104 PHYS

Ch. 27

Current and Resistance

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Contents

Electric Current

- Studying the flow of electric charges through a piece of material depends on the material through which the charges are passing and the potential difference across the material. Whenever there is a net flow of charge through some region, an electric current is said to exist.
- **The current is the rate at which charge flows through this surface**.
- If ΔQ is the amount of charge that passes through this area in a time interval Δt, the **average current** I_{av} is equal to the charge that passes through A per unit time:

$$I_{av} = \frac{\Delta Q}{\Delta t}$$

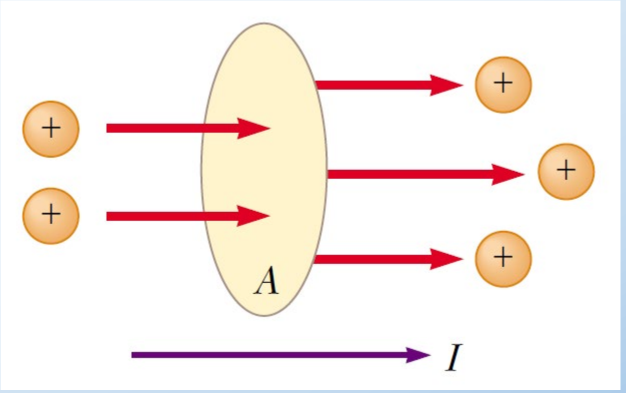
- If the rate at which charge flows varies in time, then the current varies in time; we define the **instantaneous current** I as the differential limit of average current:

$$I = \frac{dQ}{dt}$$

- The SI unit of current is the **ampere** (A):

$$1\ \text{A} = \frac{1\ \text{C}}{1\ \text{s}}$$

These power lines transfer energy from the power company to homes and businesses. The energy is transferred at a very high voltage, possibly hundreds of thousands of volts in some cases. Despite the fact that this makes power lines very dangerous, the high voltage results in less loss of power due to resistance in the wires.



Charges in motion through an area A. The time rate at which charge flows through the area is defined as the current I. The direction of the current is the direction in which positive charges flow when free to do so.

Electric Current

- That is, 1 A of current is equivalent to 1 C of charge passing through the surface area in 1 s.
- The charges passing through the surface can be positive or negative, or both.
- **It is conventional to assign to the current the same direction as the flow of positive charge.**
- In electrical conductors, such as copper or aluminum, the current is due to the motion of negatively charged electrons.
- In an ordinary conductor, **the direction of the current is opposite the direction of flow of electrons.**
- It is common to refer to a moving charge (positive or negative) as a mobile **charge carrier**. For example, the mobile charge carriers in a metal are electrons.

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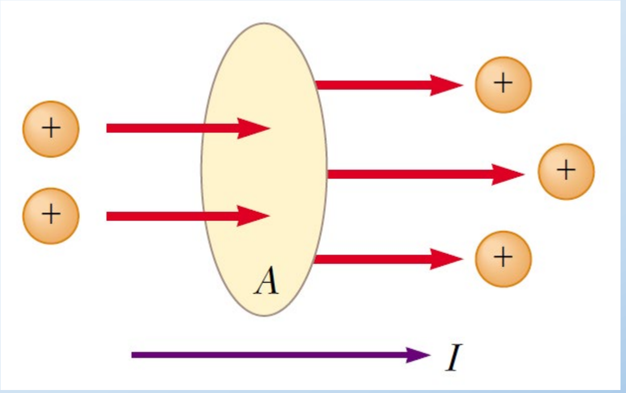


Charges in motion through an area A. The time rate at which charge flows through the area is defined as the current I. The direction of the current is the direction in which positive charges flow when free to do so.

Electric Current

Microscopic Model of Current

- The volume of a section of the conductor of length Δx is $A\Delta x$.
- If n represents the number of mobile charge carriers per unit volume (the charge carrier density), the number of carriers in the gray section is $nA\Delta x$.
- Therefore, the total charge ΔQ in this section is:

$$\Delta Q = \text{number of carriers in section} \times \text{charge per carrier} = (nA\Delta x)q$$

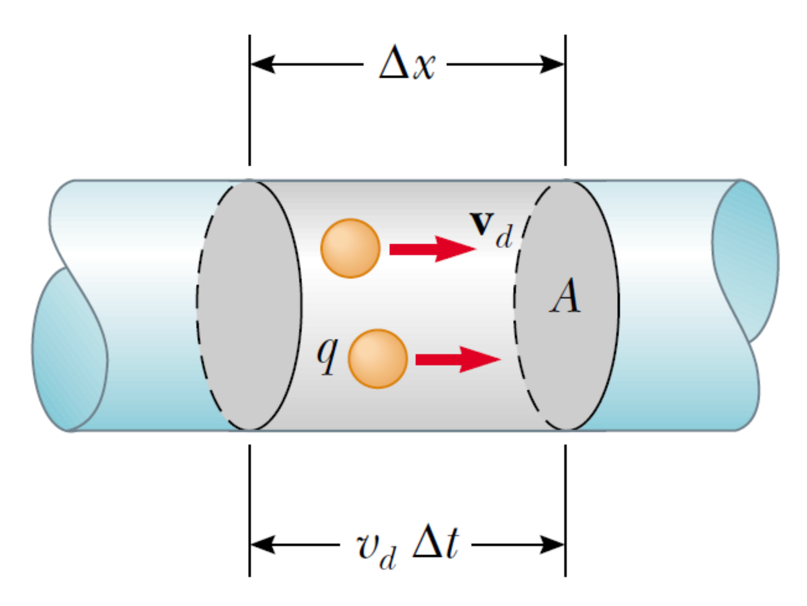
- where q is the charge on each carrier. If the carriers move with a speed v_d, the displacement they experience in the x direction in a time interval Δt is $\Delta x = v_d\Delta t$. Therefore:

$$\Delta Q = (nAv_d\Delta t)q$$

- the average current in a conductor in terms of microscopic quantities is:

$$I_{av} = \frac{\Delta Q}{\Delta t} = nqv_dA$$

- The speed of the charge carriers v_d is an average speed called the **drift speed**.

