104 PHYS

Ch. 30

Sources of the Magnetic Field

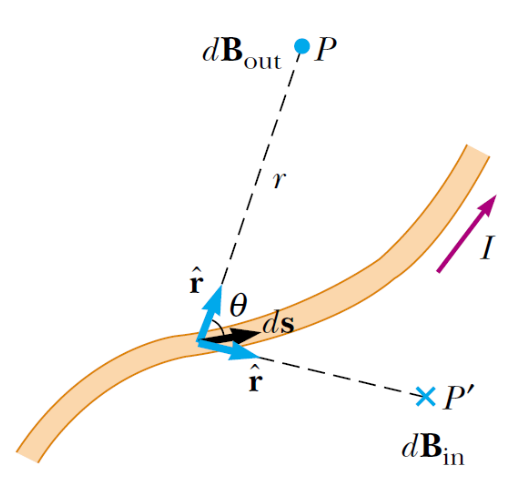
Dr. Ayman Alismail ©

Contents

The Biot–Savart Law

- Shortly after Oersted's discovery in 1819 that a compass needle is deflected by a current-carrying conductor, Jean-Baptiste Biot (1774–1862) and Félix Savart (1791–1841) performed quantitative experiments on the force exerted by an electric current on a nearby magnet.
- From their experimental results, Biot and Savart arrived at a mathematical expression that gives the magnetic field at some point in space in terms of the current that produces the field.
- That expression is based on the following experimental observations for the magnetic field $d\vec{B}$ at a point P associated with a length element $d\vec{s}$ of a wire carrying a steady current I:
  - The vector $d\vec{B}$ is perpendicular both to $d\vec{s}$ (which points in the direction of the current) and to the unit vector $\hat{r}$ directed from $d\vec{s}$ toward P.
  - The magnitude of $d\vec{B}$ is inversely proportional to r^2, where r is the distance from $d\vec{s}$ to P.
  - The magnitude of $d\vec{B}$ is proportional to the current and to the magnitude ds of the length element $d\vec{s}$.
  - The magnitude of $d\vec{B}$ is proportional to $\sin\theta$, where θ is the angle between the vectors $d\vec{s}$ and $\hat{r}$.

A proposed method for launching future payloads into space is the use of rail guns, in which projectiles are accelerated by means of magnetic forces. This photo shows the firing of a projectile at a speed of over 3 km/s from an experimental rail gun at Sandia National Research Laboratories, Albuquerque, New Mexico.



The magnetic field $d\vec{B}$ at a point due to the current I through a length element $d\vec{s}$ is given by the Biot–Savart law. The direction of the field is out of the page at P and into the page at P'.

The Biot–Savart Law

- These observations are summarized in the mathematical expression known today as the **Biot–Savart law**:

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{I d\vec{s} \times \hat{r}}{r^2}$$

- where μ_0 is a constant called the **permeability of free space**:

$$\mu_0 = 4\pi \times 10^{-7}\ \mathrm{T \cdot m/A}$$

- Note that the field $d\vec{B}$ is the field created by the current in only a small length element $d\vec{s}$ of the conductor.

- To find the <u>total</u> magnetic field $\vec{B}$ created at some point by a current of finite size, we must sum up contributions from all current elements $Id\vec{s}$ that make up the current.

$$\vec{B} = \frac{\mu_0 I}{4\pi}\int \frac{d\vec{s} \times \hat{r}}{r^2}$$

- where the integral is taken over the entire current distribution.

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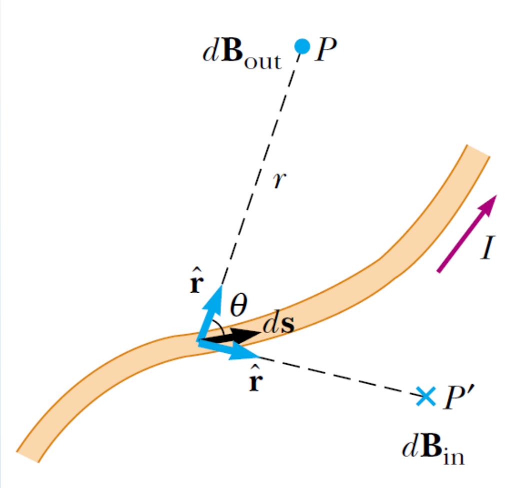

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Example 30.01

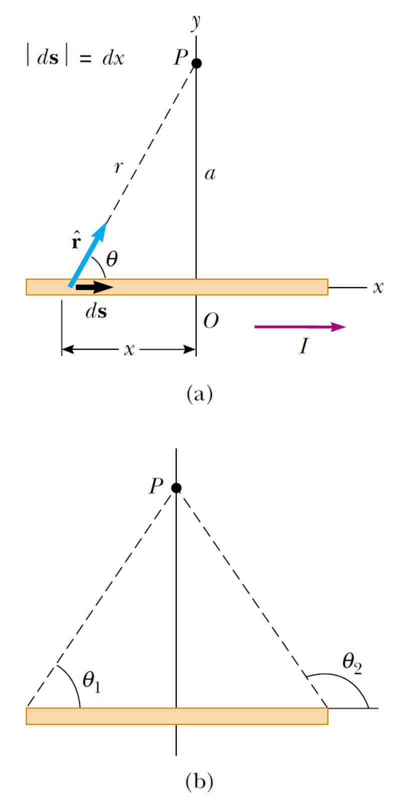
Consider a thin, straight wire carrying a constant current I and placed along the x axis as shown in the following figure. Determine the magnitude and direction of the magnetic field at point P due to this current.

From the Biot–Savart law, we expect that the magnitude of the field is proportional to the current in the wire and decreases as the distance a from the wire to point P increases. We start by considering a length element $d\vec{s}$ located a distance r from P. The direction of the magnetic field at point P due to the current in this element is out of the page because $d\vec{s} \times \hat{r}$ is out of the page. In fact, because all of the current elements $Id\vec{s}$ lie in the plane of the page, they all produce a magnetic field directed out of the page at point P. Thus, we have the direction of the magnetic field at point P, and we need only find the magnitude. Taking the origin at O and letting point P be along the positive y axis, with $\hat{k}$ being a unit vector pointing out of the page, we see that:

$$d\vec{s} \times \hat{r} = |d\vec{s} \times \hat{r}|\, \hat{k} = (dx \sin\theta)\, \hat{k}$$

where $|d\vec{s} \times \hat{r}|$ represents the magnitude of $d\vec{s} \times \hat{r}$. Because $\hat{r}$ is a unit vector, the magnitude of the cross product is simply the magnitude of $d\vec{s}$, which is the length dx. Substitution into the **Biot–Savart law** gives:

$$d\vec{B} = (dB)\, \hat{k} = \frac{\mu_0 I}{4\pi} \frac{dx \sin\theta}{r^2} \hat{k}$$



(a) A thin, straight wire carrying a current I. The magnetic field at point P due to the current in each element $d\vec{s}$ of the wire is out of the page, so the net field at point P is also out of the page. (b) The angles θ_1 and θ_2 used for determining the net field. When the wire is infinitely long, $\theta_1 = 0$ and $\theta_2 = 180°$.

The Biot–Savart Law

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Example 30.01

Consider a thin, straight wire carrying a constant current I and placed along the x axis as shown in the following figure. Determine the magnitude and direction of the magnetic field at point P due to this current.

