104 PHYS

Ch. 29

Magnetic Fields

Dr. Ayman Alismail ©

Contents

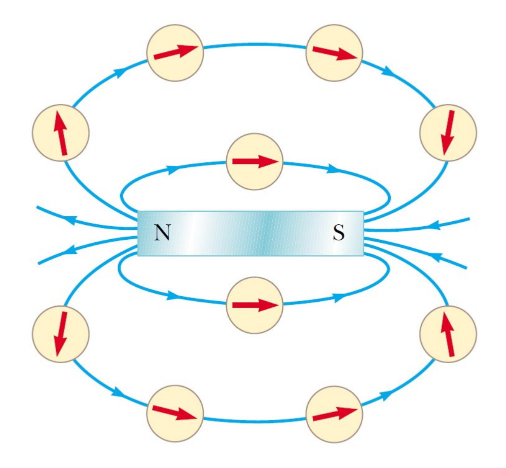
Magnetic Fields and Forces

- The early Greeks knew about magnetism as early as 800 B.C. They discovered that the stone magnetite (Fe_3O_4) attracts pieces of iron.
- Every magnet, regardless of its shape, has two poles, called north (N) and south (S) poles, that exert forces on other magnetic poles similar to the way that electric charges exert forces on one another.
- Poles (N − N or S − S) repel each other, and opposite poles (N − S) attract each other.
- Electric charges can be isolated (witness the electron and proton) whereas a single magnetic pole has never been isolated. That is, magnetic poles are always found in pairs.
- No matter how many times a permanent magnet is cut in two, each piece always has a north and a south pole.
- The relationship between magnetism and electricity:
    - It was discovered in 1819 when, Oersted found that an electric current in a wire deflected a nearby compass needle.
    - In the 1820s, Faraday and Henry (1797–1878) independently showed that an electric current can be produced in a circuit either by moving a magnet near the circuit or by changing the current in a nearby circuit. These observations demonstrate that a changing magnetic field creates an electric field.
    - Years later, theoretical work by Maxwell showed that the reverse is also true: a changing electric field creates a magnetic field.

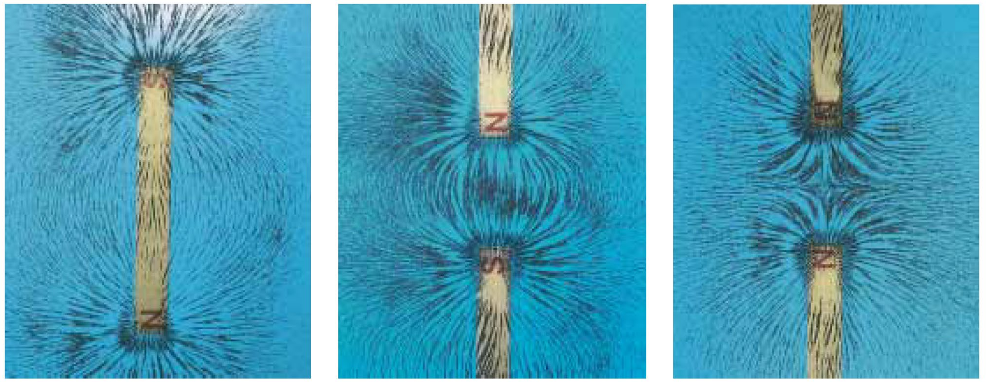
Magnetic fingerprinting allows fingerprints to be seen on surfaces that otherwise would not allow prints to be lifted. The powder spread on the surface is coated with an organic material that adheres to the greasy residue in a fingerprint. A magnetic “brush” removes the excess powder and makes the fingerprint visible.

Magnetic Fields and Forces

- The region of space surrounding any moving electric charge also contains a magnetic field.
- A magnetic field also surrounds a magnetic substance making up a permanent magnet.
- The symbol $\vec{B}$ has been used to represent a magnetic field.
- The direction of the magnetic field $\vec{B}$ at any location is the direction in which a compass needle points at that location.
- As with the electric field, we can represent the magnetic field by means of drawings with magnetic field lines.
- The magnetic field lines outside the magnet point away from north poles and toward south poles.
- A magnetic field $\vec{B}$ at some point in space can be defied in terms of the magnetic force $\vec{F}_B$ that the field exerts on a charged particle moving with a velocity $\vec{v}$.



Compass needles can be used to trace the magnetic field lines in the region outside a bar magnet.



(a) Magnetic field pattern surrounding a bar magnet as displayed with iron filings. (b) Magnetic field pattern between opposite poles (N – S) of two bar magnets. (c) Magnetic field pattern between like poles (N – N) of two bar magnets.

Magnetic Fields and Forces

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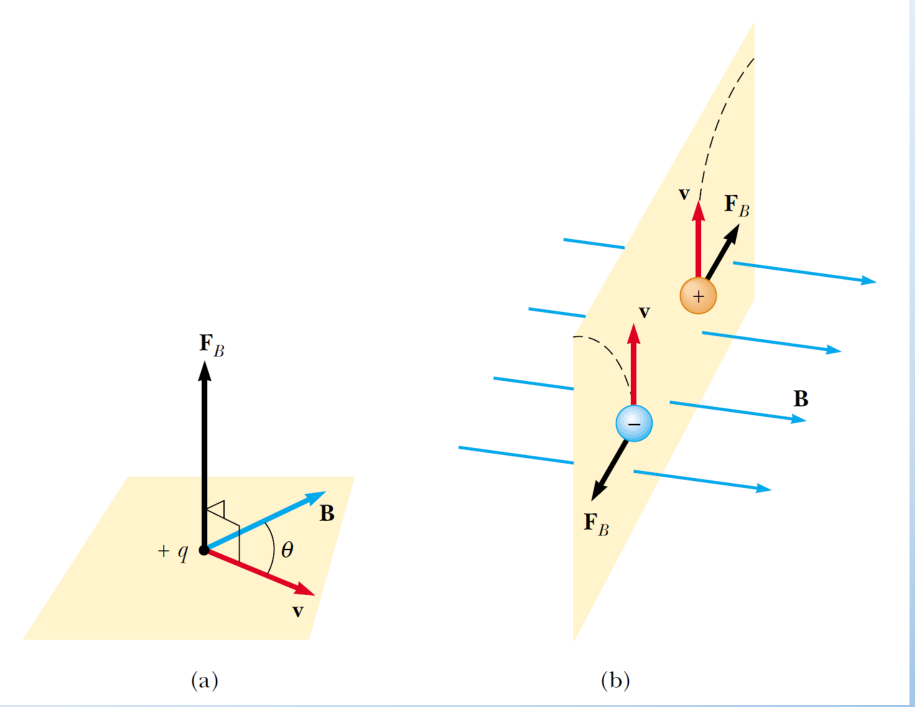
- Experiments on various charged particles moving in a magnetic field give the following results:
  - The magnitude F_B of the magnetic force exerted on the particle is proportional to the charge q and to the speed v of the particle.
  - The magnitude and direction of $\vec{F}_B$ depend on the velocity of the particle and on the magnitude and direction of the magnetic field $\vec{B}$.
  - When a charged particle moves parallel to the magnetic field vector, the magnetic force acting on the particle is zero.
  - When the particle's velocity vector makes any angle $\theta \neq 0$ with the magnetic field, the magnetic force acts in a direction perpendicular to both $\vec{v}$ and $\vec{B}$; that is, $\vec{F}_B$ is perpendicular to the plane formed by $\vec{v}$ and $\vec{B}$ (Fig. (a)).
  - The magnetic force exerted on a positive charge is in the direction opposite the direction of the magnetic force exerted on a negative charge moving in the same direction (Fig. (b)).
  - The magnitude of the magnetic force exerted on the moving particle is proportional to $\sin\theta$, where θ is the angle the particle's velocity vector makes with the direction of $\vec{B}$.
- We can summarize these observations by writing the magnetic force in the form:

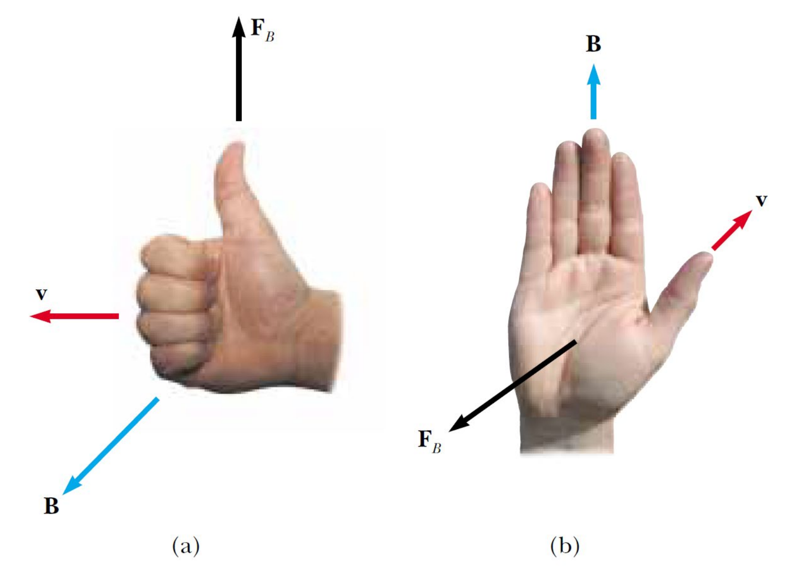
$$\vec{F}_B = q\vec{v} \times \vec{B}$$



The direction of the magnetic force $\vec{F}_B$ acting on a charged particle moving with a velocity $\vec{v}$ in the presence of a magnetic field $\vec{B}$. (a) The magnetic force is perpendicular to both $\vec{v}$ and $\vec{B}$. (b) Oppositely directed magnetic forces $\vec{F}_B$ are exerted on two oppositely charged particles moving at the same velocity in a magnetic field. The dashed lines show the paths of the particles.

Magnetic Fields and Forces

- Two right-hand rules for determining the direction of the cross product $\vec{v} \times \vec{B}$ and determining the direction of $\vec{F}_B$.
- First rule in Fig. (a):
  - Point the four fingers of your right hand along the direction of $\vec{v}$ with the palm facing $\vec{B}$ and curl them toward $\vec{B}$.
  - The extended thumb, which is at a right angle to the fingers, points in the direction of $\vec{v} \times \vec{B}$.
  - Because $\vec{F}_B = q\vec{v} \times \vec{B}$, $\vec{F}_B$ is in the direction of your thumb if q is positive and opposite the direction of your thumb if q is negative.
- An alternative rule in Fig. (b):
  - The thumb points in the direction of $\vec{v}$ and the extended fingers in the direction of $\vec{B}$.
  - The force $\vec{F}_B$ on a positive charge extends outward from your palm. The advantage of this rule is that the force on the charge is in the direction that you would push on something with your hand—outward from your palm.
  - The force on a negative charge is in the opposite direction.
- <u>Feel free to use either of these two right-hand rules</u>.



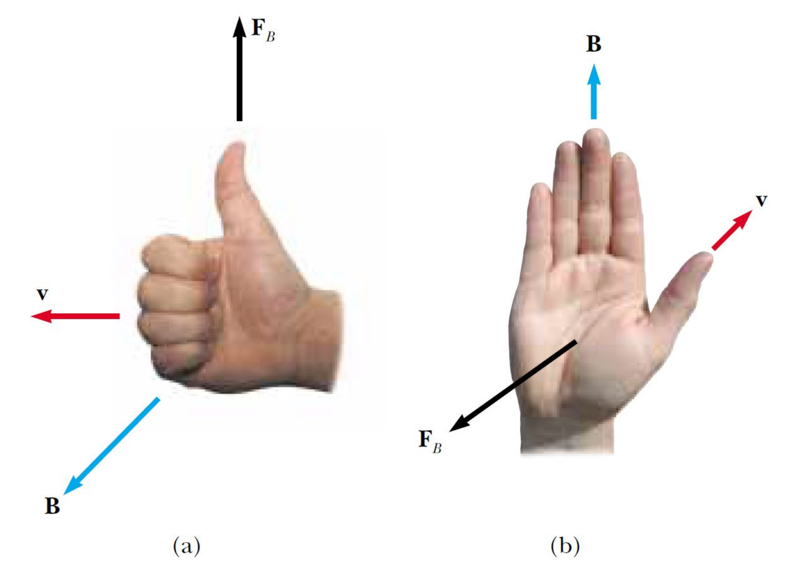
Two right-hand rules for determining the direction of the magnetic force $\vec{F}_B = q\vec{v} \times \vec{B}$ acting on a particle with charge q moving with a velocity $\vec{v}$ in a magnetic field $\vec{B}$. (a) In this rule, the fingers point in the direction of $\vec{v}$, with $\vec{B}$ coming out of your palm, so that you can curl your fingers in the direction of $\vec{B}$. The direction of $\vec{v} \times \vec{B}$, and the force on a positive charge, is the direction in which the thumb points. (b) In this rule, the vector $\vec{v}$ is in the direction of your thumb and $\vec{B}$ in the direction of your fingers. The force $\vec{F}_B$ on a positive charge is in the direction of your palm, as if you are pushing the particle with your hand.

Magnetic Fields and Forces

- The magnitude of the magnetic force on a charged particle is:

$$F_B = |q|vB\sin\theta$$

- Where θ is the smaller angle between $\vec{v}$ and $\vec{B}$.
- From this expression, we see that $\vec{F}_B$ is zero when $\vec{v}$ is parallel or antiparallel to $\vec{B}$ ($\theta = 0$ or $180°$) and maximum when $\vec{v}$ is perpendicular to $\vec{B}$ ($\theta = 90$).
- There are several important differences between electric and magnetic forces:
  - The electric force acts along the direction of the electric field, whereas the magnetic force acts perpendicular to the magnetic field.
  - The electric force acts on a charged particle regardless of whether the particle is moving, whereas the magnetic force acts on a charged particle only when the particle is in motion.
  - The electric force does work in displacing a charged particle, whereas the magnetic force associated with a steady magnetic field does no work when a particle is displaced because the force is perpendicular to the displacement.
- The kinetic energy of a charged particle moving through a magnetic field cannot be altered by the magnetic field alone. In other words, when a charged particle moves with a velocity $\vec{v}$ through a magnetic field, the field can alter the direction of the velocity vector but cannot change the speed or kinetic energy of the particle.



Two right-hand rules for determining the direction of the magnetic force $\vec{F}_B = q\vec{v}\times\vec{B}$ acting on a particle with charge q moving with a velocity $\vec{v}$ in a magnetic field $\vec{B}$. (a) In this rule, the fingers point in the direction of $\vec{v}$, with $\vec{B}$ coming out of your palm, so that you can curl your fingers in the direction of $\vec{B}$. The direction of $\vec{v}\times\vec{B}$, and the force on a positive charge, is the direction in which the thumb points. (b) In this rule, the vector $\vec{v}$ is in the direction of your thumb and $\vec{B}$ in the direction of your fingers. The force $\vec{F}_B$ on a positive charge is in the direction of your palm, as if you are pushing the particle with your hand.

Magnetic Fields and Forces

- From the equation $F_B = |q|vB \sin\theta$, we see that the SI unit of magnetic field is the newton per coulomb-meter per second, which is called the **tesla** (T):

$$1\ \mathrm{T} = 1\ \frac{\mathrm{N}}{\mathrm{C \cdot m/s}}$$

- Because a coulomb per second is defined to be an ampere, we see that:

$$1\ \mathrm{T} = 1\ \frac{\mathrm{N}}{\mathrm{A \cdot m}}$$

- A non-SI magnetic-field unit in common use, called the gauss (G), is related to the tesla through the conversion $1\ \mathrm{T} = 10^4\ \mathrm{G}$.

$$\hat{\mathbf{i}} \times \hat{\mathbf{i}} = \hat{\mathbf{j}} \times \hat{\mathbf{j}} = \hat{\mathbf{k}} \times \hat{\mathbf{k}} = 0$$

$$\hat{\mathbf{i}} \times \hat{\mathbf{j}} = -\hat{\mathbf{j}} \times \hat{\mathbf{i}} = \hat{\mathbf{k}}$$

$$\hat{\mathbf{j}} \times \hat{\mathbf{k}} = -\hat{\mathbf{k}} \times \hat{\mathbf{j}} = \hat{\mathbf{i}}$$

$$\hat{\mathbf{k}} \times \hat{\mathbf{i}} = -\hat{\mathbf{i}} \times \hat{\mathbf{k}} = \hat{\mathbf{j}}$$

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Example 29.01

An electron in a television picture tube moves toward the front of the tube with a speed of 8.0×10^6 m/s along the x axis (see the following figure). Surrounding the neck of the tube are coils of wire that create a magnetic field of magnitude $0.025\ T$, directed at an angle of 60° to the x axis and lying in the xy plane. (a) Calculate the magnetic force on the electron using the equation $F_B = |q|vB \sin\theta$. (b) Find a vector expression for the magnetic force on the electron using the equation $\vec{F}_B = q\vec{v} \times \vec{B}$.

