104 PHYS

Ch. 26

Capacitance and Dielectrics

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Contents

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➤ A **capacitor** consists of two conductors carrying charges of equal magnitude and opposite sign.

➤ A potential difference ΔV exists between the conductors due to the presence of the charges.

➤ The quantity of charge Q on a capacitor is linearly proportional to the potential difference ΔV between the conductors:

$$Q \propto \Delta V$$

➤ The proportionality constant C (known as the **capacitance**) depends on the shape and separation of the conductors. We can write this relationship as: $Q = C\Delta V$.

➤ The capacitance C of any capacitor is the ratio of the charge Q on either conductor to the potential difference ΔV between the conductors:

$$C = \frac{Q}{\Delta V}$$

➤ **The capacitance is always a positive quantity.**

➤ The capacitance is a measure of a capacitor's ability to store charge. Because positive and negative charges are separated in the system of two conductors in a capacitor, there is electric potential energy stored in the system.

➤ The SI unit of capacitance is coulombs per volt, or the **farad** (F), $1 \text{ F} = 1 \text{ C/V}$.


All of these devices are capacitors, which store electric charge and energy. A capacitor is one type of circuit element that we can combine with others to make electric circuits.
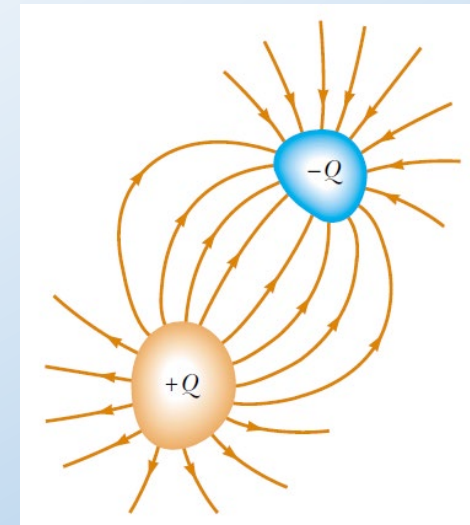
A capacitor consists of two conductors. When the capacitor is charged, the conductors carry charges of equal magnitude and opposite sign.

Definition of Capacitance

➢ Let us consider a capacitor formed from a pair of parallel plates.

- Each plate is connected to one terminal of a battery, which acts as a source of potential difference.

- If the capacitor is initially uncharged, the battery establishes an electric field in the connecting wires when the connections are made.

- On the plate connected to the negative terminal of the battery, the electric field applies a force on electrons in the wire just outside this plate; this force causes the electrons to move onto the plate.

- This movement continues until the plate, the wire, and the terminal are all at the same electric potential.

- Once this equilibrium point is attained, a potential difference no longer exists between the terminal and the plate, and as a result no electric field is present in the wire, and the movement of electrons stops. The plate now carries a negative charge.

- A similar process occurs at the other capacitor plate, with electrons moving from the plate to the wire, leaving the plate positively charged.

- In this final configuration, the potential difference across the capacitor plates is the same as that between the terminals of the battery.
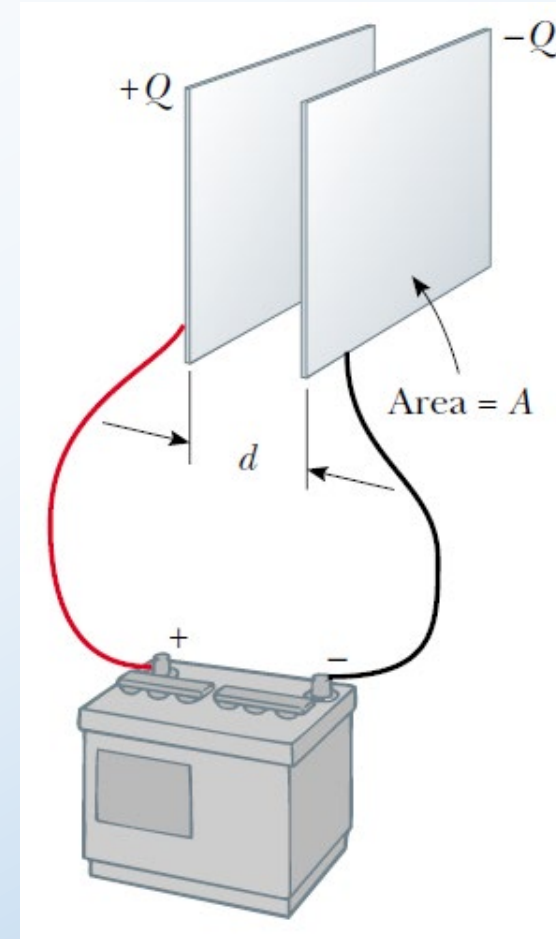
A parallel-plate capacitor consists of two parallel conducting plates, each of area A, separated by a distance d. When the capacitor is charged by connecting the plates to the terminals of a battery, the plates carry equal amounts of charge. One plate carries positive charge, and the other carries negative charge.

Problem 26.01

(a) How much charge is on each plate of a 4.00 μF capacitor when it is connected to a 12.0 V battery? (b) If this same capacitor is connected to a 1.50 V battery, what charge is stored?

(a) $Q = C\Delta V = 4.00 \times 10^{-6} \times 12.0 = 4.80 \times 10^{-5}$ C = 48.0 μC

(b) $Q = C\Delta V = 4.00 \times 10^{-6} \times 1.50 = 6.00 \times 10^{-6}$ C = 6.00 μC

Calculating Capacitance

➤ The capacitance depends only on the geometry of the conductors and not on an external source of charge or potential difference.

➤ There are three familiar geometries of the capacitors, namely, <u>parallel plates</u>, concentric cylinders, and concentric spheres.

Parallel-Plate Capacitors

▪ Two parallel metallic plates of equal area A are separated by a distance d (assuming that the charged conductors are separated by a <u>vacuum</u>).

▪ One plate carries a charge Q, and the other carries a charge $-Q$.

▪ The amount of charge that can be stored on a plate for a given potential difference increases as the plate area is increased. Thus, we expect the capacitance to be proportional to the plate area A.

▪ Moving the plates closer together increases the potential difference between the plates and causes the charge on the capacitor to increase. If d is increased, the charge decreases. As a result, we expect the capacitance of the pair of plates to be inversely proportional to d.
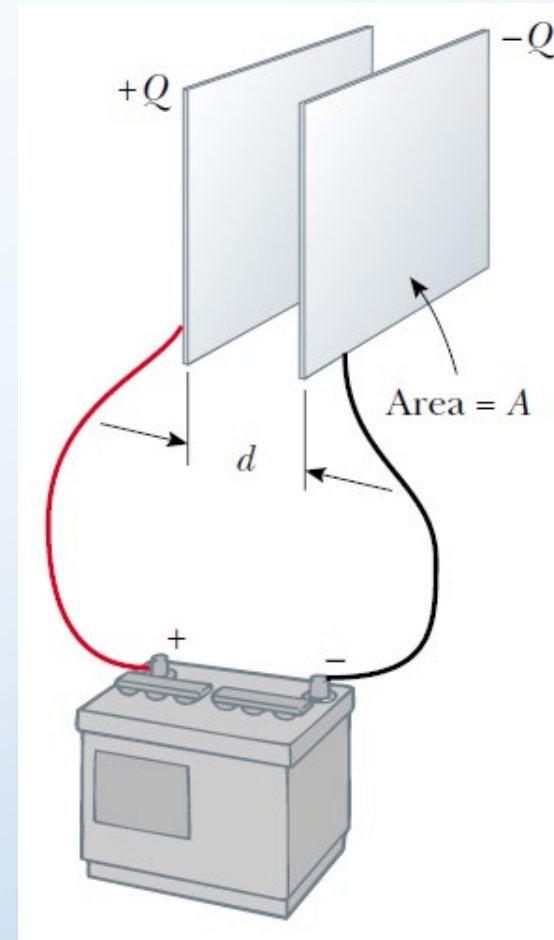
A parallel-plate capacitor consists of two parallel conducting plates, each of area A, separated by a distance d. When the capacitor is charged by connecting the plates to the terminals of a battery, the plates carry equal amounts of charge. One plate carries positive charge, and the other carries negative charge.