Because all current elements produce a magnetic field in the $\hat{k}$ direction, let us restrict our attention to the magnitude of the field due to one current element, which is:

$$dB = \frac{\mu_0 I}{4\pi}\frac{dx \sin\theta}{r^2} \qquad (1)$$

To integrate this expression, we must relate the variables θ, x, and r. One approach is to express x and r in terms of θ. From the geometry in Fig. (a), we have:

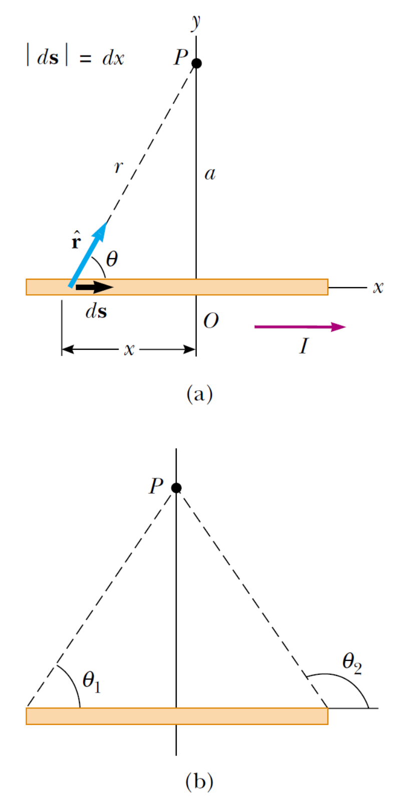
$$r = \frac{a}{\sin\theta} = a\csc\theta \qquad (2)$$

Because $\tan\theta = a/(-x)$ from the right triangle in Fig. (a) (the negative sign is necessary because $d\vec{s}$ is located at a negative value of x), we have:

$$x = -a\cot\theta$$

Taking the derivative of this expression gives:

$$dx = a\csc^2\theta\, d\theta \qquad (3)$$



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Example 30.01

Consider a thin, straight wire carrying a constant current I and placed along the x axis as shown in the following figure. Determine the magnitude and direction of the magnetic field at point P due to this current.

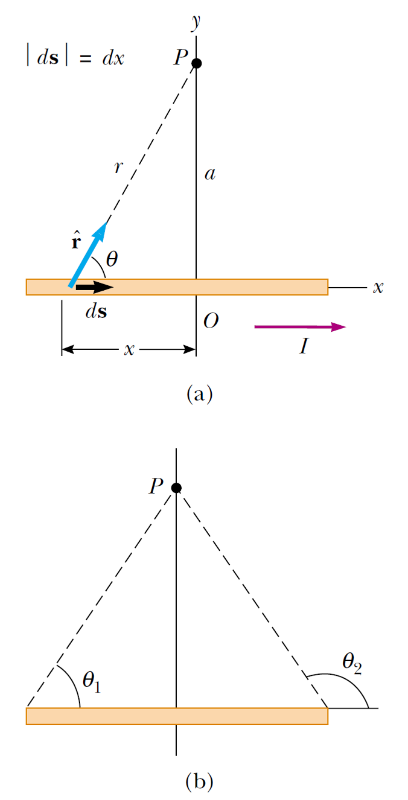
Substitution of Equations (2) and (3) into Equation (1) gives:

$$dB = \frac{\mu_0 I}{4\pi}\frac{a\csc^2\theta \sin\theta\, d\theta}{a^2\csc^2\theta} = \frac{\mu_0 I}{4\pi a}\sin\theta\, d\theta \qquad (4)$$

an expression in which the only variable is θ. We now obtain the magnitude of the magnetic field at point P by integrating Equation (4) over all elements, where the subtending angles range from θ_1 to θ_2 as defined in Fig. (b):

$$B = \frac{\mu_0 I}{4\pi a}\int_{\theta_1}^{\theta_2}\sin\theta\, d\theta = \frac{\mu_0 I}{4\pi a}(\cos\theta_1 - \cos\theta_2)$$

We can use this result to find the magnetic field of <u>any</u> straight current-carrying wire if we know the geometry and hence the angles θ_1 and θ_2. Consider the special case of an infinitely long, straight wire. If we let the wire in Fig. (b) become infinitely long, we see that $\theta_1 = 0$ and $\theta_2 = \pi$ for length elements ranging between positions $x = -\infty$ and $x = +\infty$. Because $(\cos\theta_1 - \cos\theta_2) = (\cos 0 - \cos\pi) = 2$, therefore, the magnetic field becomes:



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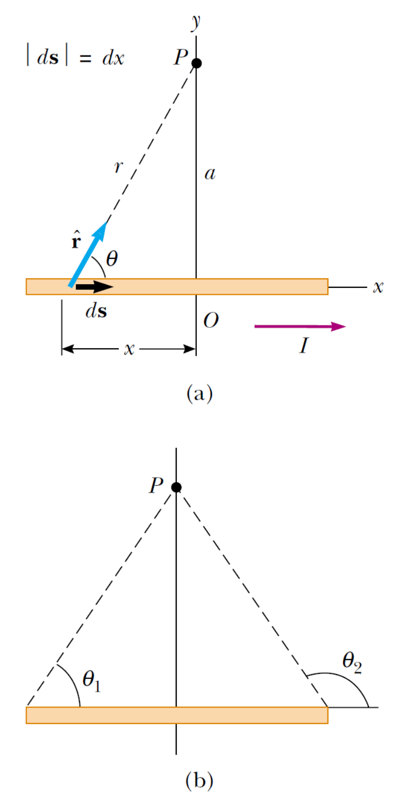
The Biot–Savart Law

Example 30.01

Consider a thin, straight wire carrying a constant current I and placed along the x axis as shown in the following figure. Determine the magnitude and direction of the magnetic field at point P due to this current.

$$B = \frac{\mu_0 I}{2\pi a}$$

We find that the magnitude of the magnetic field is proportional to the current and decreases with increasing distance from the wire, as we expected.



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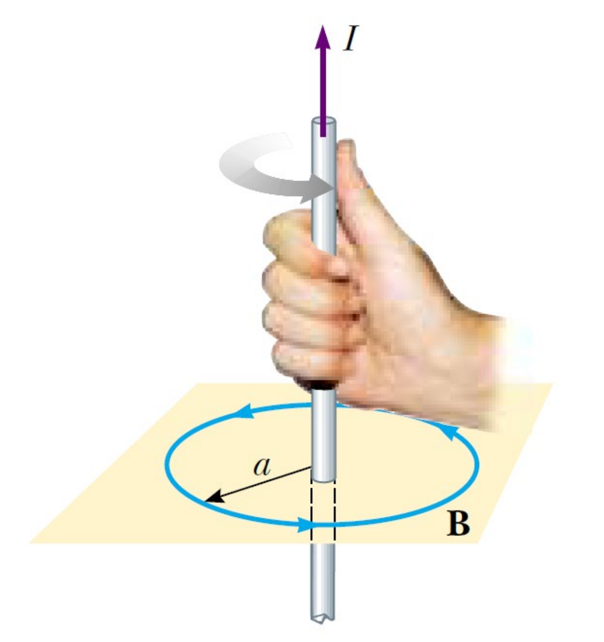
The Biot–Savart Law

- The right hand rule for determining the direction of the magnetic field surrounding a long, straight current-carrying wire.
- Because of the symmetry of the wire, the magnetic field lines are circles concentric with the wire and lie in planes perpendicular to the wire.
- The magnitude of $\vec{B}$ is constant on any circle of radius a and is given by $B = \mu_0 I/2\pi a$.
- A convenient rule for determining the direction of $\vec{B}$ is:
  - Grasping the wire with the right hand.
  - Positioning the thumb along the direction of the current.
  - Wrapping the four fingers wrap in the direction of the magnetic field.



The right-hand rule for determining the direction of the magnetic field surrounding a long, straight wire carrying a current. Note that the magnetic field lines form circles around the wire.