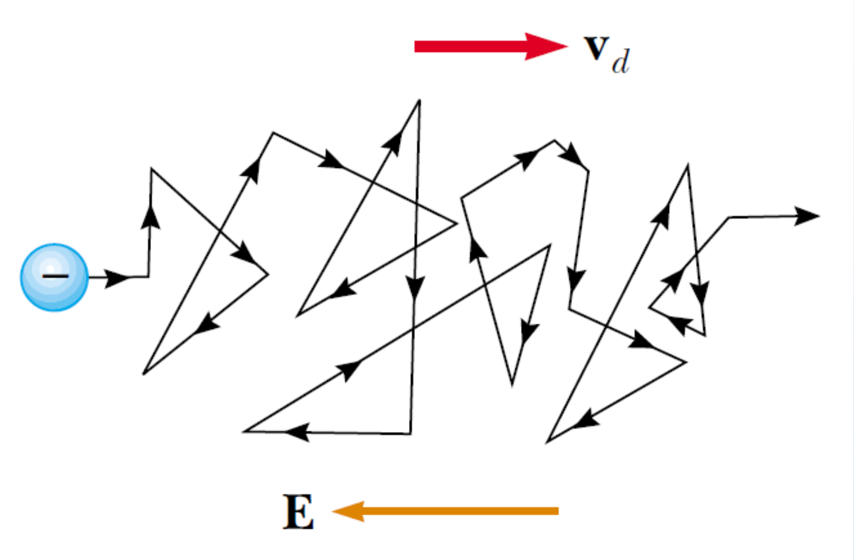


A section of a uniform conductor of cross-sectional area A. The mobile charge carriers move with a speed v_d, and the displacement they experience in the x direction in a time interval Δt is $\Delta x = v_d\Delta t$. If we choose Δt to be the time interval during which the charges are displaced, on the average, by the length of the cylinder, the number of carriers in the section of length Δx is $nAv_d\Delta t$, where n is the number of carriers per unit volume.

Electric Current

Microscopic Model of Current

- When a potential difference is applied across the conductor (for example, by means of a battery), an electric field is set up in the conductor; this field exerts an electric force on the electrons, producing a current.
- The electrons do not move in straight lines along the conductor. Instead, they collide repeatedly with the metal atoms, and their resultant motion is complicated and zigzag.
- The energy transferred from the electrons to the metal atoms during collisions causes an increase in the vibrational energy of the atoms and a corresponding increase in the temperature of the conductor.



A schematic representation of the zigzag motion of an electron in a conductor. The changes in direction are the result of collisions between the electron and atoms in the conductor. Note that the net motion of the electron is opposite the direction of the electric field. Because of the acceleration of the charge carriers due to the electric force, the paths are actually parabolic. However, the drift speed is much smaller than the average speed, so the parabolic shape is not visible on this scale.

Electric Current

Example 27.01

The 12-gauge copper wire in a typical residential building has a cross-sectional area of $3.31 \times 10^{-6}\ \mathrm{m}^2$. If it carries a current of 10.0 A, what is the drift speed of the electrons? Assume that each copper atom contributes one free electron to the current. The density of copper is $8.95\ \mathrm{g/cm^3}$.

We use $I = nqAv_d$, where n is the number of charge carriers per unit volume, and is identical to the number of atoms per unit volume. We assume a contribution of 1 free electron per atom in the relationship above. For copper, which has a molar mass of 63.5, we know that Avogadro's number of atoms, $N_A = 6.02 \times 10^{23}\ \mathrm{mol}^{-1}$, has a mass of 63.5 g. Thus, the mass per atom is:

$$\mathrm{V} = \frac{m}{\rho} = \frac{63.5}{8.95} = 7.09\ \mathrm{cm}^3 = 7.09 \times 10^{-6}\ \mathrm{m}^3$$

Because each copper atom contributes one free electron to the current, we have:

$$n = \frac{N_A}{\mathrm{V}} = \frac{6.02 \times 10^{23}}{7.09 \times 10^{-6}} = 8.49 \times 10^{28}\ \ \mathrm{electrons/m^3}$$

We find that the drift speed is:

$$v_d = \frac{I}{nqA} = \frac{10.0}{8.49 \times 10^{28} \times 1.60 \times 10^{-19} \times 3.31 \times 10^{-6}} = 2.22 \times 10^{-4}\ \ \mathrm{m/s}$$

Electric Current

Problem 27.01

In a particular cathode ray tube, the measured beam current is 30.0 μA. How many electrons strike the tube screen every 40.0 s?

$$I = \frac{\Delta Q}{\Delta t}$$

$$\Delta Q = I\Delta t = 30.0 \times 10^{-6} \times 40.0 = 1.20 \times 10^{-3}\ \text{C}$$

$$N = \frac{\Delta Q}{e} = \frac{1.20 \times 10^{-3}}{1.60 \times 10^{-19}} = 7.50 \times 10^{15}\ \text{electrons}$$

Electric Current

Problem 27.11

Additional problem

An aluminum wire having a cross-sectional area of $4.00 \times 10^{-6}\ \text{m}^2$ carries a current of 5.00 A. Find the drift speed of the electrons in the wire. The density of aluminum is $2.70\ \text{g/cm}^3$. Assume that one conduction electron is supplied by each atom.

We use $I = nqAv_d$, where n is the number of charge carriers per unit volume, and is identical to the number of atoms per unit volume. We assume a contribution of 1 free electron per atom in the relationship above. For aluminum, which has a molar mass of 27, we know that Avogadro's number of atoms, $N_A = 6.02 \times 10^{23}\ \text{mol}^{-1}$, has a mass of 27.0 g. Thus, the mass per atom is:

$$\text{V} = \frac{m}{\rho} = \frac{27.0}{2.70} = 10.0\ \text{cm}^3 = 1.0 \times 10^{-5}\ \text{m}^3$$

Because each aluminum atom contributes one free electron to the current, we have:

$$n = \frac{N_A}{\text{V}} = \frac{6.02 \times 10^{23}}{1.0 \times 10^{-5}} = 6.02 \times 10^{28}\ \text{electrons/m}^3$$

We find that the drift speed is:

$$v_d = \frac{I}{nqA} = \frac{5.00}{6.02 \times 10^{28} \times 1.60 \times 10^{-19} \times 4.00 \times 10^{-6}} = 1.30 \times 10^{-4}\ \text{m/s}$$

Electric Current

Problem 27.56

A high-voltage transmission line with a diameter of 2.00 cm and a length of 200 km carries a steady current of 1000 A. If the conductor is copper wire with a free charge density of 8.49×10^{28} electrons/m^3, how long does it take one electron to travel the full length of the line?