➢ The value of the electric field between the plates is:

$$E = \frac{\sigma}{\epsilon_0} = \frac{Q}{\epsilon_0 A}$$

Because the field between the plates is uniform, the magnitude of the potential difference between the plates equals Ed; therefore:
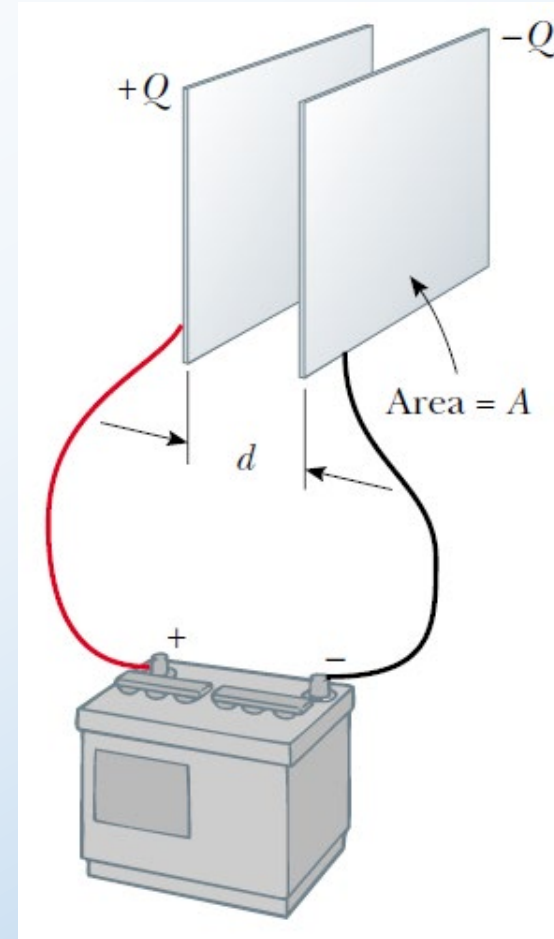
$$\Delta V = Ed = \frac{Qd}{\epsilon_0 A}$$

➢ We find that the capacitance is:

$$C = \frac{Q}{\Delta V} = \frac{Q}{Qd/\epsilon_0 A}$$

$$C = \frac{\epsilon_0 A}{d}$$

➢ That is, the capacitance of a parallel-plate capacitor is proportional to the area of its plates and inversely proportional to the plate separation.

A parallel-plate capacitor consists of two parallel conducting plates, each of area A, separated by a distance d. When the capacitor is charged by connecting the plates to the terminals of a battery, the plates carry equal amounts of charge. One plate carries positive charge, and the other carries negative charge.

Calculating Capacitance

➢ The electric field is uniform in the central region between the plates. However, the field is nonuniform at the edges of the plates. Such edge effects can be neglected if the plate separation is small compared with the length of the plates.
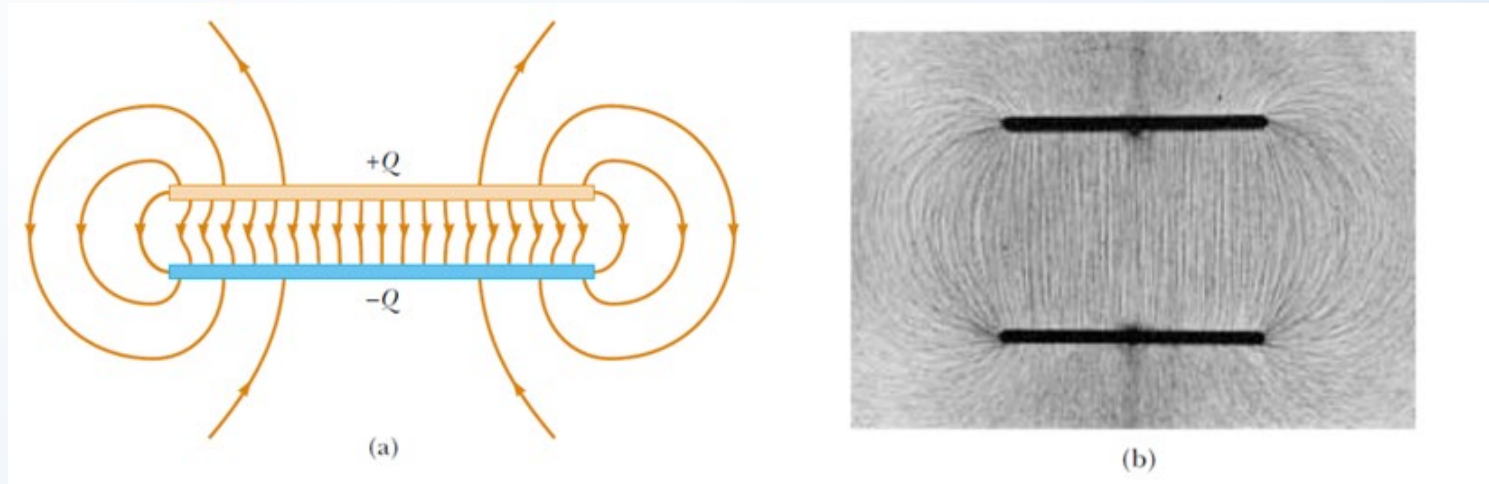
The electric field between the plates of a parallel-plate capacitor is uniform near the center but nonuniform near the edges. (b) Electric field pattern of two oppositely charged conducting parallel plates. Small pieces of thread on an oil surface align with the electric field.

Calculating Capacitance

➢ Let us assume a battery connected to a single parallel-plate capacitor with a switch in the circuit.

- When the switch is closed, the battery establishes an electric field in the wires and charges flow between the wires and the capacitor. As this occurs, there is a transformation of energy within the circuit.

- Before the switch is closed, energy is stored as chemical energy in the battery.

- This energy is transformed during the chemical reaction that occurs within the battery when it is operating in an electric circuit.

- When the switch is closed, some of the chemical energy in the battery is converted to electric potential energy related to the separation of positive and negative charges on the plates.

- As a result, we can describe a capacitor as a device that stores energy as well as charge.
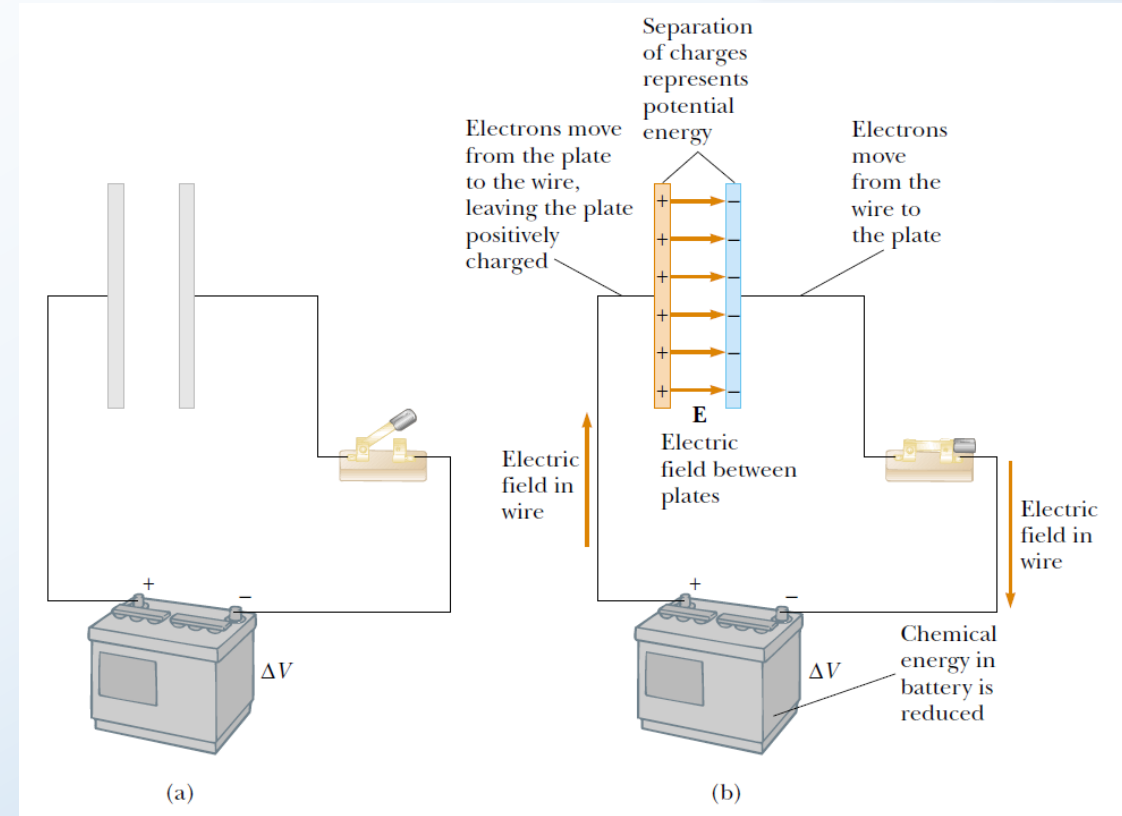
(a) A circuit consisting of a capacitor, a battery, and a switch. (b) When the switch is closed, the battery establishes an electric field in the wire that causes electrons to move from the left plate into the wire and into the right plate from the wire. As a result, a separation of charge exists on the plates, which represents an increase in electric potential energy of the system of the circuit. This energy in the system has been transformed from chemical energy in the battery.

Example 26.01

A parallel-plate capacitor with air between the plates has an area $A = 2.00 \times 10^{-4}$ m^2 and a plate separation $d = 1.00$ mm. Find its capacitance.

$$C = \frac{\epsilon_0 A}{d}$$

$$C = \frac{8.85 \times 10^{-12} \times 2.00 \times 10^{-4}}{1.00 \times 10^{-3}}$$

$$C = 1.77 \times 10^{-12} \text{ F}$$

$$C = 1.77 \text{ pF}$$

Calculating Capacitance

Problem 26.07

Additional problem

An air-filled capacitor consists of two parallel plates, each with an area of 7.60 cm², separated by a distance of 1.80 mm. A 20.0 V potential difference is applied to these plates. Calculate (a) the electric field between the plates, (b) the surface charge density, (c) the capacitance, and (d) the charge on each plate.