The Biot–Savart Law

Problem 30.04

Calculate the magnitude of the magnetic field at a point 100 cm from a long, thin conductor carrying a current of 1.00 A.

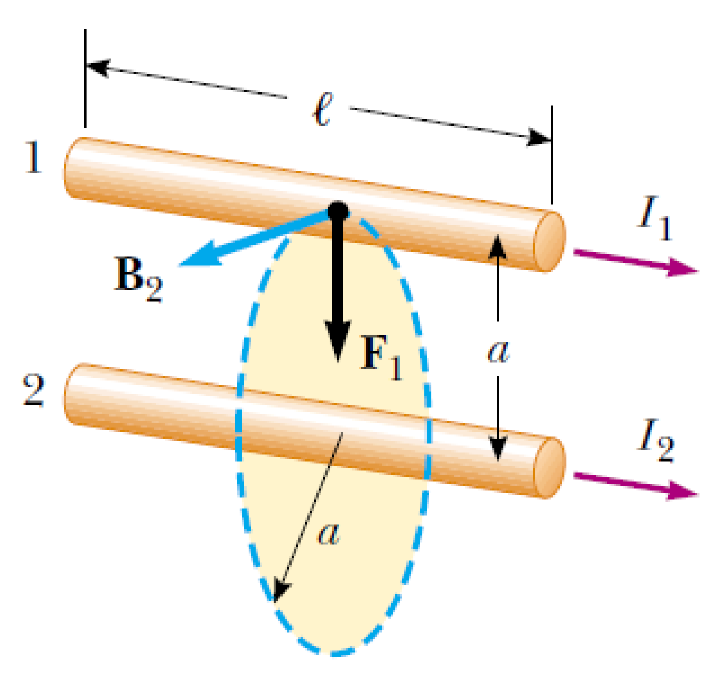
$$B = \frac{\mu_0 I}{2\pi a} = \frac{4\pi \times 10^{-7} \times 1.00}{2\pi \times 1.00} = 2 \times 10^{-7}\ \mathrm{T}$$

The Magnetic Force Between Two Parallel Conductors

- Because a current in a conductor sets up its own magnetic field, it is easy to understand that two current-carrying conductors exert magnetic forces on each other.
- Consider two long, straight, parallel wires separated by a distance a and carrying currents I_1 and I_2 in the same direction.
- We can determine the force exerted on one wire due to the magnetic field set up by the other wire.
- Wire 2, which carries a current I_2 and is identified arbitrarily as the source wire, creates a magnetic field $\vec{B}_2$ at the location of wire 1, the test wire. The direction of $\vec{B}_2$ is perpendicular to wire 1.
- The magnetic force on a length l of wire 1 is $\vec{F}_1 = I_1\vec{l} \times \vec{B}_2$. Because $\vec{l}$ is perpendicular to $\vec{B}_2$ in this situation, the magnitude of $\vec{F}_1$ is:

$$F_1 = I_1 l B_2 = I_1 l \left(\frac{\mu_0 I_2}{2\pi a}\right) = \frac{\mu_0 I_1 I_2}{2\pi a} l$$

- If the field set up at wire 2 by wire 1 is calculated, the force $\vec{F}_2$ acting on wire 2 is found to be equal in magnitude and opposite in direction to $\vec{F}_1$.
- When the currents are in opposite directions (that is, when one of the currents is reversed), the forces are reversed and the wires repel each other.
- Hence, **parallel conductors carrying currents in the same direction attract each other, and parallel conductors carrying currents in opposite directions repel each other.**



Two parallel wires that each carry a steady current exert a magnetic force on each other. The field B_2 due to the current in wire 2 exerts a magnetic force of magnitude $F_1 = I_1 l B_2$ on wire 1. The force is attractive if the currents are parallel (as shown) and repulsive if the currents are antiparallel.

The Magnetic Force Between Two Parallel Conductors

- Because the magnitudes of the forces are the same on both wires, we denote the magnitude of the magnetic force between the wires as simply F_B.

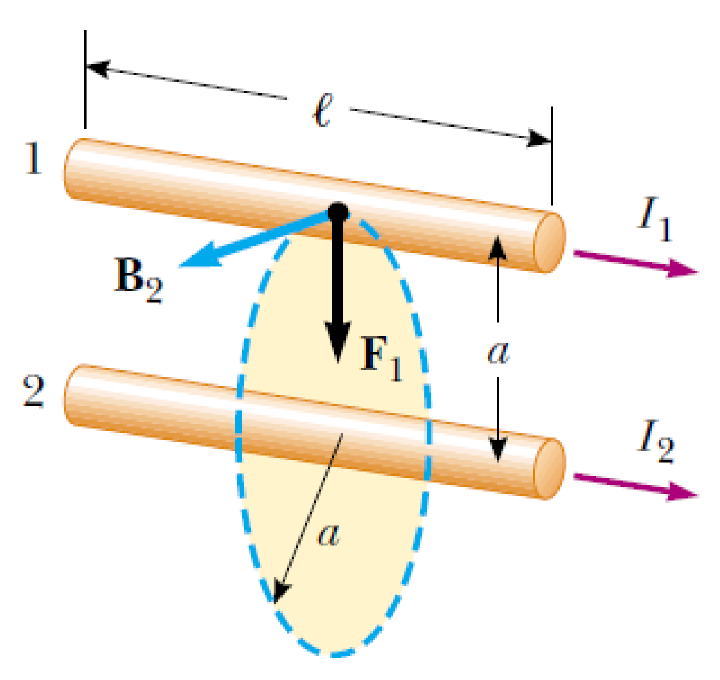
$$\frac{F_B}{l} = \frac{\mu_0 I_1 I_2}{2\pi a}$$

- The force between two parallel wires is used to define the **ampere** as follows:

When the magnitude of the force per unit length between two long parallel wires that carry identical currents and are separated by 1 m is 2×10^{-7} N/m, the current in each wire is defined to be 1 A.

- The SI unit of charge, the **coulomb**, is defined in terms of the ampere:

When a conductor carries a steady current of 1 A, the quantity of charge that flows through a cross section of the conductor in 1 s is 1 C.



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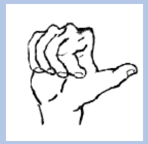
Problem 30.16

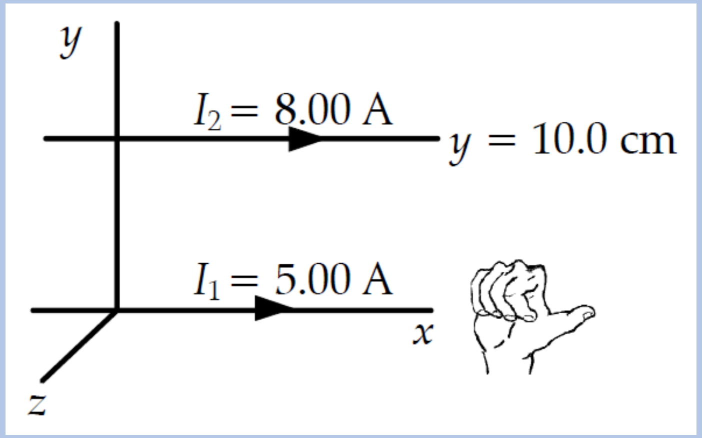
Two long, parallel conductors, separated by 10.0 cm, carry currents in the same direction. The first wire carries current $I_1 = 5.00$ A and the second carries $I_2 = 8.00$ A. (a) What is the magnitude of the magnetic field created by I_1 at the location of I_2? (b) What is the force per unit length exerted by I_1 on I_2? (c) What is the magnitude of the magnetic field created by I_2 at the location of I_1? (d) What is the force per length exerted by I_2 on I_1?