We find the drift velocity from:

$$I = nqv_d A = nqv_d(\pi r^2)$$

$$v_d = \frac{I}{nq(\pi r^2)}$$

$$v_d = \frac{1000}{8.49 \times 10^{28} \times 1.60 \times 10^{-19} \times 3.14 \times (1.00 \times 10^{-2})^2}$$

$$v_d = 2.34 \times 10^{-4}\ \text{m/s}$$

$$v_d = \frac{x}{t}$$

$$t = \frac{x}{v_d} = \frac{200 \times 10^3}{2.34 \times 10^{-4}} = 8.54 \times 10^8\ \text{s} = 27.0\ \text{years}$$

Resistance

- Consider a conductor of cross-sectional area A carrying a current I. The **current density** J in the conductor is defined as the current per unit area. Because the current $I = nqv_dA$, the current density is:

$$J = \frac{I}{A} = nqv_d$$

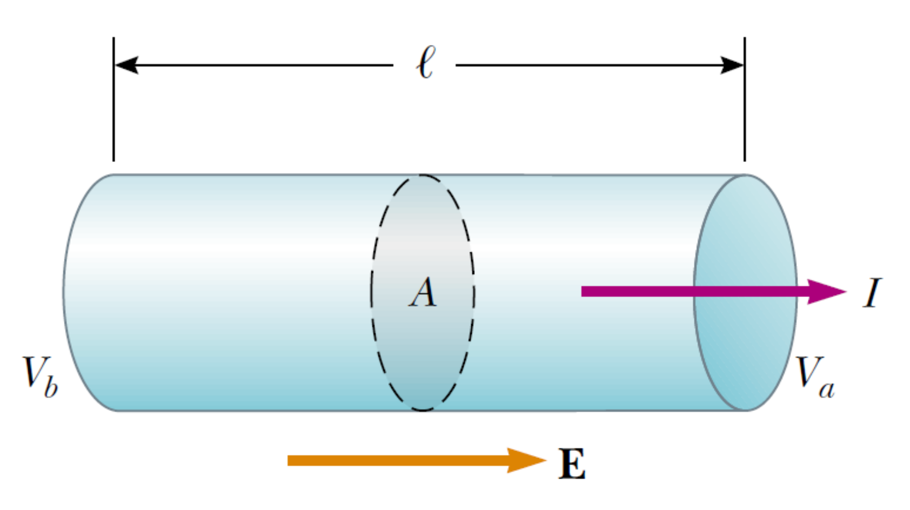
- where J has SI units of $\mathrm{A/m^2}$.
- This expression is valid only if the current density is uniform and only if the surface of cross-sectional area A is perpendicular to the direction of the current.
- In general, current density is a vector quantity:

$$\vec{J} = nq\vec{v}_d$$

- The current density is in the direction of charge motion for positive charge carriers and opposite the direction of motion for negative charge carriers.
- **A current density $\vec{J}$ and an electric field $\vec{E}$ are established in a conductor whenever a potential difference is maintained across the conductor.**
- In some materials, the current density is proportional to the electric field:

$$\vec{J} = \sigma\vec{E} \quad ^{a}$$

- where the constant of proportionality σ is called the **conductivity** of the conductor.



A uniform conductor of length l and cross-sectional area A. A potential difference $\Delta V = V_b - V_a$ maintained across the conductor sets up an electric field $\vec{E}$, and this field produces a current I that is proportional to the potential difference.

[a] Do not confuse conductivity σ with surface charge density, for which the same symbol is used.

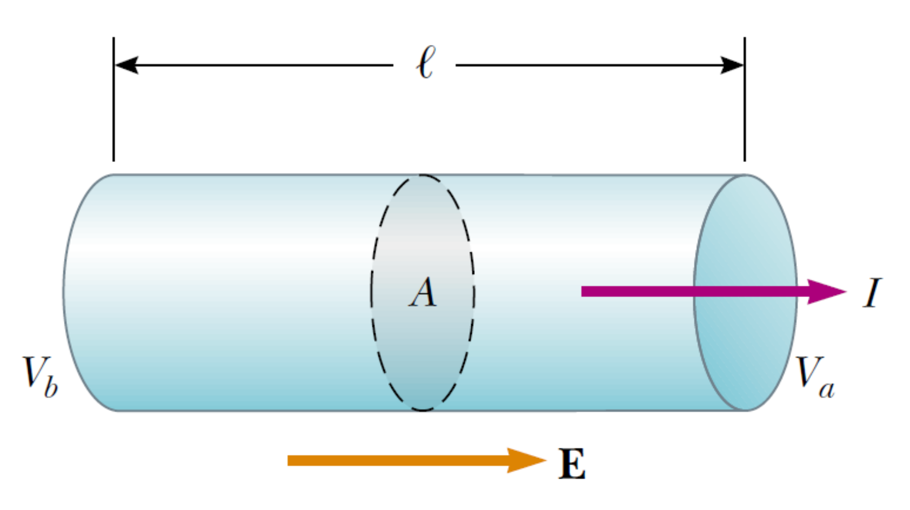
Resistance

- Materials that obey the equation $\vec{J} = \sigma\vec{E}$ are said to follow **Ohm's law**.
- For many materials (including most metals), the ratio of the current density to the electric field is a constant σ that is independent of the electric field producing the current.
- Materials that obey Ohm's law and hence demonstrate this simple relationship between $\vec{E}$ and $\vec{J}$ are said to be ohmic. Materials and devices that do not obey Ohm's law are said to be nonohmic.
- Consider a segment of straight wire of uniform cross-sectional area A and length l.
- A potential difference $\Delta V = V_b - V_a$ is maintained across the wire, creating in the wire an electric field and a current.
- If the field is assumed to be uniform, the potential difference is related to the field through the relationship:

$$\Delta V = V_b - V_a = -\int_a^b \vec{E}\cdot d\vec{s} = E\int_0^l dx = El$$

- Therefore, we can express the magnitude of the current density in the wire as:

$$J = \sigma E = \sigma\frac{\Delta V}{l}$$



A uniform conductor of length l and cross-sectional area A. A potential difference $\Delta V = V_b - V_a$ maintained across the conductor sets up an electric field $\vec{E}$, and this field produces a current I that is proportional to the potential difference.