(a) $\Delta V = Ed$

$$E = \frac{\Delta V}{d} = \frac{20.0}{1.80 \times 10^{-3}} = 1.11 \times 10^4 \text{ V/m} = \boxed{11.1 \text{ kV/m}}$$

(b) $E = \dfrac{\sigma}{\epsilon_0}$

$$\sigma = \epsilon_0 E = 8.85 \times 10^{-12} \times 1.11 \times 10^4 = 9.83 \times 10^{-8} \text{ C/m}^2 = \boxed{98.3 \text{ nC/m}^2}$$

(c) $C = \dfrac{\epsilon_0 A}{d} = \dfrac{8.85 \times 10^{-12} \times 7.60 \times 10^{-4}}{1.80 \times 10^{-3}} = 3.74 \times 10^{-12} \text{ F} = \boxed{3.74 \text{ pF}}$

(d) $\Delta V = \dfrac{Q}{C}$

$$Q = C\Delta V = 3.74 \times 10^{-12} \times 20.0 = 7.47 \times 10^{-11} \text{ C} = \boxed{74.7 \text{ pC}}$$

Problem **26.09**

When a potential difference of 150 V is applied to the plates of a parallel-plate capacitor, the plates carry a surface charge density of $30.0 \ nC/cm^2$. What is the spacing between the plates?

$$Q = \frac{\epsilon_0 A}{d} \Delta V$$

$$\frac{Q}{A} = \sigma = \frac{\epsilon_0}{d} \Delta V$$

$$d = \frac{\epsilon_0}{\sigma} \Delta V$$

$$d = \frac{8.85 \times 10^{-12}}{3.00 \times 10^{-4}} \times 150 = 4.42 \times 10^{-6} \ m = \boxed{4.42 \ \mu m}$$

Parallel Combination

If two or more capacitors are connected in **parallel:**

➤ The individual potential differences across capacitors connected in parallel are the same and are equal to the potential difference applied across the combination.

$$\Delta V = \Delta V_1 = \Delta V_2$$

➤ The total charge Q stored by the two capacitors is:

$$Q = Q_1 + Q_2$$

➤ The total charge on capacitors connected in parallel is the sum of the charges on the individual capacitors.
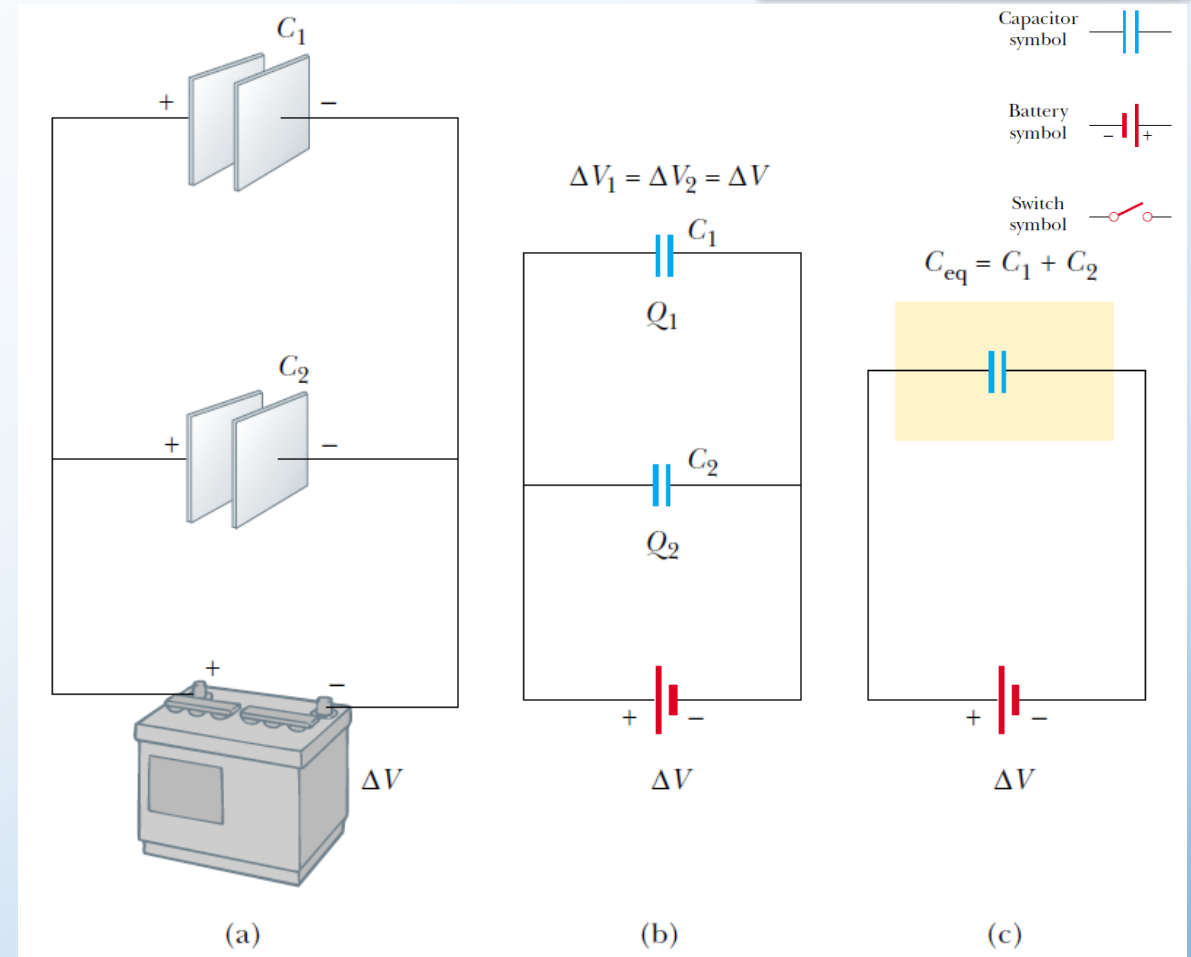
➤ The charges that the capacitors carry are:

$$Q_1 = C_1 \Delta V \qquad Q_2 = C_2 \Delta V$$

➤ For the equivalent capacitor:

$$Q = C_{eq} \Delta V$$

$$C_{eq} \Delta V = C_1 \Delta V + C_2 \Delta V$$

(a) A parallel combination of two capacitors in an electric circuit in which the potential difference across the battery terminals is ΔV. (b) The circuit diagram for the parallel combination. (c) The equivalent capacitance is $C_{eq} = C_1 + C_2$.
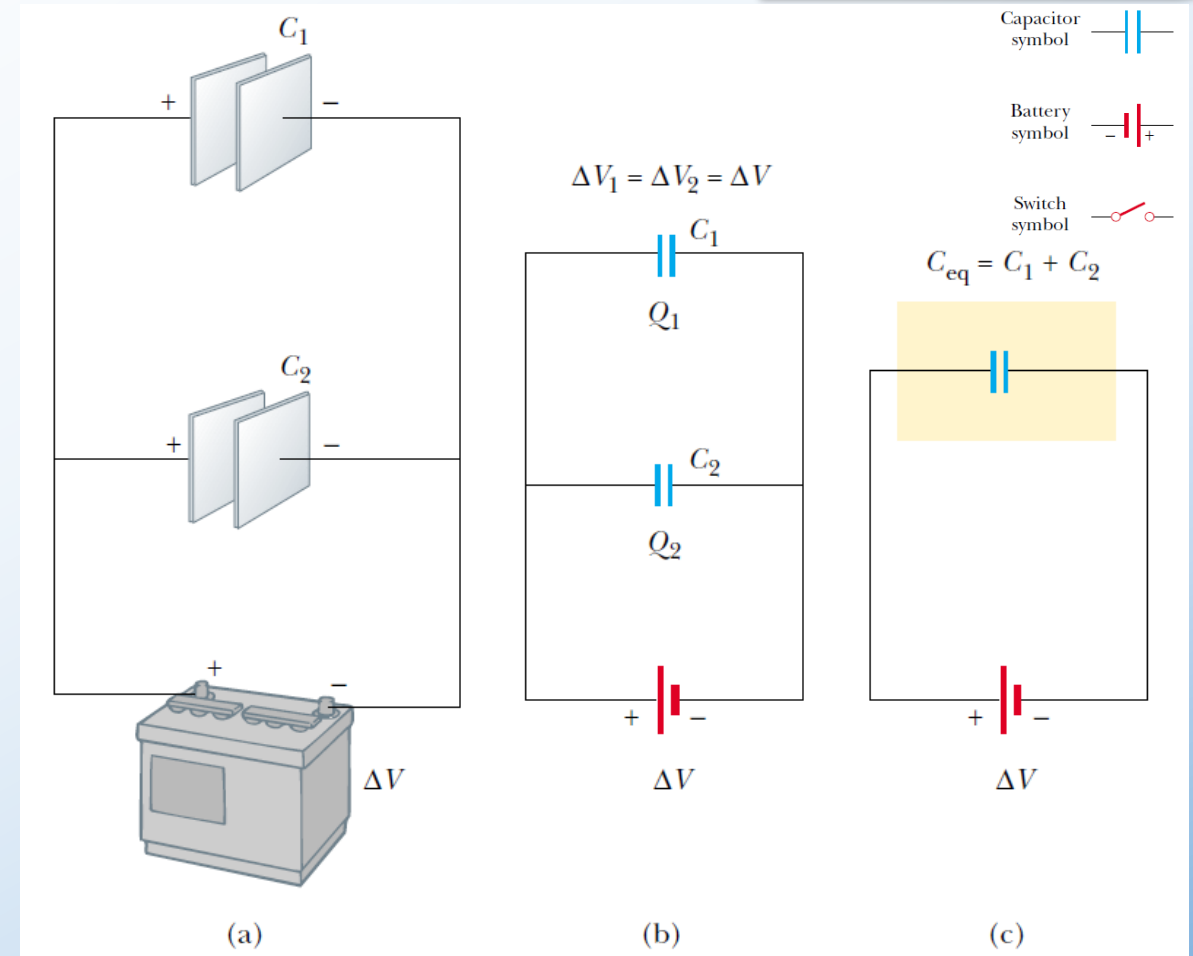
➤ The equivalent capacitance of two parallel capacitors is:

$$C_{eq} = C_1 + C_2$$

➤ If we extend this treatment to three or more capacitors connected in parallel, we find the equivalent capacitance to be:

$$C_{eq} = C_1 + C_2 + C_3 + \cdots$$

➤ The equivalent capacitance of a parallel combination of capacitors is the algebraic sum of the individual capacitances and is greater than any of the individual capacitances.

(a) A parallel combination of two capacitors in an electric circuit in which the potential difference across the battery terminals is ΔV. (b) The circuit diagram for the parallel combination. (c) The equivalent capacitance is $C_{eq} = C_1 + C_2$.

Series Combination

If two or more capacitors are connected in **series:**

➢ The charges on capacitors connected in series are the same.

$$Q = Q_1 = Q_2$$

➢ The voltage ΔV across the battery terminals is split between the two capacitors:
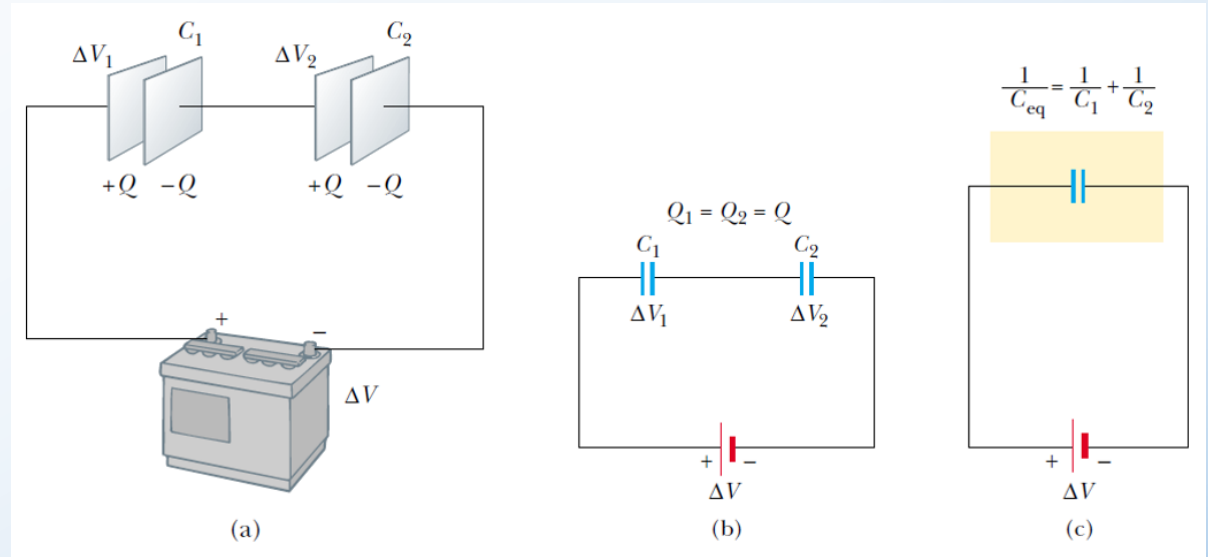
$$\Delta V = \Delta V_1 + \Delta V_2$$

➢ The total potential difference across any number of capacitors connected in series is the sum of the potential differences across the individual capacitors.

➢ For the equivalent capacitor:

$$\Delta V = \frac{Q}{C_{eq}}$$

➢ The potential differences across the capacitors are:

$$\Delta V_1 = \frac{Q}{C_1} \qquad \Delta V_2 = \frac{Q}{C_2}$$

(a) A series combination of two capacitors. The charges on the two capacitors are the same. (b) The circuit diagram for the series combination. (c) The equivalent capacitance can be calculated from the relationship $\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$.

$$\frac{Q}{C_{eq}} = \frac{Q}{C_1} + \frac{Q}{C_2}$$

➤ Canceling Q, we arrive at the relationship:

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$$

➤ When this analysis is applied to three or more capacitors connected in series, the relationship for the equivalent capacitance is:

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots$$

➤ The inverse of the equivalent capacitance is the algebraic sum of the inverses of the individual capacitances and the equivalent capacitance of a series combination is always less than any individual capacitance in the combination.
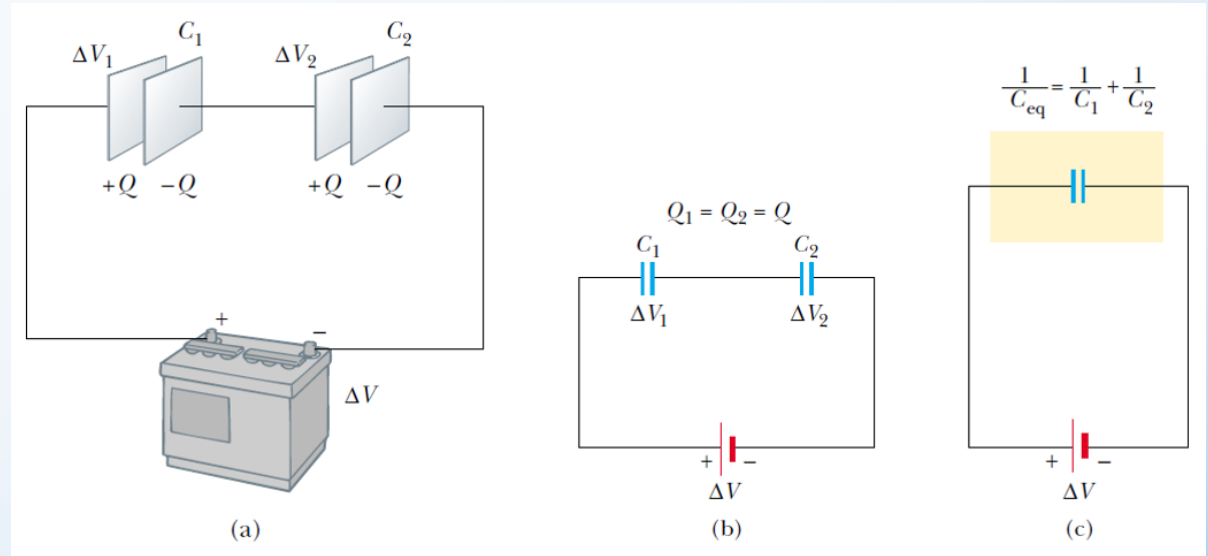
(a) A series combination of two capacitors. The charges on the two capacitors are the same. (b) The circuit diagram for the series combination. (c) The equivalent capacitance can be calculated from the relationship $\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$.

Problem Solving Hints

▪ Be careful with units. When you calculate capacitance in farads, make sure that distances are expressed in meters. When checking consistency of units, remember that the unit for electric fields can be either N/C or V/m.

▪ When two or more capacitors are connected in parallel, the potential difference across each is the same. The charge on each capacitor is proportional to its capacitance; hence, the capacitances can be added directly to give the equivalent capacitance of the parallel combination. The equivalent capacitance is always larger than the individual capacitances.

▪ When two or more capacitors are connected in series, they carry the same charge, and the sum of the potential differences equals the total potential difference applied to the combination. The sum of the reciprocals of the capacitances equals the reciprocal of the equivalent capacitance, which is always less than the capacitance of the smallest individual capacitor.

Example 26.04

Find the equivalent capacitance between a and b for the combination of capacitors shown in Fig. (a). All capacitances are in microfarads.

We reduce the combination step by step as indicated in the figure. The 1.0 µF and 3.0 µF capacitors are in parallel and combine according to the expression $C_{eq} = C_1 + C_2 = 4.0$ µF. The 2.0 µF and 6.0 µF capacitors also are in parallel and have an equivalent capacitance of 8.0 µF . Thus, the upper branch in Fig. (b) consists of two 4.0 µF capacitors in series, which combine as follows:

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{4.0 \text{ µF}} + \frac{1}{4.0 \text{ µF}} = \frac{1}{2.0 \text{ µF}}$$

$$C_{eq} = 2.0 \text{ µF}$$

The lower branch in Fig. (b) consists of two 8.0 µF capacitors in series, which combine to yield an equivalent capacitance of 4.0 µF. Finally, the 2.0 µF and 4.0 µF capacitors in Fig.(c) are in parallel and thus have an equivalent capacitance of 6.0 µF.