Let both wires carry current in the x direction, the first at $y = 0$ and the second at $y = 10.0$ cm.

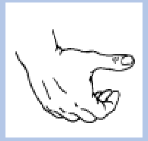
(a) $$B_1 = \frac{\mu_0 I_1}{2\pi a} = \frac{4\pi \times 10^{-7} \times 5.00}{2\pi \times 0.100} = 1.00 \times 10^{-5}\ \text{T} \quad \text{out of the page}$$



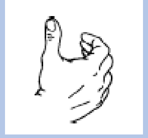
(b) $$\frac{F_B}{l} = I_2 B_1 = 8.00 \times 1.00 \times 10^{-5} = 8.00 \times 10^{-5}\ \text{N} \quad \text{toward the first wire}$$



(c) $$B_2 = \frac{\mu_0 I_2}{2\pi a} = \frac{4\pi \times 10^{-7} \times 8.00}{2\pi \times 0.100} = 1.60 \times 10^{-5}\ \text{T} \quad \text{into the page}$$



(d) $$\frac{F_B}{l} = I_1 B_2 = 5.00 \times 1.60 \times 10^{-5} = 8.00 \times 10^{-5}\ \text{N} \quad \text{toward the second wire}$$



Problem 30.17

In the following figure, the current in the long, straight wire is $I_1 = 5.00$ A and the wire lies in the plane of the rectangular loop, which carries the current $I_2 = 10.0$ A. The dimensions are $c = 0.100$ m, $a = 0.150$ m, and $l = 0.450$ m. Find the magnitude and direction of the net force exerted on the loop by the magnetic field created by the wire.

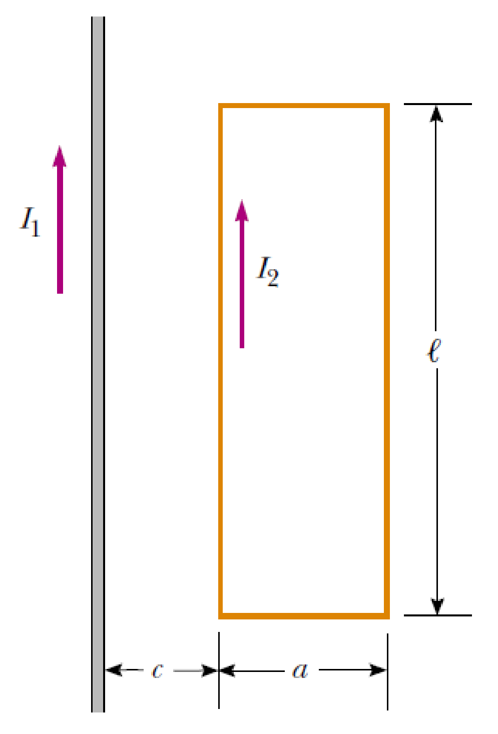
By symmetry, we note that the magnetic forces on the top and bottom segments of the rectangle cancel. The net force on the vertical segments of the rectangle is:

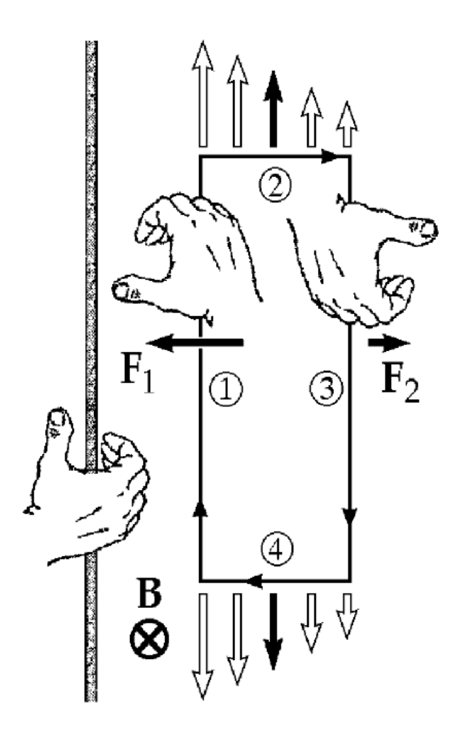
$$\vec{F}_B = \vec{F}_1 + \vec{F}_2$$

$$\vec{F}_B = \frac{\mu_0 I_1 I_2 l}{2\pi}\left(-\frac{1}{c} + \frac{1}{c+a}\right)\hat{\imath}$$

$$\vec{F}_B = \frac{4\pi \times 10^{-7} \times 5.00 \times 10.0 \times 0.450}{2\pi}\left(-\frac{1}{0.100} + \frac{1}{0.100 + 0.150}\right)\hat{\imath}$$

$$\vec{F}_B = -\left[\left(2.70 \times 10^{-5}\right)\hat{\imath}\right] \text{ N} \quad \text{toward the left}$$



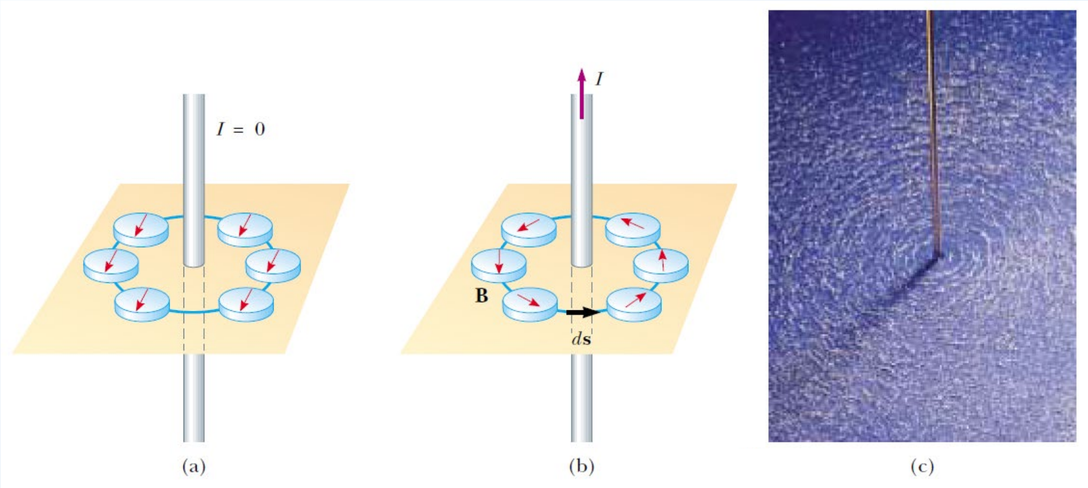


Ampère's Law

- Oersted's 1819 discovery about deflected compass needles demonstrates that a current-carrying conductor produces a magnetic field.
- Several compass needles are placed in a horizontal plane near a long vertical wire.
  - When no current is present in the wire, all the needles point in the same direction (that of the Earth's magnetic field), as expected (Fig. (a)).
  - When the wire carries a strong, steady current, the needles all deflect in a direction tangent to the circle (Fig. (b)).
- These observations demonstrate that the direction of the magnetic field produced by the current in the wire is consistent with the right-hand rule.
- When the current is reversed, the needles in Fig. (b) also reverse.
- The sum of the products Bds over the closed path, which is equivalent to the line integral of $\vec{B} \cdot d\vec{s}$, is:

$$\oint \vec{B} \cdot d\vec{s} = B \oint d\vec{s} = \frac{\mu_0 I}{2\pi r}(2\pi r) = \mu_0 I$$

- Although this result was calculated for the special case of a circular path surrounding a wire, it holds for a closed path of any shape (an amperian loop) surrounding a current that exists in an unbroken circuit.