(a) The magnitude of the magnetic force is :

$$F_B = |q|vB \sin\theta$$

$$F_B = 1.6 \times 10^{-19} \times 8.0 \times 10^6 \times 0.025 \times \sin(60)$$

$$F_B = 2.8 \times 10^{-14}\ \text{N}$$

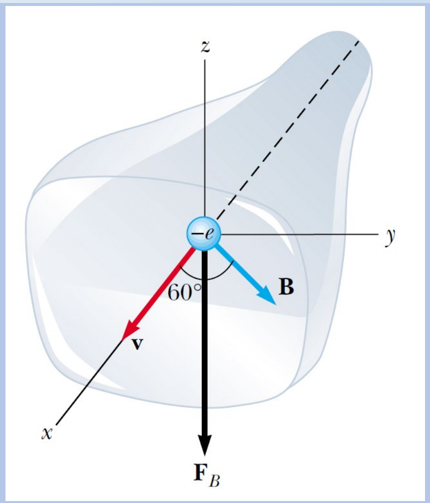
Because $\vec{v} \times \vec{B}$ is in the positive z direction (from the righthand rule) and the charge is negative, $\vec{F}_B$ is in the negative z direction.

(b) We begin by writing a vector expression for the velocity of the electron:

$$\vec{v} = \left(8.0 \times 10^6 \hat{\imath}\right)\ \text{m/s}$$

And one for the magnetic field:

$$\vec{B} = [(0.025 \times \cos(60))\hat{\imath} + (0.025 \times \sin(60))\hat{\jmath}]$$



The magnetic force $\vec{F}_B$ acting on the electron is in the negative z direction when $\vec{v}$ and $\vec{B}$ lie in the xy plane.

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Example 29.01

An electron in a television picture tube moves toward the front of the tube with a speed of 8.0×10^6 m/s along the x axis (see the following figure). Surrounding the neck of the tube are coils of wire that create a magnetic field of magnitude $0.025\ T$, directed at an angle of 60° to the x axis and lying in the xy plane. (a) Calculate the magnetic force on the electron using the equation $F_B = |q|vB\sin\theta$. (b) Find a vector expression for the magnetic force on the electron using the equation $\vec{F}_B = q\vec{v} \times \vec{B}$.

$$\vec{B} = (0.013\hat{\imath} + 0.022\hat{\jmath})\ \mathrm{T}$$

The force on the electron is:

$$\vec{F}_B = q\vec{v} \times \vec{B}$$

$$\vec{F}_B = (-e)\left[\left(8.0 \times 10^6\hat{\imath}\right) \times (0.013\hat{\imath} + 0.022\hat{\jmath})\right]$$

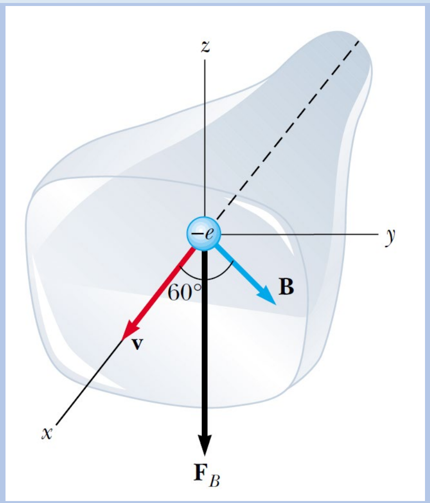
$$\vec{F}_B = (-e)\left[\left(8.0 \times 10^6\hat{\imath}\right) \times (0.013\hat{\imath})\right] + (-e)\left[\left(8.0 \times 10^6\hat{\imath}\right) \times (0.022\hat{\jmath})\right]$$

$$\vec{F}_B = \left[\left(-1.6 \times 10^{-19} \times 8.0 \times 10^6 \times 0.013\right)(\hat{\imath} \times \hat{\imath})\right] + \left[\left(-1.6 \times 10^{-19} \times 8.0 \times 10^6 \times 0.022\right)(\hat{\imath} \times \hat{\jmath})\right]$$

From $\hat{\imath} \times \hat{\imath} = 0$ and $\hat{\imath} \times \hat{\jmath} = \hat{k}$, we find:

$$\vec{F}_B = \left(-2.8 \times 10^{-14}\ \mathrm{N}\right)\hat{k}$$

This expression agrees with the result in part (a). The magnitude is the same as we found there, and the force vector is in the negative z direction.



The magnetic force $\vec{F}_B$ acting on the electron is in the negative z direction when $\vec{v}$ and $\vec{B}$ lie in the xy plane.

Magnetic Fields and Forces

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Problem 29.01

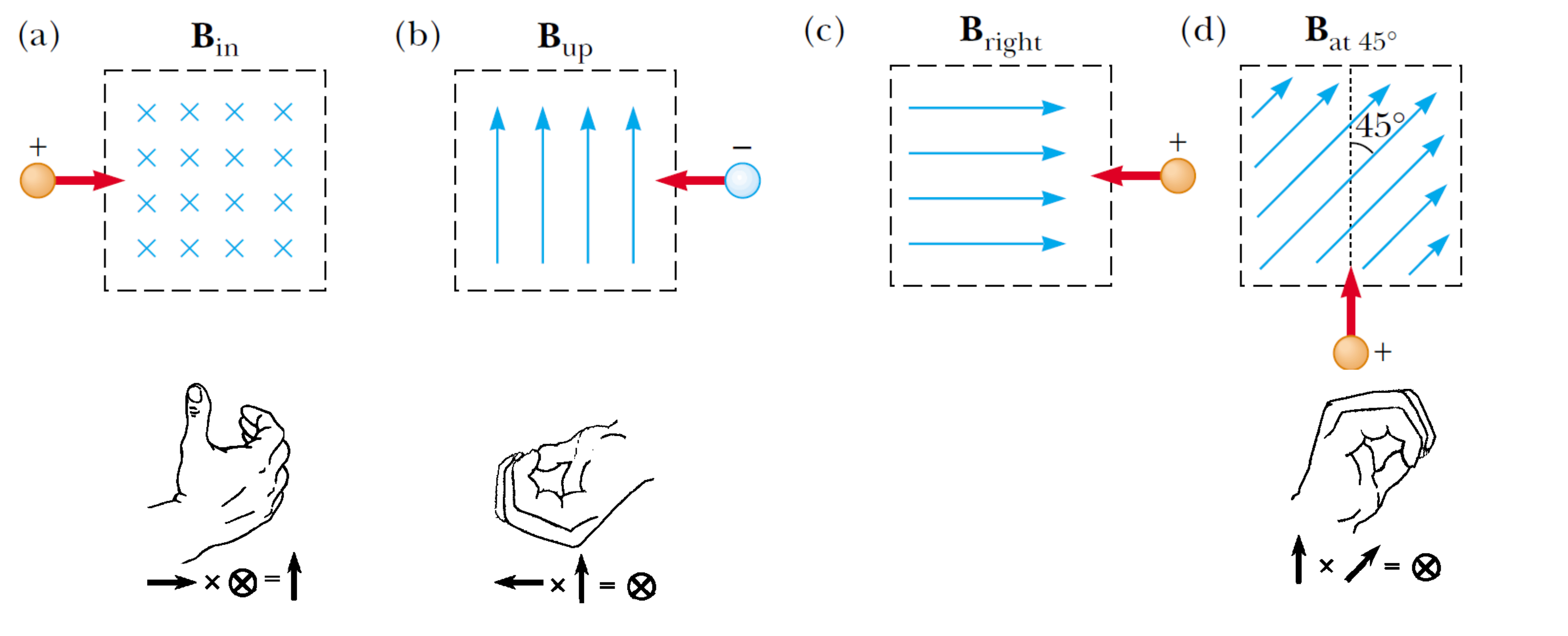
Determine the initial direction of the deflection of charged particles as they enter the magnetic fields as shown in the following figure.

(a) up

(b) out of the page, since the charge is negative.

(c) no deflection.

(d) into the page.