Resistance

- Because $J = I/A$, we can write the potential difference as:

$$\Delta V = \frac{l}{\sigma} J = \left(\frac{l}{\sigma A}\right) I = RI$$

- The quantity $R = l/\sigma A$ is called the **resistance** of the conductor. We can define the resistance as the ratio of the potential difference across a conductor to the current in the conductor:

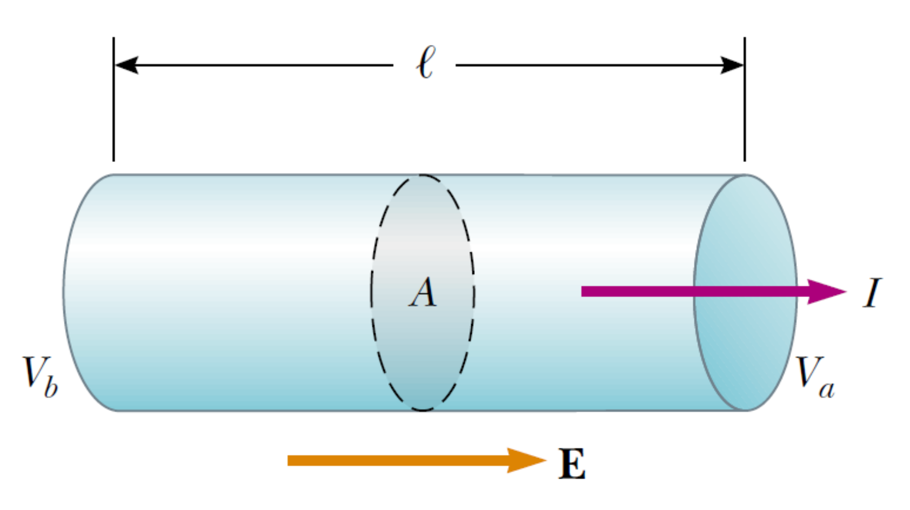
$$R = \frac{\Delta V}{I}$$

- The resistance has SI units of volts per ampere. One volt per ampere is defined to be one **ohm** (Ω):

$$1\ \Omega = \frac{1\ \text{V}}{1\ \text{A}}$$

- The inverse of conductivity is **resistivity** ρ:

$$\rho = \frac{1}{\sigma}$$



A uniform conductor of length l and cross-sectional area A. A potential difference $\Delta V = V_b - V_a$ maintained across the conductor sets up an electric field $\vec{E}$, and this field produces a current I that is proportional to the potential difference.

Resistance

- where ρ has the units ohm-meters ($\Omega \cdot$ m). Because $R = l/\sigma A$, we can express the resistance of a uniform block of material along the length l as:

$$R = \rho \frac{l}{A} \quad {}^{d}$$

- The resistance of a sample depends on geometry as well as on resistivity.
- The resistance of a wire is proportional to its length and inversely proportional to its cross-sectional area.
- Every ohmic material has a characteristic resistivity that depends on the properties of the material and on temperature.
- An ideal conductor would have zero resistivity, and an ideal insulator would have infinite resistivity.

Resistivities and Temperature Coefficients of Resistivity for Various Materials

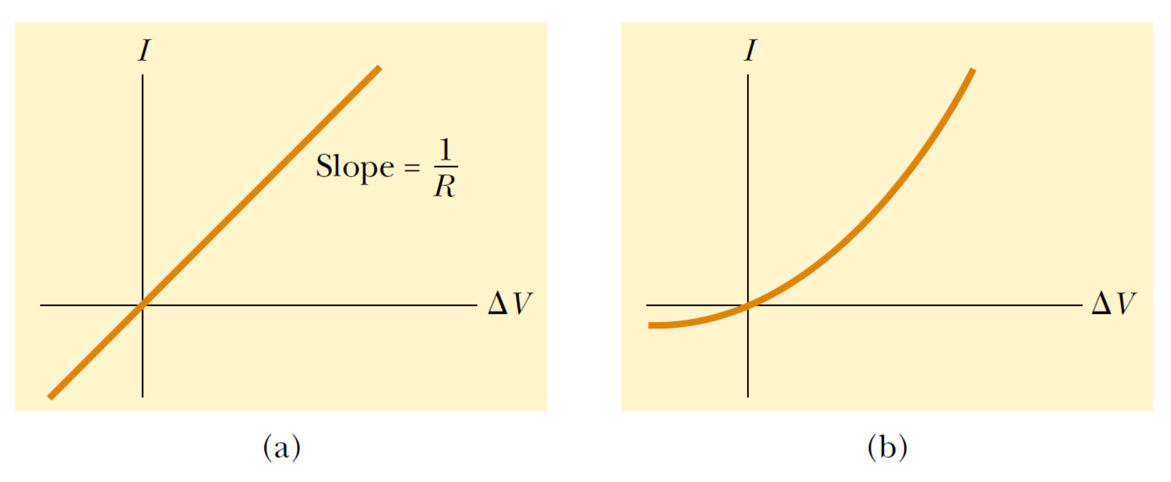
Material	Resistivity[a] ($\Omega \cdot$ m)	Temperature Coefficient[b] $\alpha[(°C)^{-1}]$
Silver	1.59×10^{-8}	3.8×10^{-3}
Copper	1.7×10^{-8}	3.9×10^{-3}
Gold	2.44×10^{-8}	3.4×10^{-3}
Aluminum	2.82×10^{-8}	3.9×10^{-3}
Tungsten	5.6×10^{-8}	4.5×10^{-3}
Iron	10×10^{-8}	5.0×10^{-3}
Platinum	11×10^{-8}	3.92×10^{-3}
Lead	22×10^{-8}	3.9×10^{-3}
Nichrome[c]	1.50×10^{-6}	0.4×10^{-3}
Carbon	3.5×10^{-5}	-0.5×10^{-3}
Germanium	0.46	-48×10^{-3}
Silicon	640	-75×10^{-3}
Glass	10^{10} to 10^{14}	
Hard rubber	$\sim 10^{13}$	
Sulfur	10^{15}	
Quartz (fused)	75×10^{16}	