(a) When no current is present in the wire, all compass needles point in the same direction (toward the Earth's north pole). (b) When the wire carries a strong current, the compass needles deflect in a direction tangent to the circle, which is the direction of the magnetic field created by the current. (c) Circular magnetic field lines surrounding a current-carrying conductor, displayed with iron filings.

Ampère's Law

- The general case, known as **Ampère's law**, can be stated as follows:

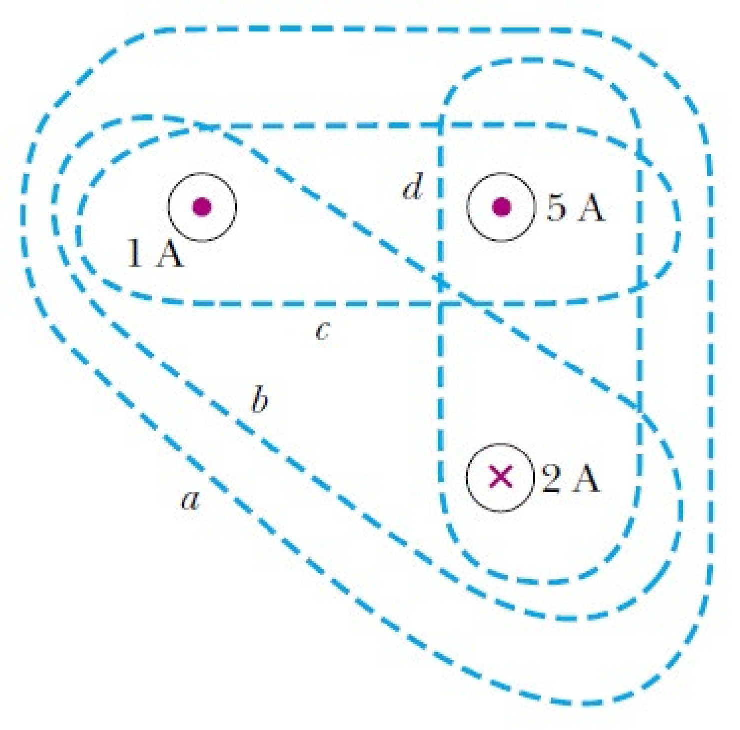
The line integral of $\vec{B}\cdot d\vec{s}$ around any closed path equals $\mu_0 I$, where I is the total steady current passing through any surface bounded by the closed path.

$$\oint \vec{B}\cdot d\vec{s} = \mu_0 I$$

- Ampère's law describes the creation of magnetic fields by all continuous current configurations, but at our mathematical level it is useful only for calculating the magnetic field of current configurations having a high degree of symmetry.

Quick Quiz 30.4 Rank the magnitudes of $\oint \vec{B}\cdot d\vec{s}$ for the closed paths in the following figure, from least to greatest.

(a) a, b, c, d. (b) b, d, a, c. (c) c, d, b, a. (d) c, b, a, d.



Four closed paths around three current-carrying wires.

Ampère's Law

Example 30.04

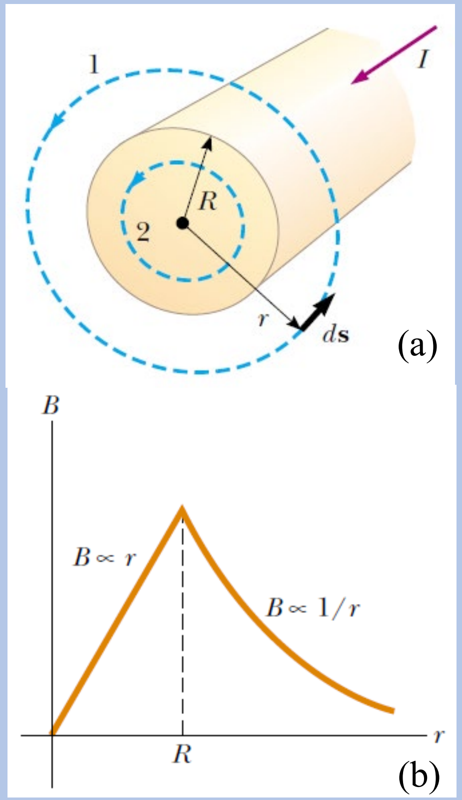
A long, straight wire of radius R carries a steady current I that is uniformly distributed through the cross section of the wire (Fig. (a)). Calculate the magnetic field a distance r from the center of the wire in the regions $r \geq R$ and $r < R$.

Fig. (a) helps us to conceptualize the wire and the current. Because the wire has a high degree of symmetry, we categorize this as an Ampère's law problem. For the $r \geq R$ case, we should arrive at the same result we obtained in Example 30.1, in which we applied the Biot–Savart law to the same situation. To analyze the problem, let us choose for our path of integration circle 1 in Fig. (a). From symmetry, $\vec{B}$ must be constant in magnitude and parallel to $d\vec{s}$ at every point on this circle. Because the total current passing through the plane of the circle is I, Ampère's law gives:

$$\oint \vec{B} \cdot d\vec{s} = B \oint d\vec{s} = B(2\pi r) = \mu_0 I$$

$$B = \frac{\mu_0 I}{2\pi r} \qquad (\text{for} \quad r \geq R\,) \qquad (1)$$

Note how much easier it is to use Ampère's law than to use the Biot–Savart law. This is often the case in highly symmetric situations.



(a) A long, straight wire of radius R carrying a steady current I uniformly distributed across the cross section of the wire. The magnetic field at any point can be calculated from Ampère's law using a circular path of radius r, concentric with the wire. (b) Magnitude of the magnetic field versus r for the wire shown in Fig. (a). The field is proportional to r inside the wire and varies as $1/r$ outside the wire.

Ampère's Law

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Example 30.04

A long, straight wire of radius R carries a steady current I that is uniformly distributed through the cross section of the wire (Fig. (a)). Calculate the magnetic field a distance r from the center of the wire in the regions $r \geq R$ and $r < R$.

Now consider the interior of the wire, where $r < R$. Here the current I' passing through the plane of circle 2 is less than the total current I. Because the current is uniform over the cross section of the wire, the fraction of the current enclosed by circle 2 must equal the ratio of the area πr^2 enclosed by circle 2 to the cross-sectional area πR^2 of the wire:

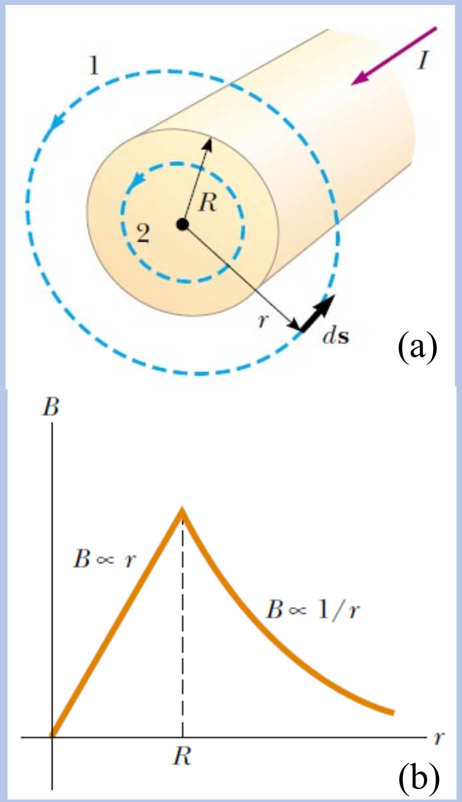
$$\frac{I'}{I} = \frac{\pi r^2}{\pi R^2}$$

$$I' = \frac{r^2}{R^2} I$$

Following the same procedure as for circle 1, we apply Ampère's law to circle 2:

$$\oint \vec{B} \cdot d\vec{s} = B(2\pi r) = \mu_0 I' = \mu_0 \left(\frac{r^2}{R^2} I \right)$$

$$B = \left(\frac{\mu_0 I}{2\pi R^2} \right) r \qquad (\text{for} \quad r < R) \qquad (2)$$