(a)   (b)   (c)   (d)
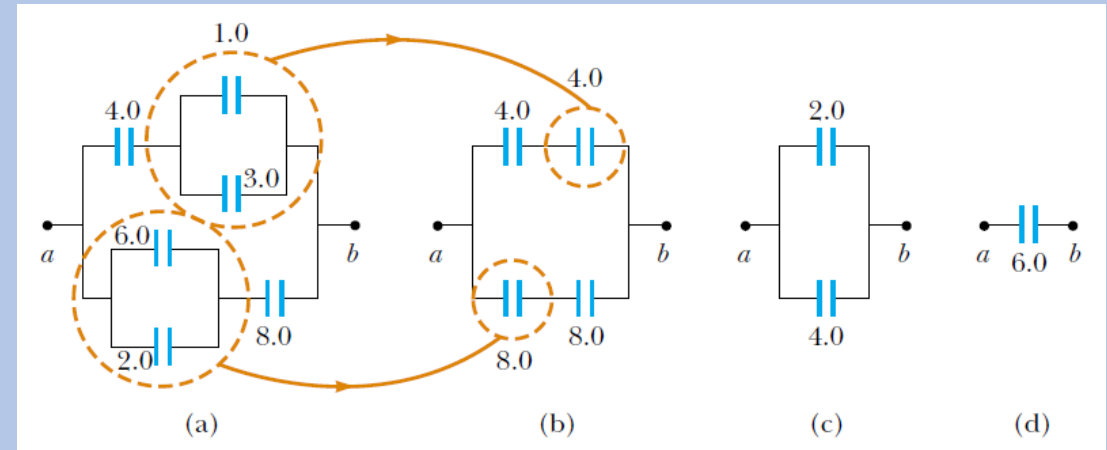
To find the equivalent capacitance of the capacitors in part (a), we reduce the various combinations in steps as indicated in parts (b), (c), and (d), using the series and parallel rules described in the text.

Problem 26.18

Evaluate the equivalent capacitance of the configuration shown the following figure. All the capacitors are identical, and each has capacitance C.

The circuit reduces first according to the rule for capacitors in series, as shown in the figure, then according to the rule for capacitors in parallel, shown below.

$$C_{eq} = C\left(1 + \frac{1}{2} + \frac{1}{3}\right) = \frac{11}{6}C = 1.83C$$
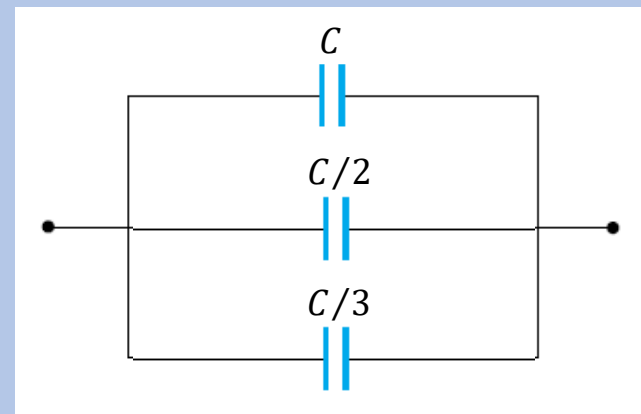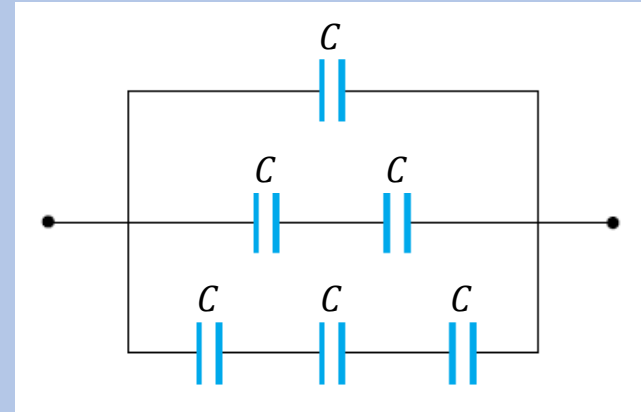
Problem 26.21

Four capacitors are connected as shown in the following figure. (a) Find the equivalent capacitance between points a and b. (b) Calculate the charge on each capacitor if ΔV_{ab} = 15.0 V.

(a)

$$\frac{1}{C_{12}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{15.0} + \frac{1}{3.00}$$

$$C_{12} = 2.50\mu F$$
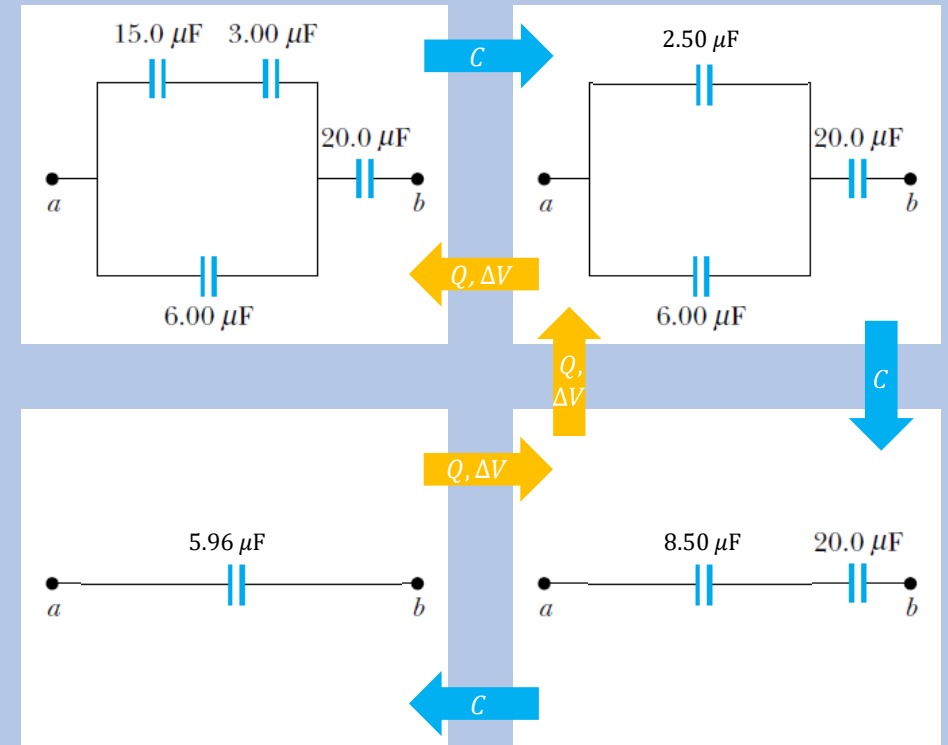
$$C_{123} = C_{12} + C_3 = 2.50 + 6.00 = 8.50 \; \mu F$$

$$C_{eq} = \left(\frac{1}{C_{123}} + \frac{1}{C_4}\right)^{-1} = \left(\frac{1}{8.50} + \frac{1}{20.0}\right)^{-1} = \boxed{5.96 \; \mu F}$$

(b)

$$Q = C_{eq}\Delta V = 5.96 \times 10^{-6} \times 15.0 = 89.5 \times 10^{-6} \; C = 89.5 \; \mu C$$

$$Q = Q_{123} = Q_4$$

$$\Rightarrow \; Q_4 = \boxed{89.5 \; \mu C}$$

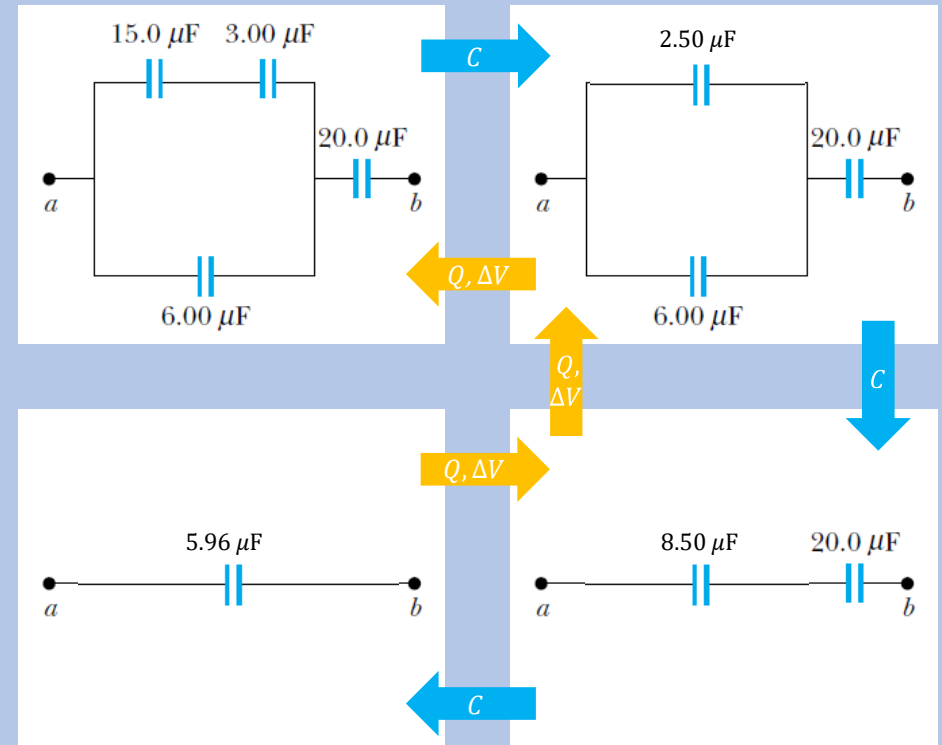$$\Delta V_{123} = \frac{Q_{123}}{C_{123}} = \frac{89.5 \times 10^{-6}}{8.5 \times 10^{-6}} = 10.53 \; V$$

Problem 26.21

Four capacitors are connected as shown in the following figure. (a) Find the equivalent capacitance between points a and b. (b) Calculate the charge on each capacitor if $\Delta V_{ab} = 15.0$ V.