[a] All values at 20°C.
[b] See Section 27.4.
[c] A nickel–chromium alloy commonly used in heating elements.

[d] Do not confuse resistivity ρ with mass density or charge density, for which the same symbol is used.

Resistance

- Ohmic materials and devices have a linear current–potential difference relationship over a broad range of applied potential differences.
- The slope of the I-versus-ΔV curve in the linear region yields a value for $1/R$.
- Nonohmic materials have a nonlinear current–potential difference relationship.



(a) The current–potential difference curve for an ohmic material. The curve is linear, and the slope is equal to the inverse of the resistance of the conductor.
A nonlinear current–potential difference curve for a junction diode. This device does not obey Ohm's law.

Resistance

King Saud University

Example 27.02

Calculate the resistance of an aluminum cylinder that has a length of 10.0 cm and a cross-sectional area of $2.00 \times 10^{-4}\ \text{m}^2$ (resistivity of aluminum is $2.82 \times 10^{-8}\ \Omega \cdot \text{m}$). Repeat the calculation for a cylinder of the same dimensions and made of glass having a resistivity of $3.0 \times 10^{10}\ \Omega \cdot \text{m}$.

We can calculate the resistance of the aluminum cylinder as follows:

$$R = \rho \frac{l}{A} = 2.82 \times 10^{-8} \times \frac{10.0 \times 10^{-2}}{2.00 \times 10^{-4}} = 1.41 \times 10^{-5}\ \Omega$$

Similarly, for glass we find that:

$$R = \rho \frac{l}{A} = 3.0 \times 10^{10} \times \frac{10.0 \times 10^{-2}}{2.00 \times 10^{-4}} = 1.5 \times 10^{13}\ \Omega$$

As you might guess from the large difference in resistivities, the resistances of identically shaped cylinders of aluminum and glass differ widely. The resistance of the glass cylinder is 18 orders of magnitude greater than that of the aluminum cylinder.

Resistance

Example 27.03

(a) Calculate the resistance per unit length of a 22-gauge Nichrome wire, which has a radius of 0.321 mm. (b) If a potential difference of 10 V is maintained across a 1.0-m length of the Nichrome wire, what is the current in the wire? (resistivity of Nichrome is $1.5 \times 10^{-6}\ \Omega \cdot \text{m}$).

(a) The cross-sectional area of this wire is:

$$A = \pi r^2 = 3.14 \times \left(0.321 \times 10^{-3}\right)^2 = 3.24 \times 10^{-7}\ \text{m}^2$$

The resistance per unit length is:

$$\frac{R}{l} = \frac{\rho}{A} = \frac{1.5 \times 10^{-6}}{3.24 \times 10^{-7}} = 4.6\ \Omega/\text{m}$$

(b) Because a 1.0-m length of this wire has a resistance of 4.6 Ω, therefore:

$$I = \frac{\Delta V}{R} = \frac{10}{4.6} = 2.2\ \text{A}$$

Note that the resistivity of Nichrome wire is about 100 times that of copper. A copper wire of the same radius would have a resistance per unit length of only $0.052\ \Omega/\text{m}$. A 1.0-m length of copper wire of the same radius would carry the same current (2.2 A) with an applied potential difference of only 0.11 V.

Because of its high resistivity and its resistance to oxidation, Nichrome is often used for heating elements in toasters, irons, and electric heaters.

Resistance

Problem 27.12

Additional problem

Calculate the current density in a gold wire at 20 °C, if an electric field of $0.740\ \mathrm{V/m}$ exists in the wire. (resistivity of gold is $2.44 \times 10^{-8}\ \Omega \cdot \mathrm{m}$).

$$J = \sigma E = \frac{E}{\rho} = \frac{0.740}{2.44 \times 10^{-8}} = 3.03 \times 10^{7}\ \mathrm{A/m^2}$$

Resistance

Problem 27.15

Additional problem

A 0.900 V potential difference is maintained across a 1.50 m length of tungsten wire that has a cross-sectional area of 0.600 mm^2. What is the current in the wire? (resistivity of tungsten is $5.60 \times 10^{-8}\ \Omega \cdot \text{m}$).

$$\Delta V = IR$$

$$R = \rho \frac{l}{A}$$

$$\Delta V = I\rho \frac{l}{A}$$

$$I = \Delta V \frac{A}{\rho l} = 0.900 \times \frac{6.00 \times 10^{-7}}{5.60 \times 10^{-8} \times 1.50} = 6.43\ \text{A}$$

Resistance

Problem 27.16

A conductor of uniform radius 1.20 cm carries a current of 3.00 A produced by an electric field of 120 V/m . What is the resistivity of the material?

$$J = \frac{I}{A} = \frac{I}{\pi r^2} = \sigma E = \frac{E}{\rho}$$

$$\rho = \frac{\pi r^2 E}{I} = \frac{3.14 \times \left(1.20 \times 10^{-2}\right)^2 \times 120}{3.00} = 0.0181\ \Omega \cdot \mathrm{m}$$

Resistance

King Saud University

Problem 27.22

Aluminum and copper wires of equal length are found to have the same resistance. What is the ratio of their radii? (resistivity of Aluminum is $2.82 \times 10^{-8}\ \Omega \cdot \mathrm{m}$ and resistivity of copper is $1.70 \times 10^{-8}\ \Omega \cdot \mathrm{m}$).

