99 \times 10^9} = -2.00 \times 10^{-6}\ \text{C} = \boxed{-2.00\ \mu\text{C}}$

Problem 25.19

Additional problem

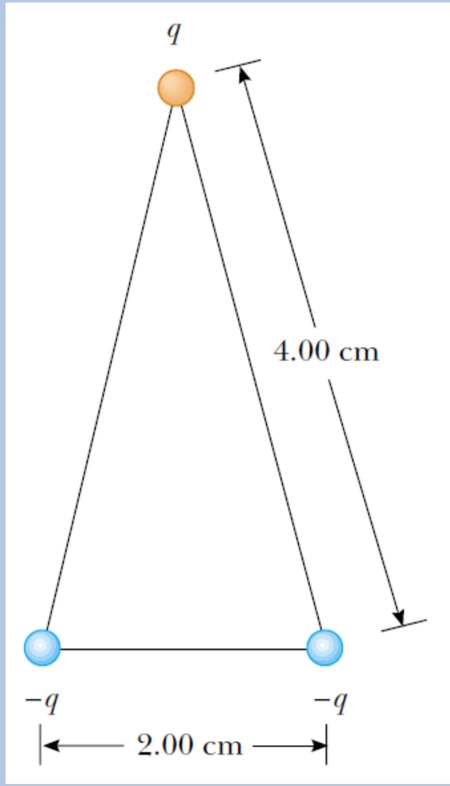
The three charges in the following figure are at the vertices of an isosceles triangle. Calculate the electric potential at the midpoint of the base, taking $q = 7.00\ \mu\text{C}$.

$$V = \sum_i k_e \frac{q_i}{r_i}$$

$$V = 8.99 \times 10^9 \times 7.00 \times 10^{-6} \times \left(\frac{-1}{0.0100} - \frac{1}{0.0100} + \frac{1}{0.0387} \right)$$

$$V = -1.10 \times 10^7\ \text{V}$$

$$V = -11.0\ \text{MV}$$



Problem 25.20

Additional problem

Two point charges, $Q_1 = +5.00\ \text{nC}$ and $Q_2 = -3.00\ \text{nC}$, are separated by 35.0 cm. (a) What is the potential energy of the pair? What is the significance of the algebraic sign of your answer? (b) What is the electric potential at a point midway between the charges?

(a) $$U = k_e \frac{Q_1 Q_2}{r}$$

$$U = 8.99 \times 10^9 \times \frac{5.00 \times 10^{-9} \times (-3.00 \times 10^{-9})}{0.350}$$

$$U = -3.86 \times 10^{-7}\ \text{J}$$

The minus sign means it takes 3.86×10^{-7} to pull the two charges apart from 35 cm to a much larger separation.

(b) $$V = k_e \left(\frac{Q_1}{r_1} + \frac{Q_2}{r_2}\right)$$

$$V = 8.99 \times 10^9 \times \left(\frac{5.00 \times 10^{-9}}{0.175} + \frac{(-3.00 \times 10^{-9})}{0.175}\right)$$

$$V = 103\ \text{V}$$

Problem 25.28

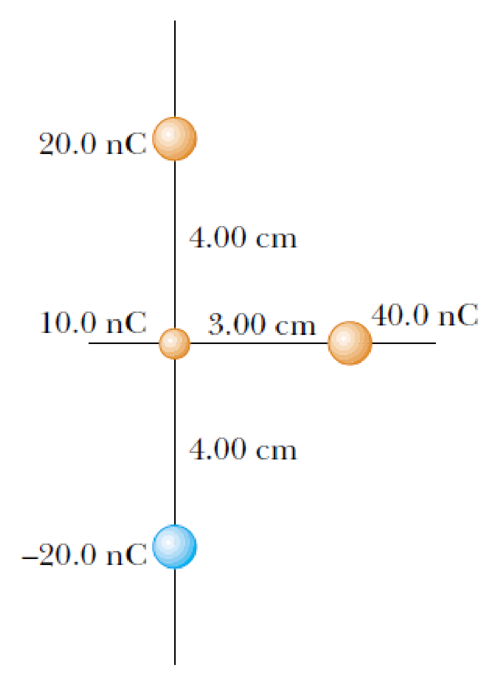
Two particles, with charges of 20.0 nC and −20.0 nC, are placed at the points with coordinates $(0, 4.00\text{ cm})$ and $(0, -4.00\text{ cm})$, as shown in the following figure. A particle with charge 10.0 nC is located at the origin. (a) Find the electric potential energy of the configuration of the three fixed charges. (b) A fourth particle, with a mass of 2.00×10^{-13} kg and a charge of 40.0 nC , is released from rest at the point $(3.00\text{ cm}, 0)$. Find its speed after it has moved freely to a very large distance away.