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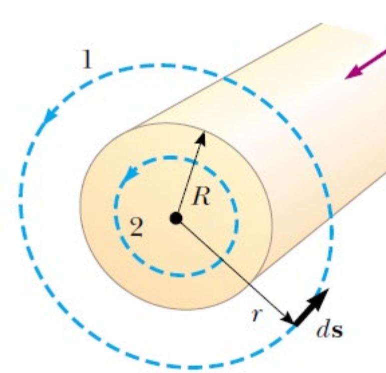
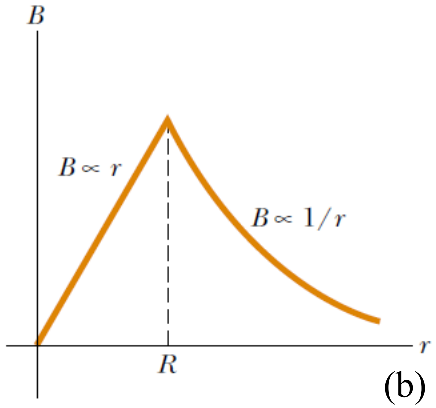
Example 30.04

A long, straight wire of radius R carries a steady current I that is uniformly distributed through the cross section of the wire (Fig. (a)). Calculate the magnetic field a distance r from the center of the wire in the regions $r \geq R$ and $r < R$.

To finalize this problem, note that this result is similar in form to the expression for the electric field inside a uniformly charged sphere (see Example 24.5). The magnitude of the magnetic field versus r for this configuration is plotted in Fig. (b). Note that inside the wire, $B \longrightarrow 0$ as $r \longrightarrow 0$. Furthermore, we see that Eqs. 1 and 2 give the same value of the magnetic field at $r = R$, demonstrating that the magnetic field is continuous at the surface of the wire.



(a)

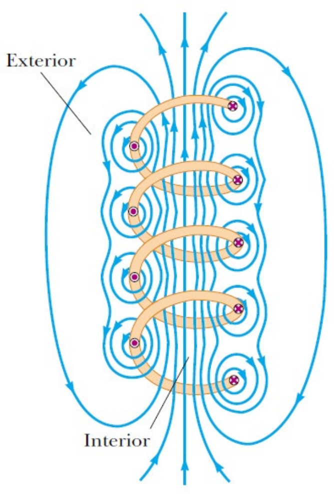


(b)

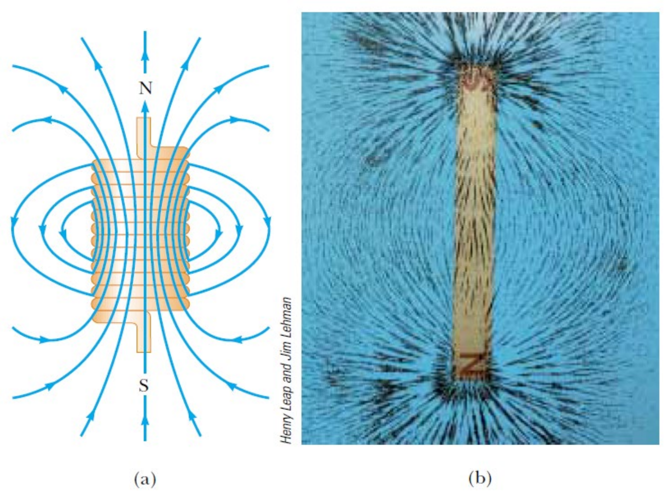
(a) A long, straight wire of radius R carrying a steady current I uniformly distributed across the cross section of the wire. The magnetic field at any point can be calculated from Ampère’s law using a circular path of radius r, concentric with the wire. (b) Magnitude of the magnetic field versus r for the wire shown in Fig. (a). The field is proportional to r inside the wire and varies as $1/r$ outside the wire.

The Magnetic Field of a Solenoid

- A **solenoid** is a long wire wound in the form of a helix. With this configuration, a reasonably uniform magnetic field can be produced in the space surrounded by the turns of wire—which we shall call the interior of the solenoid—when the solenoid carries a current.
- When the turns are closely spaced, each can be approximated as a circular loop, and the net magnetic field is the vector sum of the fields resulting from all the turns.
- This field line distribution is similar to that surrounding a bar magnet. Hence, one end of the solenoid behaves like the north pole of a magnet, and the opposite end behaves like the south pole.
- As the length of the solenoid increases, the interior field becomes more uniform and the exterior field becomes weaker.



The magnetic field lines for a loosely wound solenoid.



(a) Magnetic field lines for a tightly wound solenoid of finite length, carrying a steady current. The field in the interior space is strong and nearly uniform. Note that the field lines resemble those of a bar magnet, meaning that the solenoid effectively has north and south poles. (b) The magnetic field pattern of a bar magnet, displayed with small iron filings on a sheet of paper.

The Magnetic Field of a Solenoid

- An ideal solenoid is approached when the turns are closely spaced and the length is much greater than the radius of the turns.
- In this case, the external field is close to zero, and the interior field is uniform over a great volume.
- We can use Ampère's law to obtain a quantitative expression for the interior magnetic field in an ideal solenoid.
- Consider the rectangular path of length l and width w. We can apply Ampère's law to this path by evaluating the integral of $\vec{B} \cdot d\vec{s}$ over each side of the rectangle, as:

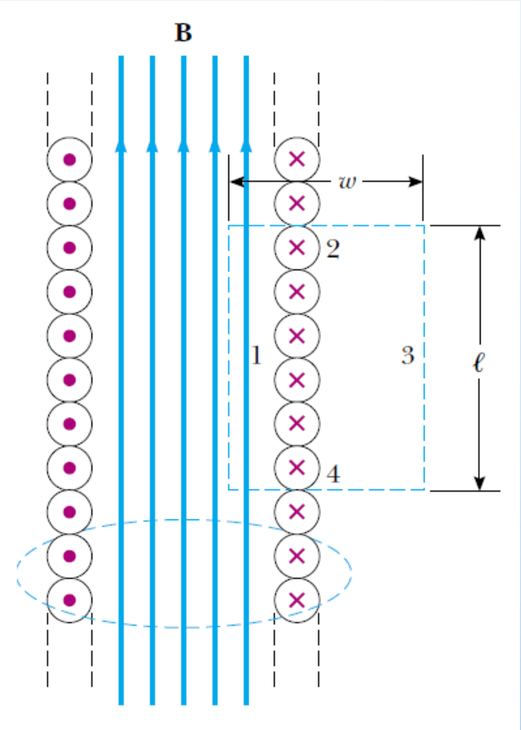
$$\oint \vec{B} \cdot d\vec{s} = \int_{\text{path 1}} \vec{B} \cdot d\vec{s} = B \int_{\text{path 1}} ds = Bl$$

- Ampère's law applied to this path gives

$$\oint \vec{B} \cdot d\vec{s} = Bl = \mu_0 NI$$

- where $n = N/l$ is the number of turns per unit length.

$$B = \mu_0 \frac{N}{l} I = \mu_0 nI$$



Cross-sectional view of an ideal solenoid, where the interior magnetic field is uniform and the exterior field is close to zero. Ampère's law applied to the circular path near the bottom whose plane is perpendicular to the page can be used to show that there is a weak field outside the solenoid. Ampère's law applied to the rectangular dashed path in the plane of the page can be used to calculate the magnitude of the interior field.