Magnetic Fields and Forces

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Problem 29.07

A proton moving at 4.00×10^6 m/s through a magnetic field of 1.70 T experiences a magnetic force of magnitude 8.20×10^{-13} N. What is the angle between the proton's velocity and the field?

$$F_B = |q|vB\sin\theta$$

$$\theta = sin^{-1}\left(\frac{F_B}{|q|vB}\right)$$

$$\theta = sin^{-1}\left(\frac{8.20 \times 10^{-13}}{1.60 \times 10^{-19} \times 4.00 \times 10^6 \times 1.70}\right)$$

$$\theta = 49° \quad or \quad 131°$$

$$\sin(180 - \theta) = \sin\theta$$

$$\cos(180 - \theta) = -\cos\theta$$

Magnetic Fields and Forces

Problem 29.09

A proton moves with a velocity of $\vec{v} = (2\hat{\imath} - 4\hat{\jmath} + \hat{k})$ m/s in a region in which the magnetic field is $\vec{B} = (\hat{\imath} + 2\hat{\jmath} - 3\hat{k})$ T. What is the magnitude of the magnetic force this charge experiences?

$$\vec{F}_B = q\vec{v} \times \vec{B}$$

$$\vec{v} \times \vec{B} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ +2 & -4 & +1 \\ +1 & +2 & -3 \end{vmatrix}$$

$$\vec{v} \times \vec{B} = \begin{vmatrix} -4 & +1 \\ +2 & -3 \end{vmatrix} \hat{\imath} - \begin{vmatrix} +2 & +1 \\ +1 & -3 \end{vmatrix} \hat{\jmath} + \begin{vmatrix} +2 & -4 \\ +1 & +2 \end{vmatrix} \hat{k}$$

$$\vec{v} \times \vec{B} = (12 - 2)\hat{\imath} + (6 + 1)\hat{\jmath} + (4 + 4)\hat{k}$$

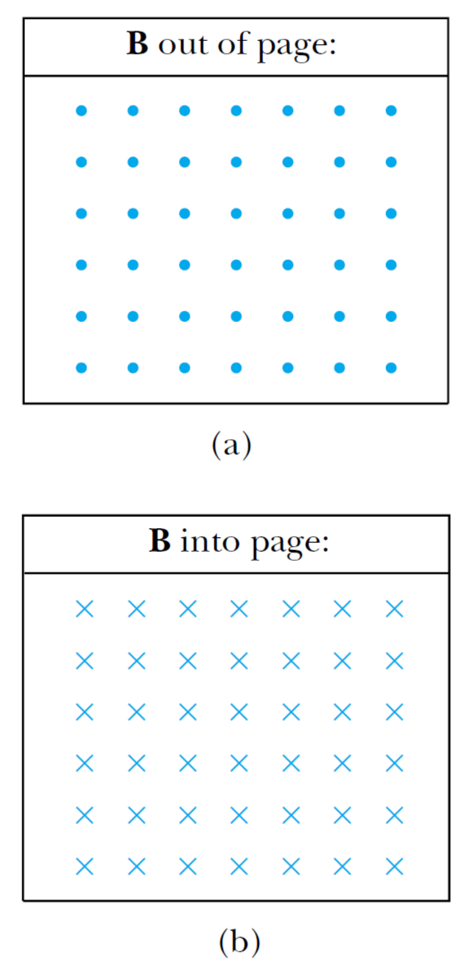
$$\vec{v} \times \vec{B} = 10\hat{\imath} + 7\hat{\jmath} + 8\hat{k}$$

$$\left|\vec{v} \times \vec{B}\right| = \sqrt{10^2 + 7^2 + 8^2} = 14.6\ \mathrm{T \cdot m/s}$$

$$\left|\vec{F}_B\right| = q\left|\vec{v} \times \vec{B}\right| = 1.60 \times 10^{-19} \times 14.6 = 2.34 \times 10^{-18}\ N$$

Magnetic Force Acting on a Current-Carrying Conductor

- A current-carrying wire experiences a magnetic force when placed in a magnetic field similar to the single charged particle when the particle moves through a magnetic field.
- The current is a collection of many charged particles in motion; hence, the resultant force exerted by the field on the wire is the vector sum of the individual forces exerted on all the charged particles making up the current.
- The force exerted on the particles is transmitted to the wire when the particles collide with the atoms making up the wire.
- A magnetic field perpendicular to and directed out of the page is depicted with a series of blue dots, which represent the tips of arrows coming toward you.
- If $\vec{B}$ is directed perpendicularly into the page, we use blue crosses, which represent the feathered tails of arrows fired away from you.
- The same notation with crosses and dots is also used for other quantities that might be perpendicular to the page, such as forces and current directions.



(a) Magnetic field lines coming out of the paper are indicated by dots, representing the tips of arrows coming outward. (b) Magnetic field lines going into the paper are indicated by crosses, representing the feathers of arrows going inward.

Magnetic Force Acting on a Current-Carrying Conductor

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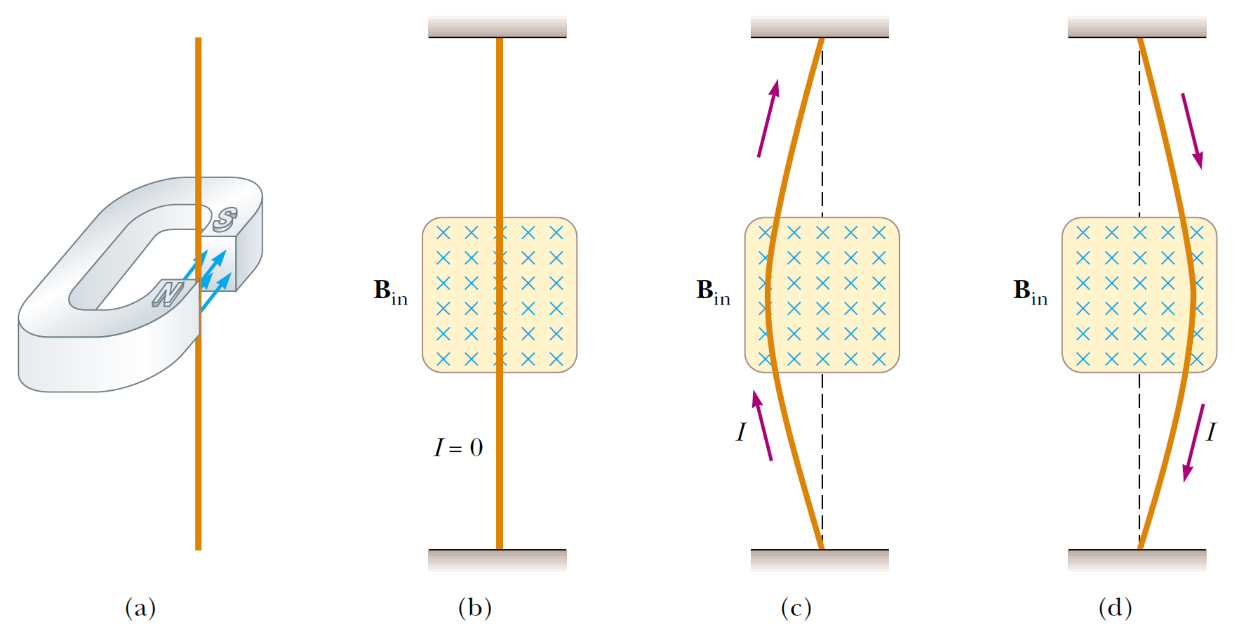
- To demonstrate the magnetic force acting on a current-carrying conductor by hanging a wire between the poles of a magnet.
- For ease in visualization, the magnetic field is directed into the page and covers the region within the shaded squares.
  - When the current in the wire is zero, the wire remains vertical.
  - When the wire carries a current directed upward, the wire deflects to the left.
  - If we reverse the current, the wire deflects to the right.



(a) A wire suspended vertically between the poles of a magnet. (b) The setup shown in part (a) as seen looking at the south pole of the magnet, so that the magnetic field (blue crosses) is directed into the page. When there is no current in the wire, it remains vertical. (c) When the current is upward, the wire deflects to the left. (d) When the current is downward, the wire deflects to the right.