(a) $U = k_e \sum \frac{q_i q_j}{r_{ij}}$, summed over all pairs of (i, j) where $i \neq j$.

$$U = U_{12} + U_{13} + U_{23}$$

$$U = k_e \left[\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}}\right]$$

$$U = 9 \times 10^9 \times \left[\frac{20.0 \times 10^{-9} \times 10.0 \times 10^{-9}}{0.04} + \frac{20.0 \times 10^{-9} \times (-20.0 \times 10^{-9})}{0.08} + \frac{10.0 \times 10^{-9} \times (-20.0 \times 10^{-9})}{0.04}\right] = -4.50 \times 10^{-5}\text{ J}$$



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Problem 25.28

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(b) The three fixed charges create this potential at the location where the fourth is released:

$$V_4 = V_{14} + V_{24} + V_{34}$$

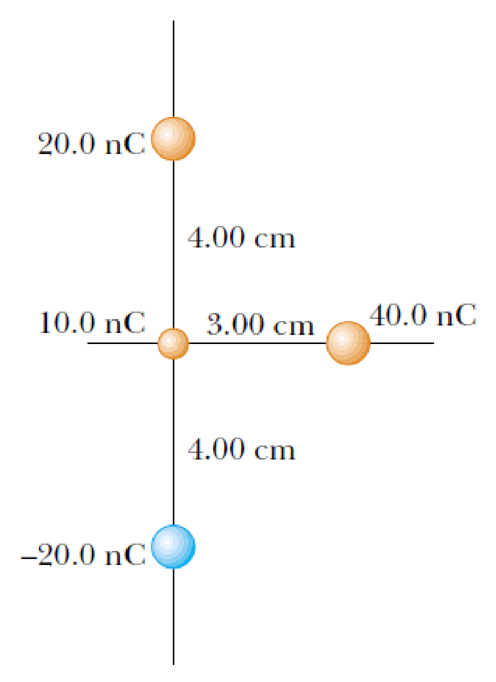
$$V_4 = k_e\left(\frac{q_1}{r_{14}} + \frac{q_2}{r_{24}} + \frac{q_3}{r_{34}}\right)$$

$$V_4 = 8.99 \times 10^9 \times \left(\frac{20.0 \times 10^{-9}}{\sqrt{0.04^2 + 0.03^2}} + \frac{10.0 \times 10^{-9}}{0.03} - \frac{20.0 \times 10^{-9}}{\sqrt{0.04^2 + 0.03^2}}\right)$$

$$V_4 = 3.00 \times 10^3\ \text{V}$$

Energy of the system of four charged objects is conserved as the fourth charge flies away:

$$\Delta K + \Delta U = 0$$



Problem 25.28

Two particles, with charges of 20.0 nC and −20.0 nC, are placed at the points with coordinates $(0, 4.00\text{ cm})$ and $(0, -4.00\text{ cm})$, as shown in the following figure. A particle with charge 10.0 nC is located at the origin. (a) Find the electric potential energy of the configuration of the three fixed charges. (b) A fourth particle, with a mass of 2.00×10^{-13} kg and a charge of 40.0 nC , is released from rest at the point $(3.00\text{ cm}, 0)$. Find its speed after it has moved freely to a very large distance away.

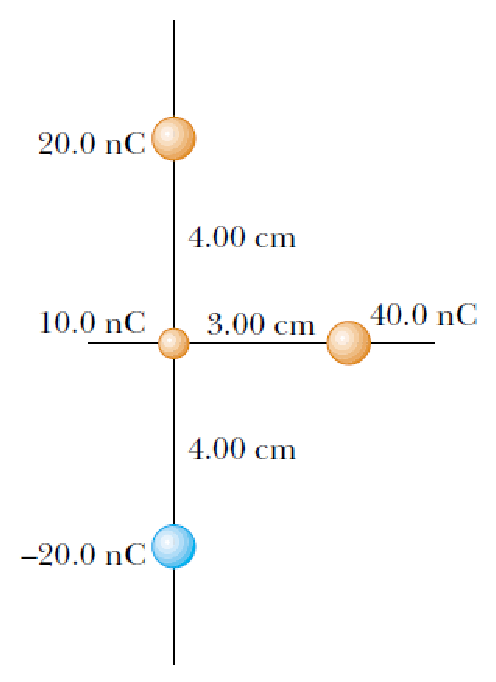
$$\left(\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2\right) + (q_4V_\infty - q_4V_4) = 0$$

$$\left(\frac{1}{2}mv_f^2 - 0\right) + (0 - q_4V_4) = 0$$

$$v_f = \sqrt{\frac{2q_4V_4}{m}}$$

$$v_f = \sqrt{\frac{2 \times 40.0 \times 10^{-9} \times 3.00 \times 10^{3}}{2.00 \times 10^{-13}}}$$

$$v_f = 3.46 \times 10^{4}\ \text{m/s}$$



Problem 25.34

Additional problem

Calculate the energy required to assemble the array of charges shown in the following figure, where $a = 0.200$ m , $b = 0.400$ m, and $q = 6.00$ μC.

$$U = k_e \sum \frac{q_i q_j}{r_{ij}}$$, summed over all pairs of (i, j) where $i \neq j$.

$$U = k_e \left[\frac{q(-2q)}{b} + \frac{q(3q)}{\sqrt{a^2 + b^2}} + \frac{(-2q)(3q)}{a} + \frac{q(2q)}{a} + \frac{(-2q)(2q)}{\sqrt{a^2 + b^2}} + \frac{(3q)(2q)}{b} \right]$$

$$U = k_e q^2 \left[\frac{-2}{0.400} + \frac{3}{0.447} - \frac{6}{0.200} + \frac{2}{0.200} - \frac{4}{0.447} + \frac{6}{0.400} \right]$$

$$U = 8.99 \times 10^9 \times (6.00 \times 10^{-6})^2 \times [-12.24]$$

$$U = -3.96 \text{ J}$$