The Magnetic Field of a Solenoid

Problem 30.31

What current is required in the windings of a long solenoid that has 1000 turns uniformly distributed over a length of 0.400 m, to produce at the center of the solenoid a magnetic field of magnitude 1.00×10^{-4} T?

$$B = \mu_0 \frac{N}{l} I$$

$$I = \frac{Bl}{\mu_0 N} = \frac{1.00 \times 10^{-4} \times 0.400}{4\pi \times 10^{-7} \times 1000} = 31.8 \text{ mA}$$

Magnetic Flux

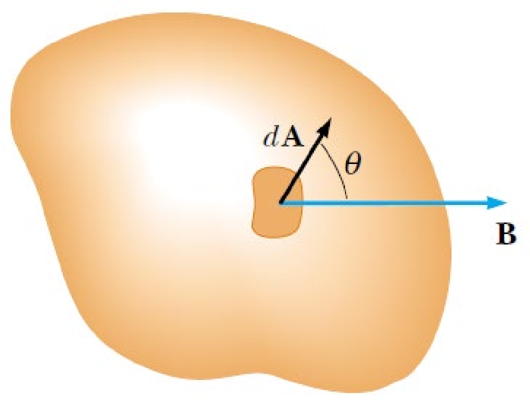
- Consider an element of area dA on an arbitrarily shaped surface.
- If the magnetic field at this element is $\vec{B}$, the magnetic flux through the element is $\vec{B} \cdot d\vec{A}$, where $d\vec{A}$ is a vector that is perpendicular to the surface and has a magnitude equal to the area dA.
- The total magnetic flux Φ_B through the surface is:

$$\Phi_B = \int \vec{B} \cdot d\vec{A}$$

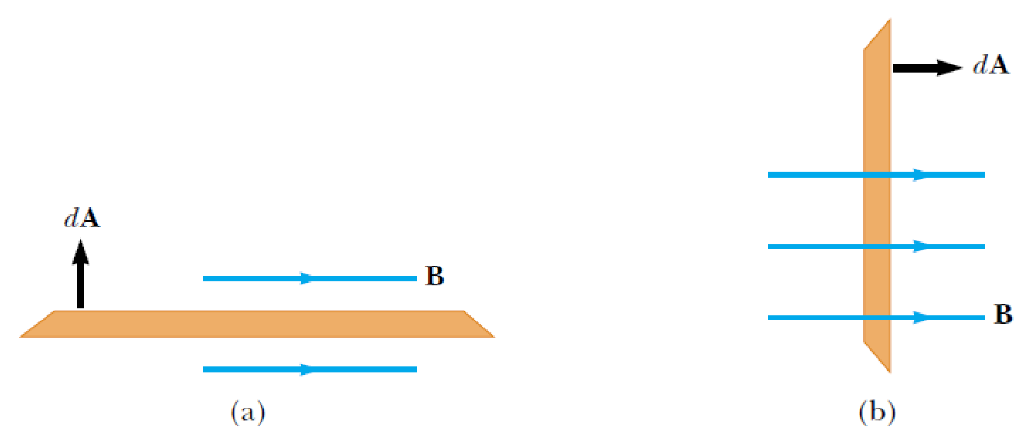
- Consider the special case of a plane of area A in a uniform field $\vec{B}$ that makes an angle θ with $d\vec{A}$. The magnetic flux through the plane in this case is:

$$\Phi_B = BA\cos\theta$$

  - If the magnetic field is parallel to the plane, then $\theta = 90°$ and the flux through the plane is zero.
  - If the field is perpendicular to the plane, then $\theta = 0$ and the flux through the plane is BA (the maximum value).
- The unit of magnetic flux is $\mathrm{T \cdot m^2}$, which is defined as a weber (Wb); $1\ \mathrm{Wb} = 1\ \mathrm{T \cdot m^2}$.



The magnetic flux through an area element dA is $\vec{B} \cdot d\vec{A} = BA\cos\theta$, where $d\vec{A}$is a vector perpendicular to the surface.



Magnetic flux through a plane lying in a magnetic field. (a) The flux through the plane is zero when the magnetic field is parallel to the plane surface. (b) The flux through the plane is a maximum when the magnetic field is perpendicular to the plane.

Magnetic Flux

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Example 30.08

A rectangular loop of width a and length b is located near a long wire carrying a current I (see the figure below). The distance between the wire and the closest side of the loop is c. The wire is parallel to the long side of the loop. Find the total magnetic flux through the loop due to the current in the wire.

We know that the magnitude of the magnetic field created by the wire at a distance r from the wire is:

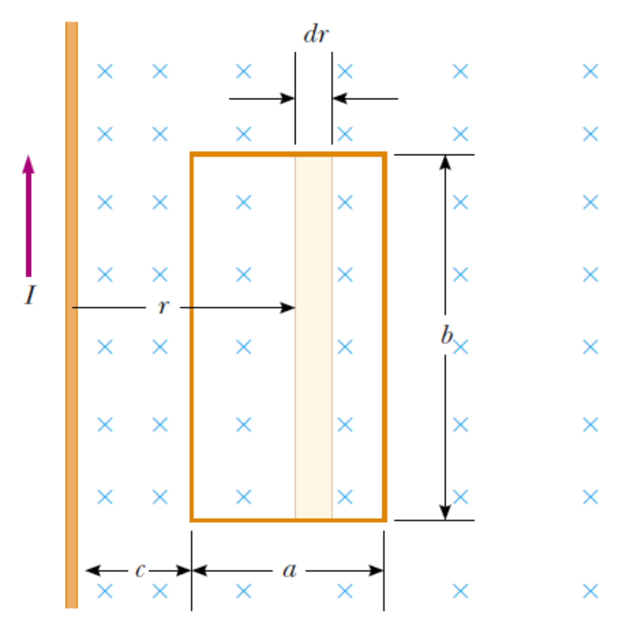
$$B = \frac{\mu_0 I}{2\pi r}$$

The factor $1/r$ indicates that the field varies over the loop, and the following figure shows that the field is directed into the page at the location of the loop. Because $\vec{B}$ is parallel to $d\vec{A}$ at any point within the loop, the magnetic flux through an area element dA is:

$$\Phi_B = \int B\, dA = \int \frac{\mu_0 I}{2\pi r} dA$$

To integrate, we first express the area element (the tan region in the following figure) as $dA = b\, dr$. Because r is now the only variable in the integral, we have:

$$\Phi_B = \frac{\mu_0 I b}{2\pi} \int_c^{a+c} \frac{dr}{r} = \frac{\mu_0 I b}{2\pi} \ln r \Big]_c^{a+c} = \frac{\mu_0 I b}{2\pi} \ln\left(\frac{a+c}{c}\right) = \frac{\mu_0 I b}{2\pi} \ln\left(1 + \frac{a}{c}\right) \qquad (1)$$



The magnetic field due to the wire carrying a current I is not uniform over the rectangular loop.

Magnetic Flux

King Saud University

Example 30.08

A rectangular loop of width a and length b is located near a long wire carrying a current I (see the figure below). The distance between the wire and the closest side of the loop is c. The wire is parallel to the long side of the loop. Find the total magnetic flux through the loop due to the current in the wire.