$\Delta V_{123} = \Delta V_{12} = \Delta V_3$

$Q_3 = C_3 \Delta V_3 = 6.00 \times 10^{-6} \times 10.53 = 63.2 \times 10^{-6}$ C $=$ 63.2 μC

$Q_1 = Q_2 = Q_{12} = Q_{123} - Q_3 = 89.5 - 63.2 =$ 26.3 μC

➢ If a capacitor is initially uncharged, no work is required to transfer a small amount of charge dq from one plate to the other.

➢ However, once this charge has been transferred, a small potential difference exists between the plates. Therefore, work must be done to move additional charge through this potential difference.

➢ As more and more charge is transferred from one plate to the other, the potential difference increases in proportion, and more work is required.

➢ The work necessary to transfer an increment of charge dq from the plate carrying charge $-q$ to the plate carrying charge q (which is at the higher electric potential) is:

$$dW = \Delta V dq = \frac{q}{C} dq$$

➢ The total work W required to charge the capacitor from $q = 0$ to some final charge $q = Q$ is:

$$W = \int_0^Q \frac{q}{C} dq = \frac{1}{C} \int_0^Q q \, dq = \frac{1}{2} \frac{Q^2}{C}$$
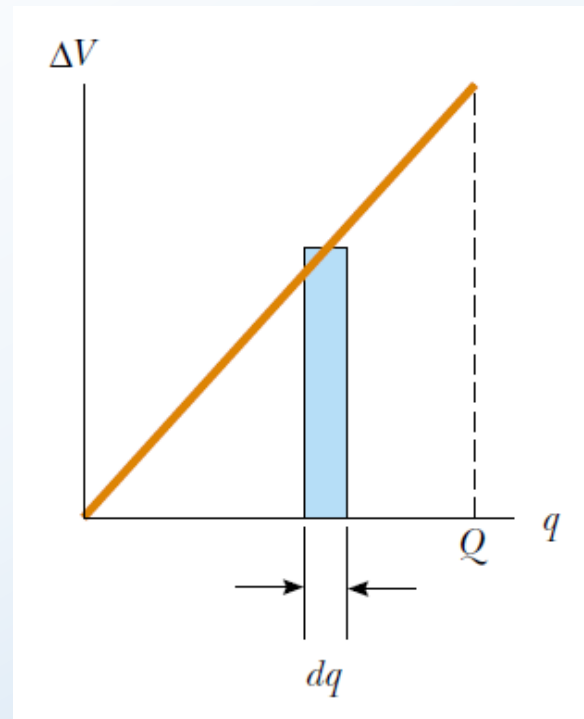
A plot of potential difference versus charge for a capacitor is a straight line having a slope $1/C$. The work required to move charge dq through the potential difference ΔV existing at the time across the capacitor plates is given approximately by the area of the shaded rectangle. The total work required to charge the capacitor to a final charge Q is the triangular area under the straight line, $W = \frac{1}{2} Q \Delta V$. (Don't forget that 1 V = J/C; hence, the unit for the triangular area is the joule.)

➤ The work done in charging the capacitor appears as electric potential energy U stored in the capacitor.

$$U = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}Q\Delta V = \frac{1}{2}C(\Delta V)^2$$

➤ For a parallel-plate capacitor, the potential difference is related to the electric field through the relationship $\Delta V = Ed$. Furthermore, its capacitance is $C = \epsilon_0 A/d$. Substituting these expressions into the previous equation, we obtain:

$$U = \frac{1}{2}\frac{\epsilon_0 A}{d}(Ed)^2 = \frac{1}{2}\epsilon_0 AdE^2$$

➤ Because the volume occupied by the electric field is Ad, the energy per unit volume $u_E = U/Ad$, known as the energy density, is:

$$u_E = \frac{1}{2}\epsilon_0 E^2$$

➤ The energy density in any electric field is proportional to the square of the magnitude of the electric field at a given point.
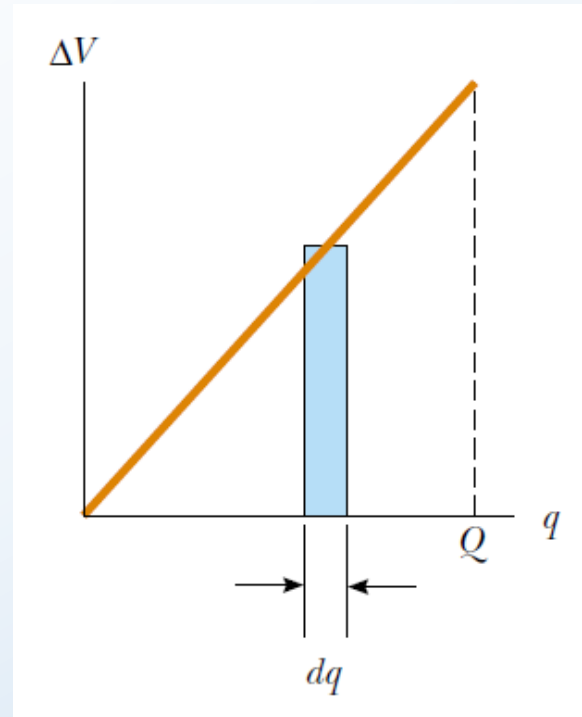
A plot of potential difference versus charge for a capacitor is a straight line having a slope $1/C$. The work required to move charge dq through the potential difference ΔV existing at the time across the capacitor plates is given approximately by the area of the shaded rectangle. The total work required to charge the capacitor to a final charge Q is the triangular area under the straight line, $W = \frac{1}{2}Q\Delta V$. (Don't forget that 1 V = J/C; hence, the unit for the triangular area is the joule.)

Problem 26.31

(a) A 3.00 μF capacitor is connected to a 12.0 V battery. How much energy is stored in the capacitor? (b) If the capacitor had been connected to a 6.00 V battery, how much energy would have been stored?

(a) $U = \frac{1}{2}C(\Delta V)^2 = \frac{1}{2} \times 3.00 \times 10^{-6} \times (12.0)^2 = 216 \times 10^{-6}$ J = 216 μJ

(b) $U = \frac{1}{2}C(\Delta V)^2 = \frac{1}{2} \times 3.00 \times 10^{-6} \times (6.00)^2 = 54.0 \times 10^{-6}$ J = 54.0 μJ

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Problem 26.36

A uniform electric field $E = 3000$ V/m exists within a certain region. What volume of space contains an energy equal to 1.00×10^{-7} J? Express your answer in cubic meters and in liters.

$$u = \frac{U}{V} = \frac{1}{2}\epsilon_0 E^2$$

$$V = \frac{2U}{\epsilon_0 E^2}$$

$$V = \frac{2 \times 1.00 \times 10^{-7}}{8.85 \times 10^{-12} \times (3000)^2}$$

$$V = 2.51 \times 10^{-3} \text{ m}^3 = 2.51 \text{ L}$$

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| Problem | 26.54 | Additional problem

For the system of capacitors shown in the following figure, find (a) the equivalent capacitance of the system, (b) the charge on each capacitor, (c) the potential across each capacitor, and (d) the total energy stored by the group.