Magnetic Force Acting on a Current-Carrying Conductor

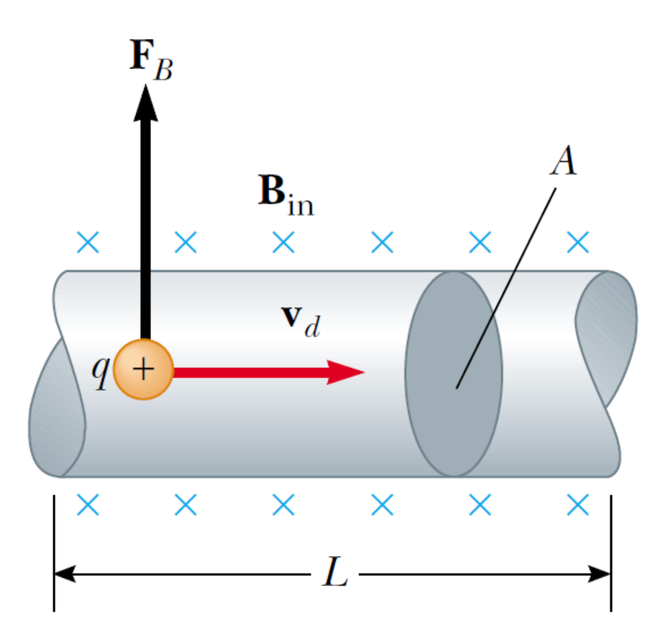
- **Considering a straight segment of wire of length L and cross-sectional area A, carrying a current I in a uniform magnetic field $\vec{B}$.**
- The magnetic force exerted on a charge q moving with a drift velocity $\vec{v}_d$ is $q\vec{v}_d \times \vec{B}$.
- The total force acting on the wire:

$$\vec{F}_B = \left(q\vec{v}_d \times \vec{B}\right) nA\,L$$

- where n is the number of charges per unit volume.
- If the current in the wire is $I = nqv_d A$. Therefore,

$$\vec{F}_B = I\vec{L} \times \vec{B}$$

- Where $\vec{L}$ is a vector that points in the direction of the current I and has a magnitude equal to the length L of the segment.
- Note that this expression applies only to a straight segment of wire in a uniform magnetic field.



A segment of a current-carrying wire in a magnetic field $\vec{B}$. The magnetic force exerted on each charge making up the current is $q\vec{v}_d \times \vec{B}$ and the net force on the segment of length L is $I\vec{L} \times \vec{B}$.

Problem 29.12

A wire carries a steady current of 2.40 A. A straight section of the wire is 0.750 m long and lies along the x axis within a uniform magnetic field, $\vec{B} = (1.60\hat{k})$ T. If the current is in the $+x$ direction, what is the magnetic force on the section of wire?

$$\vec{F}_B = I\vec{L} \times \vec{B} = 2.40 \times (0.750)\hat{\imath} \times (1.6)\hat{k} = (-2.88\hat{\jmath})\ \mathrm{N}$$

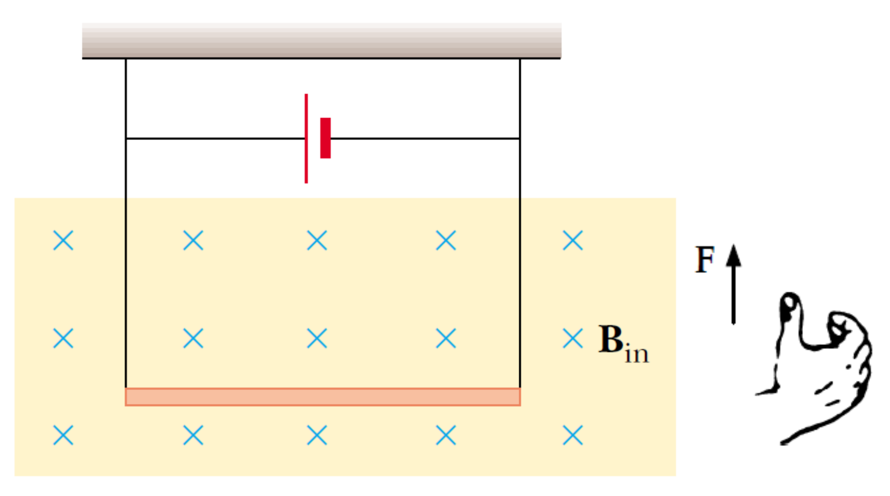
Problem 29.14

A conductor suspended by two flexible wires (see the figure below) has a mass per unit length of 0.04 kg/m. What current must exist in the conductor in order for the tension in the supporting wires to be zero when the magnetic field is 3.60 T into the page? What is the required direction for the current?

$$\frac{|\vec{F}_B|}{|\vec{L}|} = \frac{mg}{L} = \frac{I\,|\vec{l} \times \vec{B}|}{L} = \frac{ILB}{L}$$

$$I = \frac{mg}{LB} = \frac{0.0400 \times 9.80}{3.60} = 0.109 \text{ A}$$

The direction of I in the bar is to the right.



Motion of a Charged Particle in a Uniform Magnetic Field

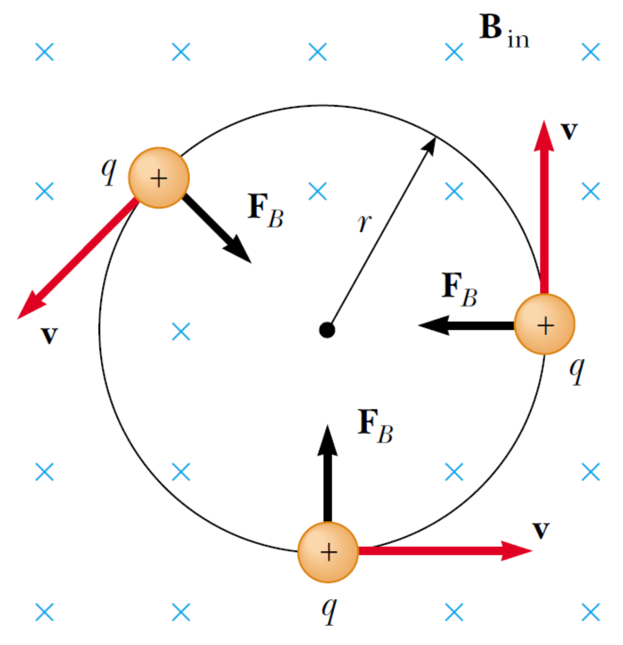
- **Now consider the special case of a positively charged particle moving in a uniform magnetic field with the initial velocity vector of the particle perpendicular to the field.**
- Let us assume that the direction of the magnetic field is into the page.
- As the particle changes the direction of its velocity in response to the magnetic force, the magnetic force remains perpendicular to the velocity.
- If the force is always perpendicular to the velocity, the path of the particle is a **circle**!
- i.e. the particle moving in a circle in a plane perpendicular to the magnetic field.
- The rotation is counterclockwise for a positive charge. If q were negative, the rotation would be clockwise.
- The magnetic force equals the product of the particle mass and the centripetal acceleration:

$$\sum F = ma_c$$

$$F_B = qvB = \frac{mv^2}{r}$$

$$r = \frac{mv}{qB}$$

- The radius r of the path is proportional to the linear momentum mv of the particle and inversely proportional to the magnitude of the charge on the particle and to the magnitude of the magnetic field.



When the velocity of a charged particle is perpendicular to a uniform magnetic field, the particle moves in a circular path in a plane perpendicular to $\vec{B}$. The magnetic force $\vec{F}_B$ acting on the charge is always directed toward the center of the circle.

- The angular speed ω of the particle is:

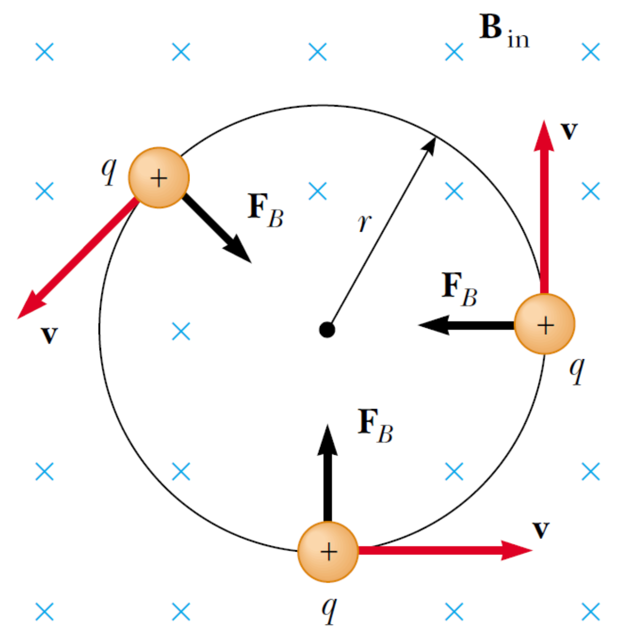
$$\omega = \frac{v}{r} = \frac{qB}{m}$$

- The period of the motion (the time interval the particle requires to complete one revolution) is equal to the circumference of the circle divided by the linear speed of the particle:

$$T = \frac{2\pi r}{v} = \frac{2\pi}{\omega} = \frac{2\pi m}{qB}$$

- The angular speed of the particle and the period of the circular motion do not depend on the linear speed of the particle or on the radius of the orbit.