$$R_{Al} = R_{Cu}$$

$$\rho_{Al}\frac{l}{A_{Al}} = \rho_{Cu}\frac{l}{A_{Cu}}$$

$$\rho_{Al}\frac{l}{\pi r_{Al}^2} = \rho_{Cu}\frac{l}{\pi r_{Cu}^2}$$

$$\frac{r_{Al}^2}{r_{Cu}^2} = \frac{\rho_{Al}}{\rho_{Cu}}$$

$$\frac{r_{Al}}{r_{Cu}} = \sqrt{\frac{\rho_{Al}}{\rho_{Cu}}} = \sqrt{\frac{2.82 \times 10^{-8}}{1.70 \times 10^{-8}}} = 1.29$$

Resistance and Temperature

- Over a limited temperature range, the resistivity of a conductor varies approximately linearly with temperature according to the expression:

$$\rho = \rho_0[1 + \alpha(T - T_0)]$$

- where ρ is the resistivity at some temperature T (in degrees Celsius), ρ_0 is the resistivity at some reference temperature T_0 (usually taken to be 20 °C), and α is **the temperature coefficient of resistivity**.
- From the previous equation, we see that the temperature coefficient of resistivity can be expressed as:

$$\alpha = \frac{1}{\rho_0}\frac{\Delta\rho}{\Delta T}$$

- where $\Delta\rho = \rho - \rho_0$ is the change in resistivity in the temperature interval $\Delta T = T - T_0$.
- Note that the unit for α is degrees Celsius^{-1} $[(°C)^{-1}]$.
- Because resistance is proportional to resistivity, we can write the variation of resistance as:

$$R = R_0[1 + \alpha(T - T_0)]$$

Resistivities and Temperature Coefficients of Resistivity for Various Materials

Material	Resistivity[a] ($\Omega \cdot m$)	Temperature Coefficient[b] $\alpha[(°C)^{-1}]$
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Nichrome[c]	1.50×10^{-6}	0.4×10^{-3}
Carbon	3.5×10^{-5}	-0.5×10^{-3}
Germanium	0.46	-48×10^{-3}
Silicon	640	-75×10^{-3}
Glass	10^{10} to 10^{14}	
Hard rubber	$\sim 10^{13}$	
Sulfur	10^{15}	
Quartz (fused)	75×10^{16}	

[a] All values at 20°C.
[b] See Section 27.4.
[c] A nickel–chromium alloy commonly used in heating elements.

Resistance and Temperature

Example 27.06

A resistance thermometer, which measures temperature by measuring the change in resistance of a conductor, is made from platinum and has a resistance of 50 Ω at 20 °C. When immersed in a vessel containing melting indium, its resistance increases to 76.8 Ω. Calculate the melting point of the indium.

$$R = R_0[1 + \alpha(T - T_0)]$$

Solving Equation ΔT and using the $\alpha = 3.92 \times 10^{-3}\ (°\mathrm{C})^{-1}$ for platinum, we obtain:

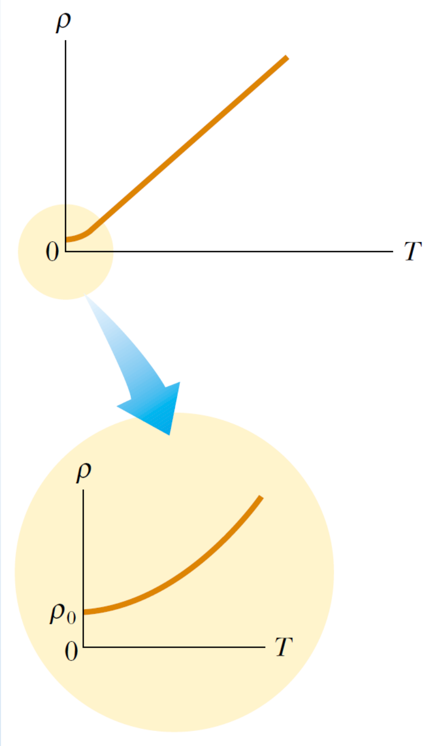
$$\Delta T = T - T_0 = \frac{R - R_0}{\alpha R_0} = \frac{76.8 - 50}{3.92 \times 10^{-3} \times 50} = 137\ °\mathrm{C}$$

Because $T_0 = 20\ °\mathrm{C}$, we find that T, the temperature of the melting indium sample, is:

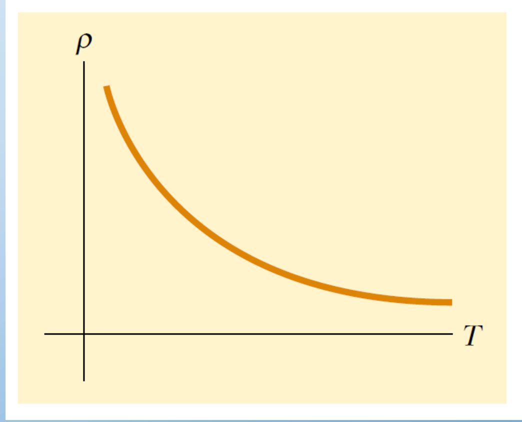
$$T = \Delta T + T_0 = 137 + 20 = 157\ °\mathrm{C}$$

Resistance and Temperature

- For metals like copper, resistivity is nearly proportional to temperature.
- However, a nonlinear region always exists at very low temperatures, and the resistivity usually reaches some finite value as the temperature approaches absolute zero.
- This residual resistivity near absolute zero is caused primarily by the collision of electrons with impurities and imperfections in the metal.
- In contrast, high temperature resistivity (the linear region) is predominantly characterized by collisions between electrons and metal atoms.
- Notice that three of the α values in the previous table are negative; this indicates that the resistivity of these materials decreases with increasing temperature, which is indicative of a class of materials called <u>semiconductors</u>.
- This behavior is due to an increase in the density of charge carriers at higher temperatures.