(a) $C_{ab} = \left(\dfrac{1}{3.00} + \dfrac{1}{6.00}\right)^{-1} = 2.00 \text{ μF}$

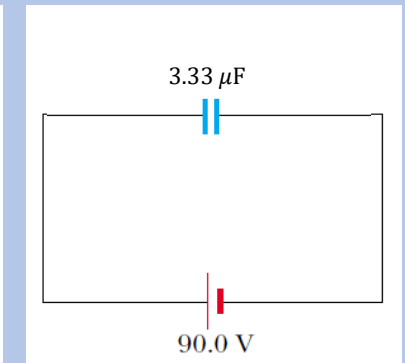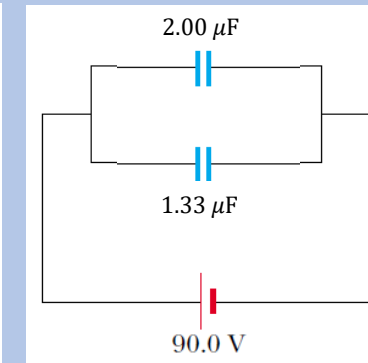$C_{cd} = \left(\dfrac{1}{2.00} + \dfrac{1}{4.00}\right)^{-1} = 1.33 \text{ μF}$
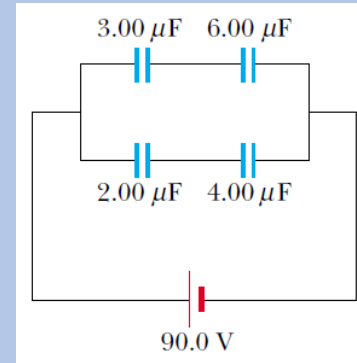
$C_{eq} = 2.00 + 1.33 = \boxed{3.33 \text{ μF}}$

(b) $Q_{ab} = C_{ab}\Delta V_{ab} = 2.00 \times 10^{-6} \times 90.0 = 180 \times 10^{-6} \text{ C} = 180 \text{ μC}$

$\Rightarrow \boxed{Q_1 = Q_2 = 180 \text{ μC}}$

$Q_{cd} = C_{cd}\Delta V_{cd} = 1.33 \times 10^{-6} \times 90.0 = 120 \times 10^{-6} \text{ C} = 120 \text{ μC}$

$\Rightarrow \boxed{Q_3 = Q_4 = 120 \text{ μC}}$

Problem 26.54

Additional problem

For the system of capacitors shown in the following figure, find (a) the equivalent capacitance of the system, (b) the charge on each capacitor, (c) the potential across each capacitor, and (d) the total energy stored by the group.
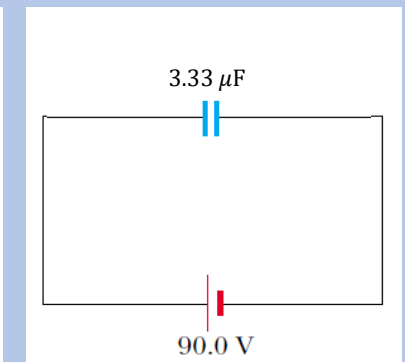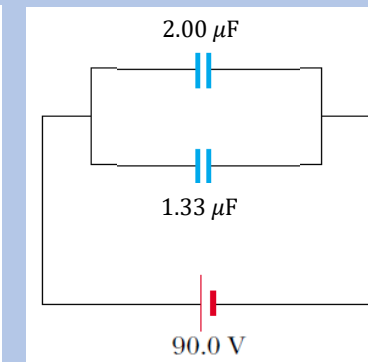
(c) $\Delta V_1 = \dfrac{Q_1}{C_1} = \dfrac{180 \times 10^{-6}}{3.00 \times 10^{-6}} = $ 60.0 V

$\Delta V_2 = \dfrac{Q_2}{C_2} = \dfrac{180 \times 10^{-6}}{6.00 \times 10^{-6}} = $ 30.0 V

$\Delta V_3 = \dfrac{Q_3}{C_3} = \dfrac{120 \times 10^{-6}}{2.00 \times 10^{-6}} = $ 60.0 V

$\Delta V_4 = \dfrac{Q_4}{C_4} = \dfrac{120 \times 10^{-6}}{4.00 \times 10^{-6}} = $ 30.0 V

(d) $U_T = \dfrac{1}{2} C_{eq}(\Delta V)^2 = \dfrac{1}{2} \times 3.33 \times 10^{-6} \times (90.0)^2 = 13.4 \times 10^{-3}$ J = 13.4 mJ

Figure (left): 3.00 μF, 6.00 μF, 2.00 μF, 4.00 μF, 90.0 V. (middle): 2.00 μF, 1.33 μF, 90.0 V. (right): 3.33 μF, 90.0 V.

Capacitors with Dielectrics
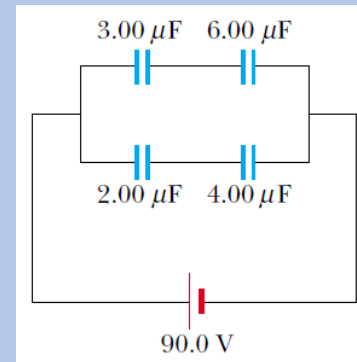
➢ A **dielectric** is a nonconducting material, such as rubber, glass, or waxed paper.

➢ When a dielectric is inserted between the plates of a capacitor, the capacitance increases.

➢ If the dielectric completely fills the space between the plates, the capacitance increases by a dimensionless factor κ, which is called the **dielectric constant** of the material.

➢ The dielectric constant varies from one material to another.

➢ Consider a parallel-plate capacitor that without a dielectric has a charge Q_0 and a capacitance C_0.

- The potential difference across the capacitor is $\Delta V_0 = Q_0/C_0$, which is measured by a voltmeter.

- When a dielectric is inserted between the plates, the voltmeter indicates that the voltage between the plates decreases to a value ΔV.

- The voltages with and without the dielectric are related by the factor κ as follows:

$$\Delta V = \frac{\Delta V_0}{\kappa}$$

➢ Because $\Delta V < \Delta V_0$, we see that $\kappa > 1$.
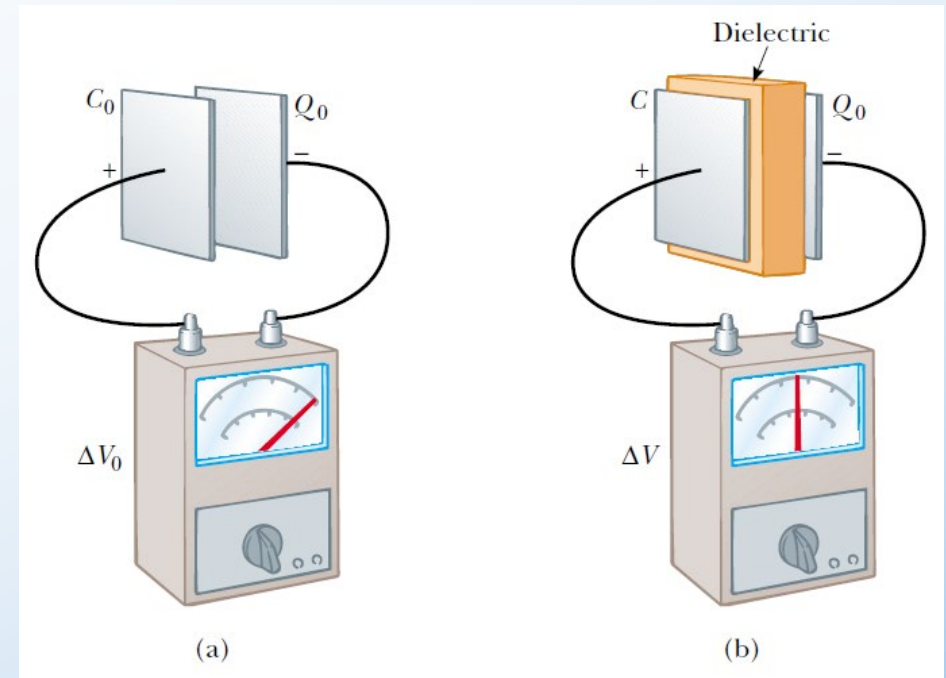
A charged capacitor (a) before and (b) after insertion of a dielectric between the plates. The charge on the plates remains unchanged, but the potential difference decreases from ΔV_0 to $\Delta V = \Delta V_0/\kappa$. Thus, the capacitance increases from C_0 to κC_0.

➤ Because the charge Q_0 on the capacitor does not change, we conclude that the capacitance must change to the value:

$$C = \frac{Q_0}{\Delta V} = \frac{Q_0}{\Delta V_0/\kappa} = \kappa \frac{Q_0}{\Delta V_0}$$

$$C = \kappa C_0$$

➤ The capacitance increases by the factor κ when the dielectric completely fills the region between the plates.

➤ For a parallel-plate capacitor, where $C_0 = \epsilon_0 A/d$, we can express the capacitance when the capacitor is filled with a dielectric as:

$$C = \kappa \frac{\epsilon_0 A}{d}$$
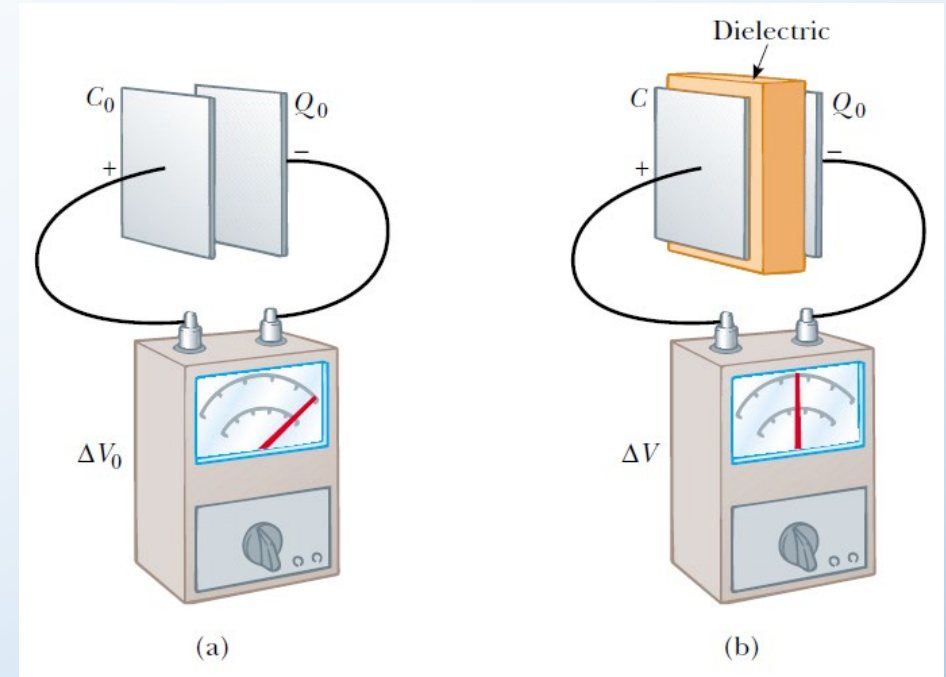
A charged capacitor (a) before and (b) after insertion of a dielectric between the plates. The charge on the plates remains unchanged, but the potential difference decreases from ΔV_0 to $\Delta V = \Delta V_0/\kappa$. Thus, the capacitance increases from C_0 to κC_0.