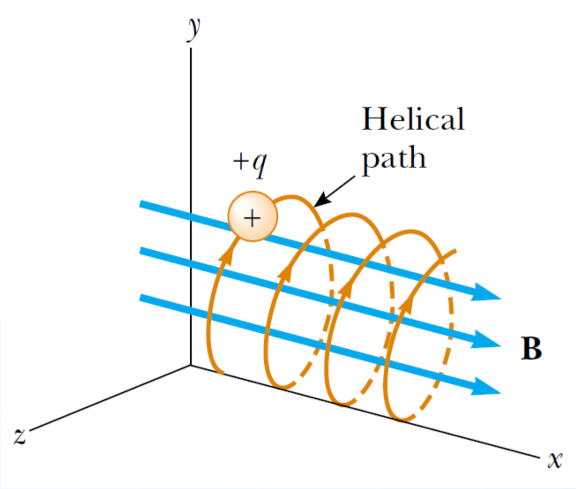
- The angular speed ω is often referred to as the **cyclotron frequency** because charged particles circulate at this angular frequency in the type of accelerator called a cyclotron.



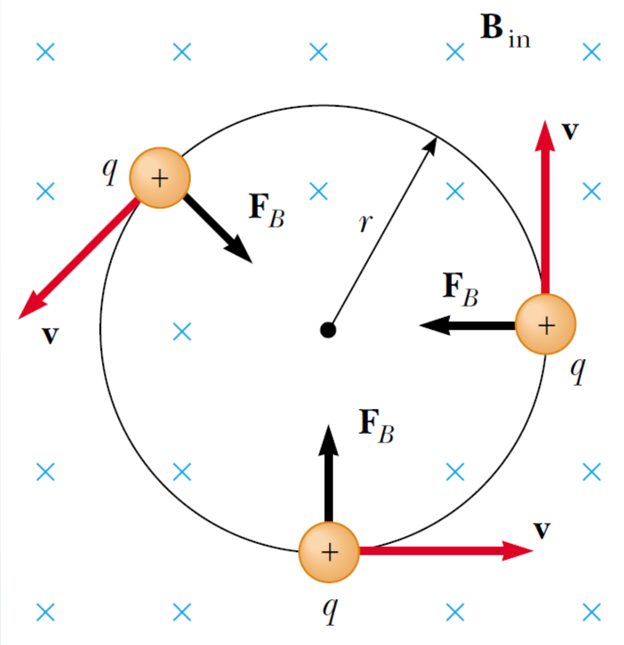
When the velocity of a charged particle is perpendicular to a uniform magnetic field, the particle moves in a circular path in a plane perpendicular to $\vec{B}$. The magnetic force $\vec{F}_B$ acting on the charge is always directed toward the center of the circle.

Motion of a Charged Particle in a Uniform Magnetic Field

- **If a charged particle moves in a uniform magnetic field with its velocity at some arbitrary angle with respect to $\vec{B}$, its path is a helix.**
- For example, if the field is directed in the x direction, there is no component of force in the x direction.
- As a result, $a_x = 0$, and the x component of velocity remains constant.
- The magnetic force $q\vec{v} \times \vec{B}$ causes the components v_y and v_z to change in time, and the resulting motion is a helix whose axis is parallel to the magnetic field.
- The projection of the path onto the yz plane (viewed along the x axis) is a circle. (The projections of the path onto the xy and xz planes are sinusoids!)
- Equations of r, ω, T still apply provided that v is replaced by $v_\perp = \sqrt{{v_y}^2 + {v_z}^2}$.



A charged particle having a velocity vector that has a component parallel to a uniform magnetic field moves in a helical path.



When the velocity of a charged particle is perpendicular to a uniform magnetic field, the particle moves in a circular path in a plane perpendicular to $\vec{B}$. The magnetic force $\vec{F}_B$ acting on the charge is always directed toward the center of the circle.

Example 29.06

A proton is moving in a circular orbit of radius 14 cm in a uniform 0.35 T magnetic field perpendicular to the velocity of the proton. Find the linear speed of the proton.

From the equation $r = mv/qB$, we have:

$$v = \frac{qBr}{m_p} = \frac{1.60 \times 10^{-19} \times 0.35 \times 0.14}{1.67 \times 10^{-27}} = 4.7 \times 10^{6}\ \text{m/s}$$

What If? What if an electron, rather than a proton, moves in a direction perpendicular to the same magnetic field with this same linear speed? Will the radius of its orbit be different?

Answer: An electron has a much smaller mass than a proton, so the magnetic force should be able to change its velocity much easier than for the proton. Thus, we should expect the radius to be smaller. Looking at the equation $r = mv/qB$, we see that r is proportional to m with q, B, and v the same for the electron as for the proton. Consequently, the radius will be smaller by the same factor as the ratio of masses m_e/m_p.

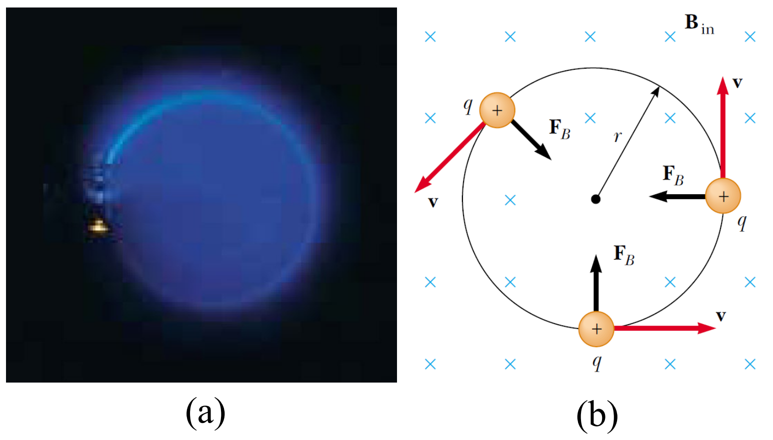
Example 29.07

In an experiment designed to measure the magnitude of a uniform magnetic field, electrons are accelerated from rest through a potential difference of 350 V. The electrons travel along a curved path because of the magnetic force exerted on them, and the radius of the path is measured to be 7.5 cm (Fig. (a) shows such a curved beam of electrons). If the magnetic field is perpendicular to the beam, (a) what is the magnitude of the field? (b) What is the angular speed of the electrons?

(a) Conceptualize the circular motion of the electrons with the help of the following figure. Looking at the equation $r = mv/qB$, we see that we need the speed v of the electron if we are to find the magnetic field magnitude, and v is not given. Consequently, we must find the speed of the electron based on the potential difference through which it is accelerated. Therefore, we also categorize this as a problem in conservation of mechanical energy for an isolated system. To begin analyzing the problem, we find the electron speed. For the isolated electron–electric field system, the loss of potential energy as the electron moves through the 350 V potential difference appears as an increase in the kinetic energy of the electron. Because $K_i = 0$ and $K_f = \frac{1}{2}m_e v^2$, we have:

$$\Delta K + \Delta U = 0$$

$$\frac{1}{2}m_e v^2 + (-e)\Delta V = 0$$



(a) The bending of an electron beam in a magnetic field. (b) When the velocity of a charged particle is perpendicular to a uniform magnetic field, the particle moves in a circular path in a plane perpendicular to $\vec{B}$. The magnetic force $\vec{F}_B$ acting on the charge is always directed toward the center of the circle.