Resistivity versus temperature for a metal such as copper. The curve is linear over a wide range of temperatures, and ρ increases with increasing temperature. As T approaches absolute zero (inset), the resistivity approaches a finite value ρ_0.



Resistivity versus temperature for a pure semiconductor, such as silicon or germanium.

Resistance and Temperature

Problem 27.32

Additional problem

An aluminum rod has a resistance of 1.234 Ω at 20 °C. Calculate the resistance of the rod at 120 °C by accounting for the changes in both the resistivity and the dimensions of the rod. (Hint: $l = l_0[1 + \alpha_l(T - T_0)]$, where α_l is the average coefficient of linear expansion).

For aluminum:

$$\alpha = 3.90 \times 10^{-3}\ (°\text{C})^{-1}$$

$$\alpha_l = 24.0 \times 10^{-6}\ (°\text{C})^{-1}$$

$$R = \frac{\rho l}{A}$$

$$R = \frac{\rho_0[1 + \alpha(T - T_0)]l_0[1 + \alpha_l(T - T_0)]}{A_0[1 + \alpha_l(T - T_0)]^2}$$

$$R = R_0 \frac{[1 + \alpha(T - T_0)]}{[1 + \alpha_l(T - T_0)]}$$

$$R = 1.234 \times \frac{[1 + (3.90 \times 10^{-3} \times (120 - 20))]}{[1 + (24.0 \times 10^{-6} \times (120 - 20))]}$$

$$R = 1.72\ \Omega$$

Resistance and Temperature

Problem 27.33

What is the rising factor of the resistance of an iron filament when its temperature changes from 25 °C to 50 °C?

For iron: $\alpha = 5.0 \times 10^{-3}\ (°\mathrm{C})^{-1}$

$$R = R_0[1 + \alpha \Delta T]$$

$$R = R_0 + R_0 \alpha \Delta T$$

$$R - R_0 = R_0 \alpha \Delta T$$

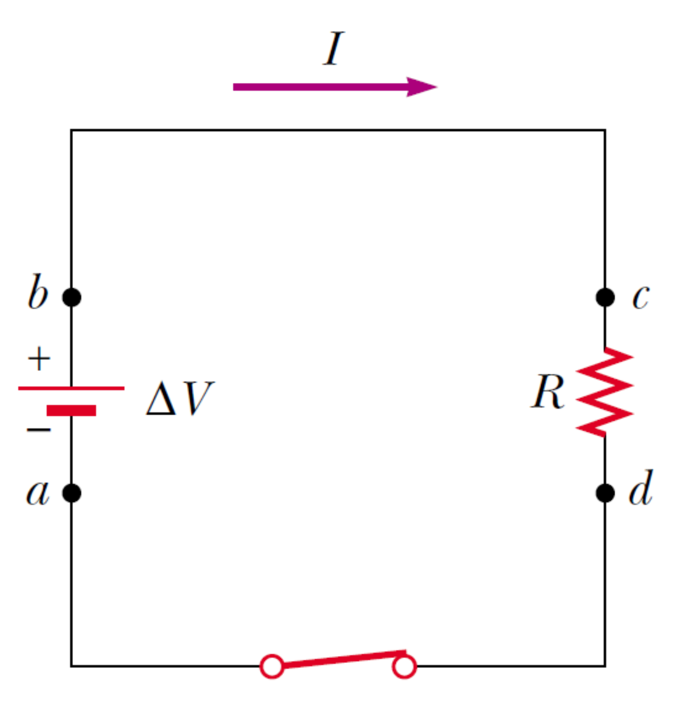
$$\frac{\Delta R}{R_0} = \alpha \Delta T = 5.0 \times 10^{-3} \times 25 = 0.125$$

Electrical Power

- If a battery is used to establish an electric current in a conductor, there is a continuous transformation of chemical energy in the battery to kinetic energy of the electrons to internal energy in the conductor, resulting in an increase in the temperature of the conductor.
- Imagine following a positive quantity of charge Q that is moving clockwise around the circuit. (the resistance of the interconnecting wires is neglected)
- As the charge moves from a to b through the battery, the electric potential energy of the system increases by an amount $Q\Delta V$ while the chemical potential energy in the battery decreases by the same amount.
- However, as the charge moves from c to d through the resistor, the system loses this electric potential energy during collisions of electrons with atoms in the resistor.
- In this process, the energy is transformed to internal energy corresponding to increased vibrational motion of the atoms in the resistor.
- The rate at which the system loses electric potential energy as the charge Q passes through the resistor:

$$\frac{dU}{dt} = \frac{d}{dt}(Q\Delta V) = \frac{dQ}{dt}\Delta V = I\Delta V$$

- The rate at which the system loses potential energy as the charge passes through the resistor is equal to the rate at which the system gains internal energy in the resistor.



A circuit consisting of a resistor of resistance R and a battery having a potential difference ΔV across its terminals. Positive charge flows in the clockwise direction.

Electrical Power

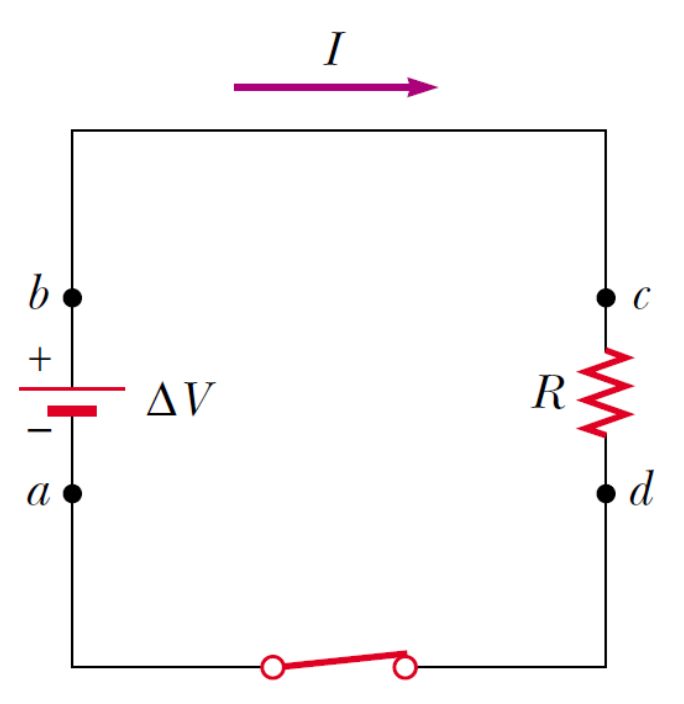
- The power $\mathcal{P}$, representing the rate at which energy is delivered to the resistor, is:

$$\mathcal{P} = I\Delta V$$

- This equation can be used to calculate the power delivered by a voltage source to any device carrying a current I and having a potential difference ΔV between its terminals.
- Using the fact that $\Delta V = IR$ for a resistor, we can express the power delivered to the resistor in the alternative forms:

$$\mathcal{P} = I^2R = \frac{(\Delta V)^2}{R}$$

- When I is expressed in amperes, ΔV in volts, and R in ohms, the SI unit of power is the watt (W).
- The process by which power is lost as internal energy in a conductor of resistance R is often called **joule heating**.
- When transporting energy by electricity through power lines, utility companies seek to minimize the power transformed to internal energy in the lines and maximize the energy delivered to the consumer.
- Because $\mathcal{P} = I\Delta V$, the same amount of power can be transported either at high currents and low potential differences or at low currents and high potential differences.
- Utility companies choose to transport energy at low currents and high potential differences primarily for economic reasons.



A circuit consisting of a resistor of resistance R and a battery having a potential difference ΔV across its terminals. Positive charge flows in the clockwise direction.

Electrical Power

Example 27.07

An electric heater is constructed by applying a potential difference of 120 V to a Nichrome wire that has a total resistance of 8.00 Ω. Find the current carried by the wire and the power rating of the heater.

Because $\Delta V = IR$, we have

$$I = \frac{\Delta V}{R} = \frac{120}{8} = 15.0\text{ A}$$

We can find the power rating using the expression $\mathcal{P} = I^2R$:

$$\mathcal{P} = I^2R = (15)^2 \times 8 = 1.80 \times 10^3\text{ W} = 1.8\text{ kW}$$

What If? What if the heater were accidentally connected to a 240 V supply? (This is difficult to do because the shape and orientation of the metal contacts in 240 V plugs are different from those in 120 V plugs.) How would this affect the current carried by the heater and the power rating of the heater?

If we doubled the applied potential difference, $I = \Delta V/R$ tells us that the current would double. According to $\mathcal{P} = (\Delta V)^2/R$, the power would be four times larger.

Electrical Power

Example 27.08

Additional example

(a) What is the required resistance of an immersion heater that will increase the temperature of 1.50 kg of water from 10.0 ℃ to 50.0 ℃ in 10.0 min while operating at 110 V? (b) Estimate the cost of heating the water. (Hint: $Q = mc\Delta T$, where c is the specific heat of a substance, for water $c = 4186\ \mathrm{J/kg \cdot ^\circ C}$).

This example allows us to link our new understanding of power in electricity with specific heat in thermodynamics. An immersion heater is a resistor that is inserted into a container of water. As energy is delivered to the immersion heater, raising its temperature, energy leaves the surface of the resistor by heat, going into the water. When the immersion heater reaches a constant temperature, the rate of energy delivered to the resistance by electrical transmission is equal to the rate of energy delivered by heat to the water.

(a) To simplify the analysis, we ignore the initial period during which the temperature of the resistor increases, and also ignore any variation of resistance with temperature. Thus, we imagine a constant rate of energy transfer for the entire 10.0 min. Setting the rate of energy delivered to the resistor equal to the rate of energy entering the water by heat, we have:

$$\mathcal{P} = \frac{(\Delta V)^2}{R} = \frac{Q}{\Delta t}$$

$$\frac{(\Delta V)^2}{R} = \frac{mc\Delta T}{\Delta t}$$

Electrical Power

Example 27.08

Additional example

(a) What is the required resistance of an immersion heater that will increase the temperature of 1.50 kg of water from 10.0 °C to 50.0 °C in 10.0 min while operating at 110 V? (b) Estimate the cost of heating the water. (Hint: $Q = mc\Delta T$, where c is the specific heat of a substance, for water $c = 4186\ \mathrm{J/kg \cdot {}^\circ C}$).

$$R = \frac{(\Delta V)^2 \Delta t}{mc\Delta T} = \frac{(110)^2 \times (10.0 \times 60)}{1.50 \times 4186 \times (50.0 - 10.0)} = 28.9\ \Omega$$

(b) Because the energy transferred equals power multiplied by time interval, the amount of energy transferred is:

$$\mathcal{P}\Delta t = \frac{(\Delta V)^2}{R}\Delta t = \frac{(110)^2}{28.9} \times \frac{10.0}{60} = 69.8\ \mathrm{Wh} = 0.0698\ \mathrm{kWh}$$

If the energy is purchased at an estimated price of 10.0 ¢ per kilowatt-hour, the cost is:

$$\text{cost} = 0.0698 \times 10.0 = 0.7\ ¢ = 0.007\ \$$$

Electrical Power

Problem 27.36

A toaster is rated at 600 W when connected to a 120 V source. What current does the toaster carry, and what is its resistance?

$$\mathcal{P} = I\Delta V$$

$$I = \frac{\mathcal{P}}{\Delta V} = \frac{600}{120} = 5.00\ \text{A}$$

$$R = \frac{\Delta V}{I} = \frac{120}{5} = 24.0\ \Omega$$

Electrical Power

Problem 27.49

Compute the cost per day of operating a lamp that draws a current of 1.70 A from a 110 V line. Assume the cost of energy from the power company is 0.060 \$/kWh.

$\mathcal{P} = I\Delta V = 1.70 \times 110 = 187\ \text{W} = 0.187\ \text{kW}$

$\text{Energy used in a 24-hour day} = 0.187 \times 24 = 4.49\ \text{kWh}$

$\text{cost} = 4.49 \times 0.060 = 0.27\ \$ = 27\ ¢$