Capacitors with Dielectrics

➢ The capacitance can be very large by decreasing d, however, the lowest value of d is limited by the electric discharge that could occur through the dielectric medium separating the plates.

➢ For any given separation d, the maximum voltage (<u>working voltage, breakdown voltage, and rated voltage</u>) that can be applied to a capacitor without causing a discharge depends on the **dielectric strength** (maximum electric field) of the dielectric.

➢ If the magnitude of the electric field in the dielectric exceeds the dielectric strength, then the insulating properties break down and the dielectric begins to conduct.

➢ A dielectric between the plates of a capacitor provides the following advantages:

  ▪ Increase in capacitance.

  ▪ Increase in maximum operating voltage.

  ▪ Possible mechanical support between the plates, which allows the plates to be close together without touching, thereby decreasing d and increasing C.

Dielectric breakdown in air. Sparks are produced when the high voltage between the wires causes the electric field to exceed the dielectric strength of air.

[a] The dielectric strength equals the maximum electric field that can exist in a dielectric without electrical breakdown. Note that these values depend strongly on the presence of impurities and flaws in the materials.

Approximate Dielectric Constants and Dielectric Strengths of Various Materials at Room Temperature

Material	Dielectric Constant κ	Dielectric Strength[a] (10^6 V/m)
Air (dry)	1.000 59	3
Bakelite	4.9	24
Fused quartz	3.78	8
Mylar	3.2	7
Neoprene rubber	6.7	12
Nylon	3.4	14
Paper	3.7	16
Paraffin-impregnated paper	3.5	11
Polystyrene	2.56	24
Polyvinyl chloride	3.4	40
Porcelain	6	12
Pyrex glass	5.6	14
Silicone oil	2.5	15
Strontium titanate	233	8
Teflon	2.1	60
Vacuum	1.000 00	—
Water	80	—

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Example 26.06

A parallel-plate capacitor has plates of dimensions 2.0 cm by 3.0 cm separated by a 1.0 mm thickness of paper. (a) Find its capacitance. (b)What is the maximum charge that can be placed on the capacitor?

(a) Because $\kappa = 3.7$ for paper (see the previous table), we have:

$$C = \kappa \frac{\epsilon_0 A}{d}$$

$$C = 3.7 \times \frac{8.85 \times 10^{-12} \times 6.0 \times 10^{-4}}{1.0 \times 10^{-3}}$$

$$C = 20 \times 10^{-12} \text{ F}$$

$$C = 20 \text{ pF}$$

Example 26.06

A parallel-plate capacitor has plates of dimensions 2.0 cm by 3.0 cm separated by a 1.0 mm thickness of paper. (a) Find its capacitance. (b)What is the maximum charge that can be placed on the capacitor?

(b) From the previous table we see that the dielectric strength of paper is 16×10^6 V/m.
Because the thickness of the paper is 1.0 mm, the maximum voltage that can be applied before breakdown is:

$\Delta V_{max} = E_{max} \, d$

$\Delta V_{max} = 16 \times 10^6 \times 1.0 \times 10^{-3}$

$\Delta V_{max} = 16 \times 10^3$ V

Hence, the maximum charge is:

$Q_{max} = C \Delta V_{max}$

$Q_{max} = 20 \times 10^{-12} \times 16 \times 10^3$

$Q_{max} = 3.2 \times 10^{-7}$ C

$Q_{max} = 0.32 \; \mu C$

Example 26.07

A parallel-plate capacitor is charged with a battery to a charge Q_0, as shown in Fig. (a). The battery is then removed, and a slab of material that has a dielectric constant κ is inserted between the plates, as shown in Fig.(b). Find the energy stored in the capacitor before and after the dielectric is inserted.

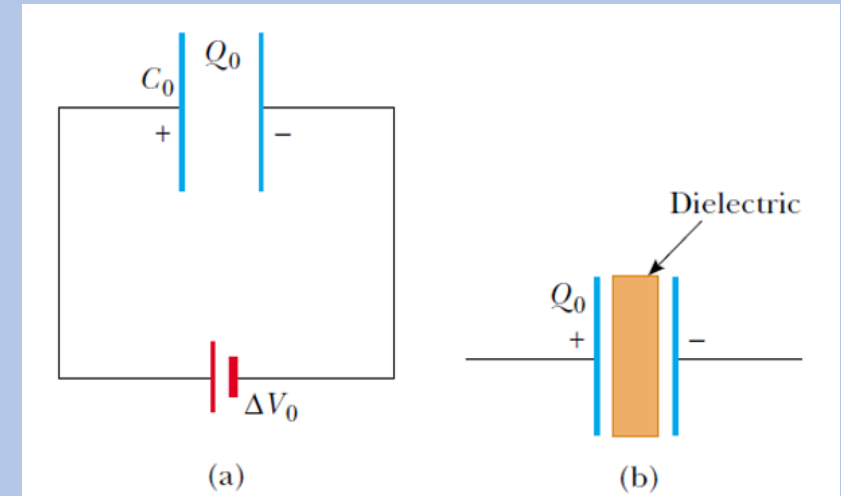We see that the energy stored in the absence of the dielectric is:

$$U_0 = \frac{1}{2}\frac{Q_0^2}{C_0}$$

After the battery is removed and the dielectric inserted, the <u>charge on the capacitor remains the same</u>. Hence, the energy stored in the presence of the dielectric is:

$$U = \frac{1}{2}\frac{Q_0^2}{C}$$

But the capacitance in the presence of the dielectric is $C = \kappa C_0$, so U becomes

$$U = \frac{Q_0^2}{2\kappa C_0} = \frac{U_0}{\kappa}$$

(a) A battery charges up a parallel-plate capacitor.
(b) The battery is removed and a slab of dielectric material is inserted between the plates.

Problem 26.47 — Additional problem

A parallel-plate capacitor in air has a plate separation of 1.50 cm and a plate area of 25.0 cm^2. The plates are charged to a potential difference of 250 V and disconnected from the source. The capacitor is then immersed in distilled water. Determine (a) the charge on the plates before and after immersion, (b) the capacitance and potential difference after immersion, and (c) the change in energy of the capacitor. Assume the liquid is an insulator. ($\kappa_{water} = 80$)

(a) The charge is the same before and after immersion, with value:

$$Q = \frac{\epsilon_0 A}{d}\Delta V_0 = \frac{8.85 \times 10^{-12} \times 25.0 \times 10^{-4}}{1.50 \times 10^{-2}} \times 250 = 369 \times 10^{-12}\ \text{C} = \boxed{369\ \text{pC}}$$

(b)
$$C = \frac{\kappa \epsilon_0 A}{d} = \frac{80.0 \times 8.85 \times 10^{-12} \times 25.0 \times 10^{-4}}{1.50 \times 10^{-2}} = 118 \times 10^{-12}\ \text{F} = \boxed{118\ \text{pF}}$$

$$\Delta V = \frac{\Delta V_0}{\kappa} = \frac{250}{80} = \boxed{3.13\ \text{V}}$$

(c) $\Delta U = U_f - U_i$

$$\Delta U = \left(\frac{1}{2}Q\Delta V\right) - \left(\frac{1}{2}Q\Delta V_0\right)$$

Problem 26.47 | Additional problem

A parallel-plate capacitor in air has a plate separation of 1.50 cm and a plate area of 25.0 cm². The plates are charged to a potential difference of 250 V and disconnected from the source. The capacitor is then immersed in distilled water. Determine (a) the charge on the plates before and after immersion, (b) the capacitance and potential difference after immersion, and (c) the change in energy of the capacitor. Assume the liquid is an insulator. ($\kappa_{water} = 80$)

$$\Delta U = \left(\frac{1}{2} Q \Delta V\right) - \left(\frac{1}{2} Q \kappa \Delta V\right)$$

$$\Delta U = \frac{1}{2} Q \Delta V (1 - \kappa)$$

$$\Delta U = \frac{1}{2} \times 369 \times 10^{-12} \times 3.13 \times (1 - 80)$$

$$\Delta U = -45.5 \times 10^{-9} \, \text{J}$$

$$\Delta U = \boxed{-45.5 \, \text{nJ}}$$