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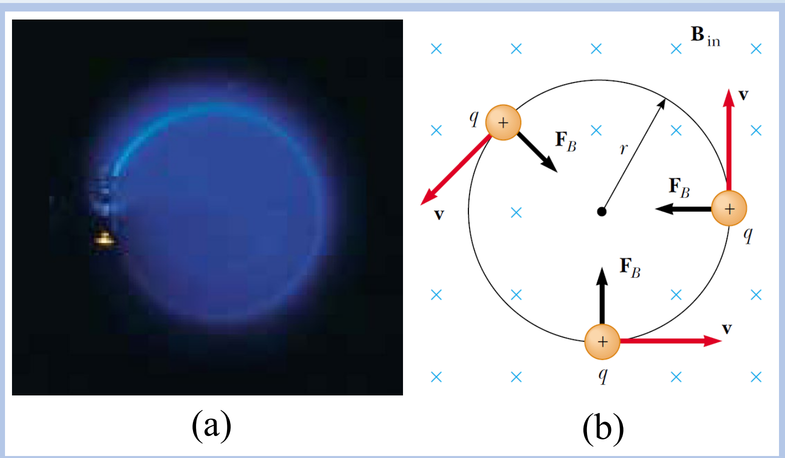
$$v = \sqrt{\frac{2e\Delta V}{m_e}} = \sqrt{\frac{2 \times 1.60 \times 10^{-19} \times 350}{9.11 \times 10^{-31}}} = 1.11 \times 10^{7}\ \mathrm{m/s}$$

Now, using Equation $r = mv/qB$, we find:

$$B = \frac{m_e v}{er} = \frac{9.11 \times 10^{-31} \times 1.11 \times 10^{7}}{1.60 \times 10^{-19} \times 0.075} = 8.4 \times 10^{-4}\ \mathrm{T}$$

(b) $$\omega = \frac{v}{r} = \frac{1.11 \times 10^{7}}{0.075} = 1.5 \times 10^{8}\ \mathrm{rad/s}$$

To finalize this problem, note that the angular speed can be represented as $\omega = (1.5 \times 10^{8}\ \mathrm{rad/s})\left(\frac{1}{2\pi}\ \mathrm{rev/rad}\right)$. The electrons travel around the circle 24 million times per second! This is consistent with the very high speed that we found in part (a).



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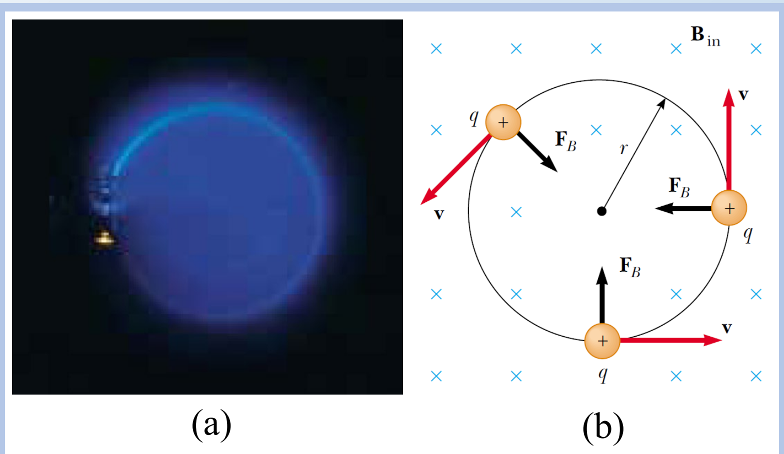
What If? What if a sudden voltage surge causes the accelerating voltage to increase to 400 V? How does this affect the angular speed of the electrons, assuming that the magnetic field remains constant?

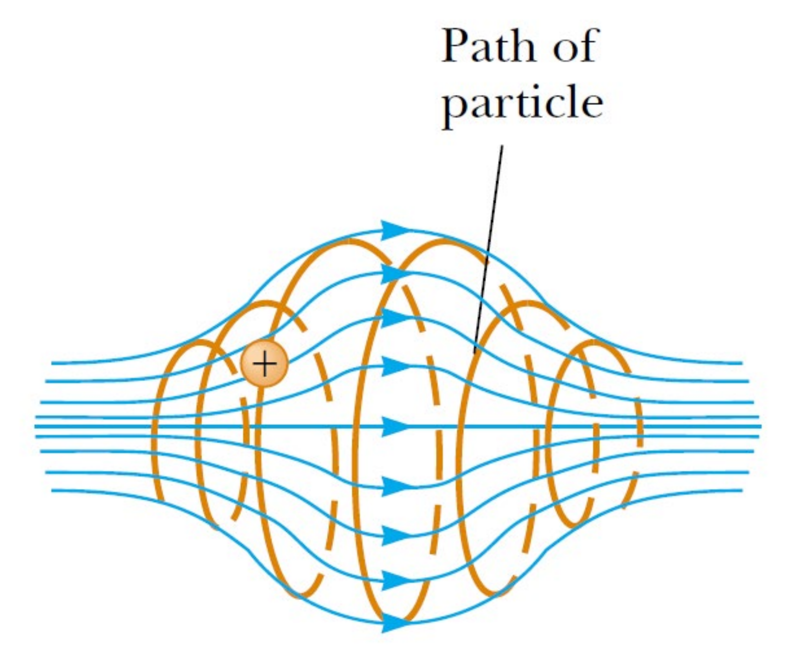
Answer: The increase in accelerating voltage ΔV will cause the electrons to enter the magnetic field with a higher speed v. This will cause them to travel in a circle with a larger radius r. The angular speed is the ratio of v to r. Both v and r increase by the same factor, so that the effects cancel and the angular speed remains the same. The equation $\omega = v/r = qB/m$ is an expression for the cyclotron frequency, which is the same as the angular speed of the electrons. The cyclotron frequency depends only on the charge q, the magnetic field B, and the mass m_e, none of which have changed. Thus, the voltage surge has no effect on the angular speed. (However, in reality, the voltage surge may also increase the magnetic field if the magnetic field is powered by the same source as the accelerating voltage. In this case, the angular speed will increase according to the equation $\omega = v/r = qB/m$.)



(a) The bending of an electron beam in a magnetic field. (b) When the velocity of a charged particle is perpendicular to a uniform magnetic field, the particle moves in a circular path in a plane perpendicular to $\vec{B}$. The magnetic force $\vec{F}_B$ acting on the charge is always directed toward the center of the circle.

Particle in a nonuniform magnetic field

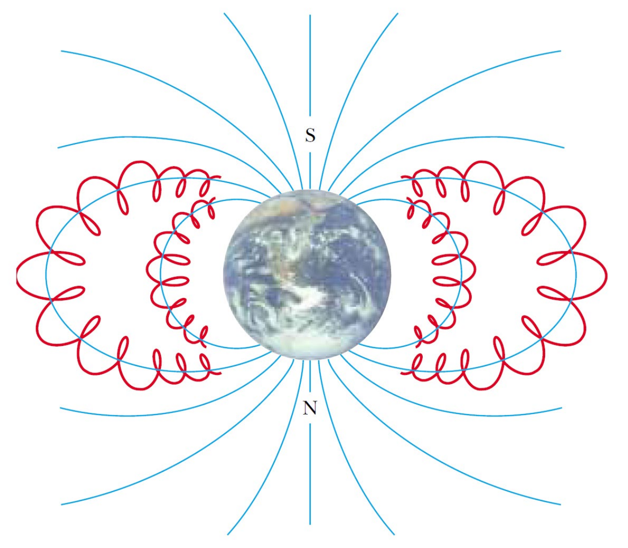
- When charged particles move in a nonuniform magnetic field, the motion is complex.
- For example, in a magnetic field that is strong at the ends and weak in the middle.
- The particles can oscillate back and forth between two positions.
- A charged particle starting at one end spirals along the field lines until it reaches the other end, where it reverses its path and spirals back.
- This configuration is known as a magnetic bottle because charged particles can be trapped within it.
- The magnetic bottle has been used to confine a plasma, a gas consisting of ions and electrons. Such a plasma-confinement scheme could fulfill a crucial role in the control of nuclear fusion, a process that could supply us with an almost endless source of energy.



A charged particle moving in a nonuniform magnetic field (a magnetic bottle) spirals about the field and oscillates between the end points. The magnetic force exerted on the particle near either end of the bottle has a component that causes the particle to spiral back toward the center.

The Van Allen radiation belts

- The Van Allen radiation belts consist of charged particles (mostly electrons and protons) surrounding the Earth in doughnut-shaped regions.
- The particles, trapped by the Earth's nonuniform magnetic field, spiral around the field lines from pole to pole, covering the distance in just a few seconds.
- These particles originate mainly from the Sun, but some come from stars and other heavenly objects. For this reason, the particles are called cosmic rays.
- Most cosmic rays are deflected by the Earth's magnetic field and never reach the atmosphere. However, some of the particles become trapped; it is these particles that make up the Van Allen belts.
- When the particles are located over the poles, they sometimes collide with atoms in the atmosphere, causing the atoms to emit visible light. Such collisions are the origin of the beautiful Aurora Borealis, or Northern Lights, in the northern hemisphere and the Aurora Australis in the southern hemisphere.



The Van Allen belts are made up of charged particles trapped by the Earth's nonuniform magnetic field. The magnetic field lines are in blue and the particle paths in red.

Aurora over Alaska due to the Earth's magnetic field.

Problem 29.30

Additional problem

A singly charged positive ion has a mass of 3.20×10^{-26} kg. After being accelerated from rest through a potential difference of 833 V, the ion enters a magnetic field of 0.920 T along a direction perpendicular to the direction of the field. Calculate the radius of the path of the ion in the field.

$$\Delta K + \Delta U = 0$$

$$\frac{1}{2}mv^2 + q\Delta V = 0$$

$$v = \sqrt{\frac{2q\Delta V}{m}} = \sqrt{\frac{2 \times 1.60 \times 10^{-19} \times 833}{3.20 \times 10^{-26}}} = 9.13 \times 10^4 \text{ m/s} = 91.3 \text{ km/s}$$

The magnetic force provides the centripetal force:

$$F_B = qvB = \frac{mv^2}{r}$$

$$r = \frac{mv}{qB} = \frac{3.20 \times 10^{-26} \times 9.13 \times 10^4}{1.60 \times 10^{-19} \times 0.920} = 1.98 \times 10^{-2} \text{ m} = 1.98 \text{ cm}$$

Problem 29.41

Singly charged uranium-238 ions are accelerated through a potential difference of 2.00 kV and enter a uniform magnetic field of 1.20 T directed perpendicular to their velocities. (a) Determine the radius of their circular path. (b) Repeat for uranium-235 ions. What If? How does the ratio of these path radii depend on the accelerating voltage and on the magnitude of the magnetic field?

$$\Delta K + \Delta U = 0$$

$$\frac{1}{2}mv^2 + q\Delta V = 0$$

$$v = \sqrt{\frac{2q\Delta V}{m}}$$

The magnetic force provides the centripetal force:

$$F_B = qvB = \frac{mv^2}{r}$$

$$r = \frac{mv}{qB} = \frac{m}{qB}\sqrt{\frac{2q\Delta V}{m}} = \frac{1}{B}\sqrt{\frac{2m\Delta V}{q}}$$

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1 u (atomic mass unit) $= 1.66 \times 10^{-27}$ kg

(a) $$r_{238} = \frac{1}{B}\sqrt{\frac{2m_{238}\Delta V}{q}} = \frac{1}{1.20}\sqrt{\frac{2 \times 238 \times 1.66 \times 10^{-27} \times 2.00 \times 10^{3}}{1.60 \times 10^{-19}}} = 8.31 \times 10^{-2}\ \text{m} = 8.31\ \text{cm}$$

(b) $$r_{235} = \frac{1}{B}\sqrt{\frac{2m_{235}\Delta V}{q}} = \frac{1}{1.20}\sqrt{\frac{2 \times 235 \times 1.66 \times 10^{-27} \times 2.00 \times 10^{3}}{1.60 \times 10^{-19}}} = 8.25 \times 10^{-2}\ \text{m} = 8.25\ \text{cm}$$

$$\frac{r_{238}}{r_{235}} = \sqrt{\frac{m_{238}}{m_{235}}} = \sqrt{\frac{238}{235}} = 1.01$$

or:

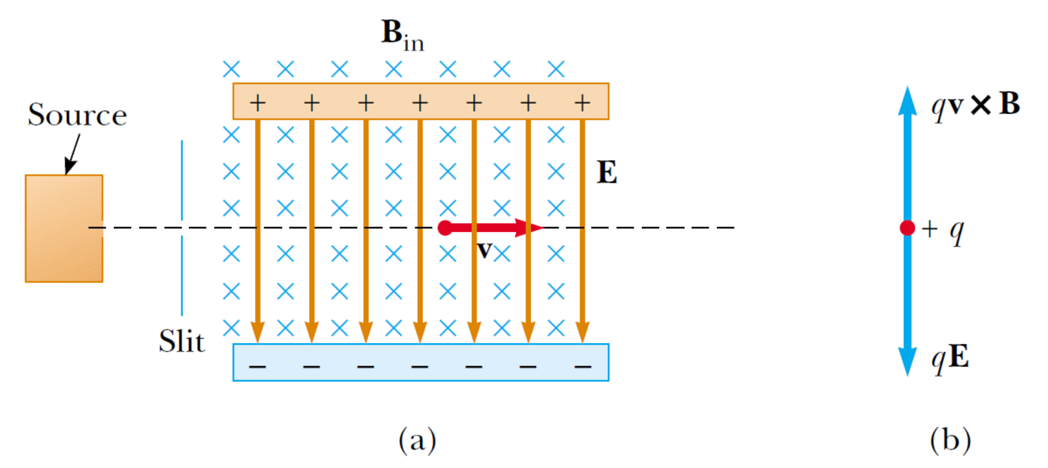
$$\frac{r_{238}}{r_{235}} = \frac{8.31}{8.25} = 1.01$$

- A charge moving with a velocity $\vec{v}$ in the presence of both an electric field $\vec{E}$ and a magnetic field $\vec{B}$ experiences both an electric force $q\vec{E}$ and a magnetic force $q\vec{v} \times \vec{B}$.
- The total force (called the Lorentz force) acting on the charge is:

$$\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$$

Velocity Selector

- In many experiments involving moving charged particles, it is important that the particles all move with essentially the same velocity.
- This can be achieved by applying a combination of an electric field and a magnetic field oriented.
- A uniform electric field is directed vertically downward (in the plane of the page in Fig. (a)), and a uniform magnetic field is applied in the direction perpendicular to the electric field (into the page in Fig. (a)).
- If q is positive and the velocity $\vec{v}$ is to the right, the magnetic force $q\vec{v} \times \vec{B}$ is upward and the electric force $q\vec{E}$ is downward.
- When the magnitudes of the two fields are chosen so that $qE = qvB$, the particle moves in a straight horizontal line through the region of the fields.

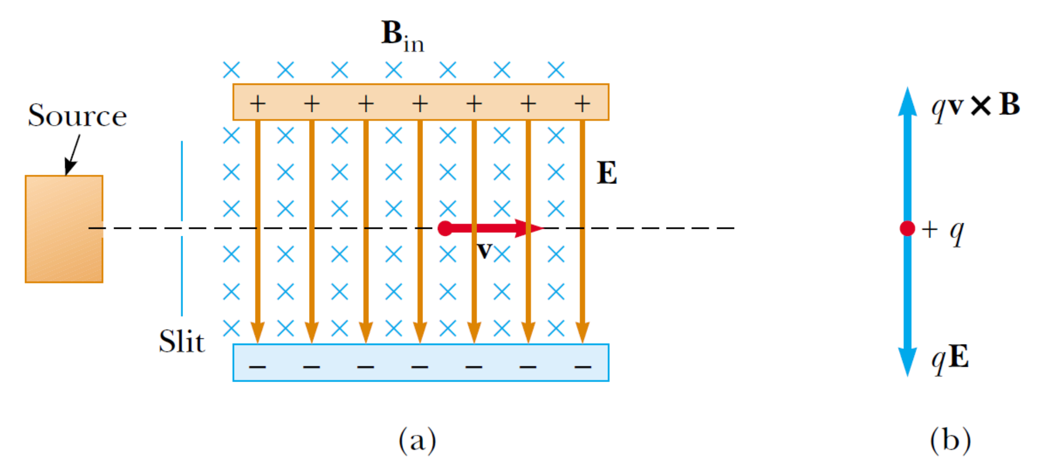


(a) A velocity selector. When a positively charged particle is moving with velocity $\vec{v}$ in the presence of a magnetic field directed into the page and an electric field directed downward, it experiences a downward electric force $q\vec{E}$ and an upward magnetic force $q\vec{v} \times \vec{B}$. (b) When these forces balance, the particle moves in a horizontal line through the fields.

- From the expression $qE = qvB$, we find that:

$$v = \frac{E}{B}$$

- Only those particles having speed v pass undeflected through the mutually perpendicular electric and magnetic fields.
- The magnetic force exerted on particles moving at speeds greater than this is stronger than the electric force, and the particles are deflected upward.
- Those moving at speeds less than this are deflected downward.



(a) A velocity selector. When a positively charged particle is moving with velocity $\vec{v}$ in the presence of a magnetic field directed into the page and an electric field directed downward, it experiences a downward electric force $q\vec{E}$ and an upward magnetic force $q\vec{v} \times \vec{B}$. (b) When these forces balance, the particle moves in a horizontal line through the fields.