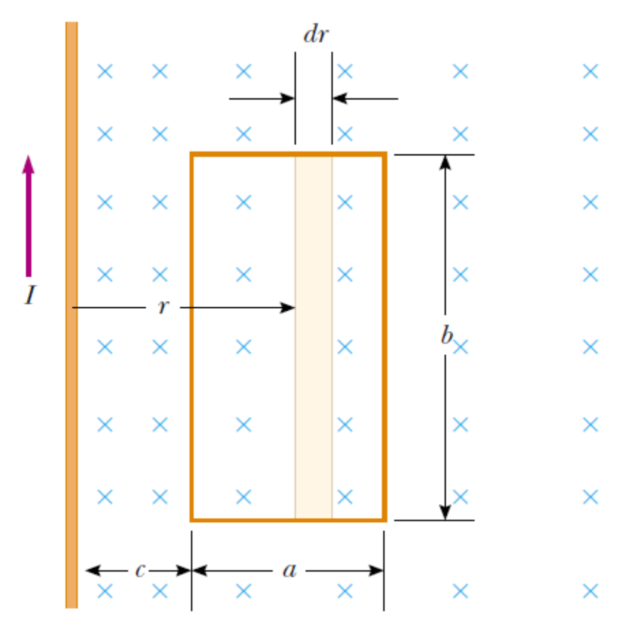
What If? Suppose we move the loop in the following figure very far away from the wire. What happens to the magnetic flux?

Answer: The flux should become smaller as the loop moves into weaker and weaker fields.

As the loop moves far away, the value of c is much larger than that of a, so that $a/c \longrightarrow 0$. Thus, the natural logarithm in Eq. (1) approaches the limit:

$$\ln\left(1+\frac{a}{c}\right) \longrightarrow \ln(1+0) = \ln(1) = 0$$

and we find that $\Phi_B \longrightarrow 0$ as we expected.



The magnetic field due to the wire carrying a current I is not uniform over the rectangular loop.

Magnetic Flux

Problem 30.35

A cube of edge length $l = 2.5$ cm is positioned as shown in the following figure. A uniform magnetic field given by $\vec{B} = \left(5\hat{\imath} + 4\hat{\jmath} + 3\hat{k}\right)$ T exists throughout the region. (a) Calculate the flux through the shaded face. (b) What is the total flux through the six faces?

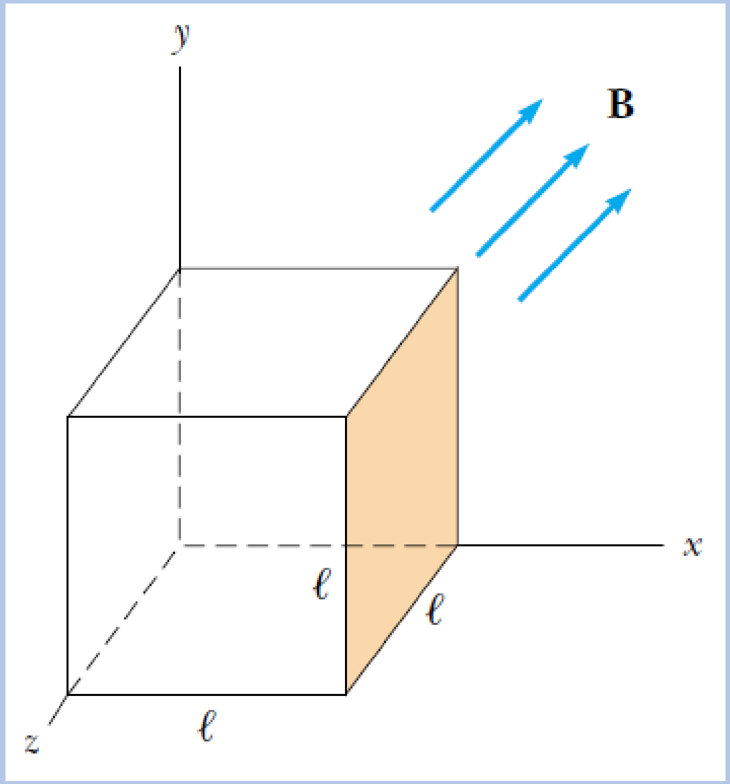
(a) $\Phi_B = \int \vec{B} \cdot d\vec{A}$

$$\Phi_B = \vec{B} \cdot \vec{A}$$

$$\Phi_B = \left(5\hat{\imath} + 4\hat{\jmath} + 3\hat{k}\right) \cdot \left(2.50 \times 10^{-2}\right)^2 \hat{\imath}$$

$$\Phi_B = 3.12 \times 10^{-3}\ \mathrm{T \cdot m^2} = 3.12 \times 10^{-3}\ \mathrm{Wb} = 3.12\ \mathrm{mWb}$$

(b) $\Phi_B = \oint \vec{B} \cdot d\vec{A} = 0$ for <u>any</u> closed surface (Gauss's law for magnetism)



Gauss's Law in Magnetism

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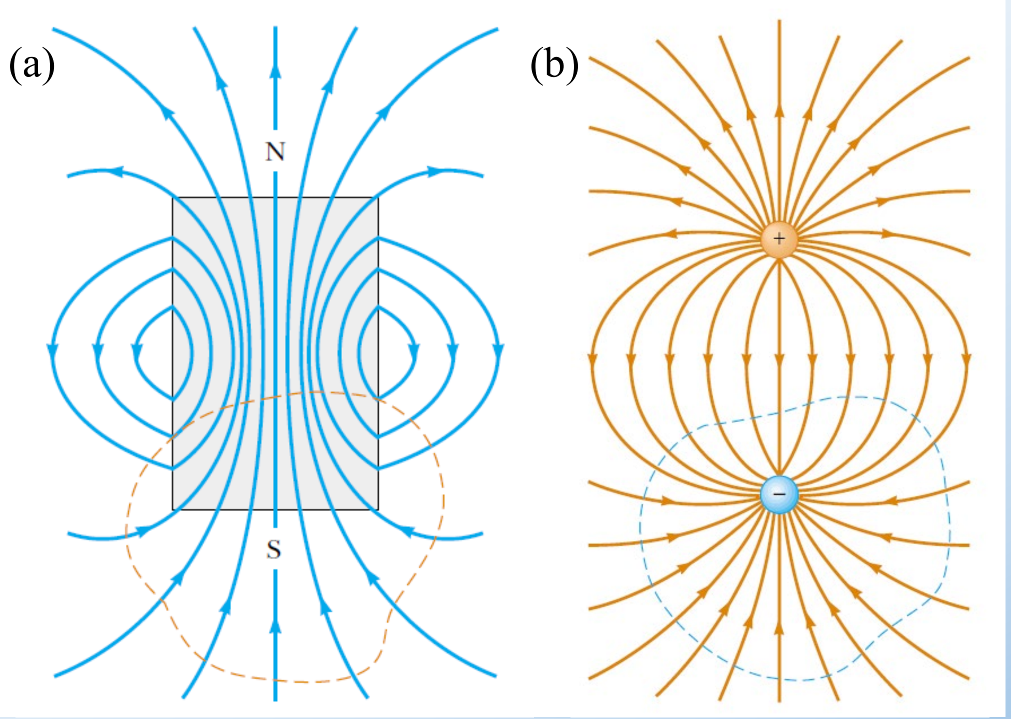
- For any closed surface, such as the one outlined by the dashed line in Fig. (a), the number of lines entering the surface equals the number leaving the surface; thus, the net magnetic flux is zero.
- In contrast, for a closed surface surrounding one charge of an electric dipole (Fig. (b)), the net electric flux is not zero.
- **Gauss's law in magnetism** states that:

The net magnetic flux through any closed surface is always zero:

$$\oint \vec{B} \cdot d\vec{A} = 0$$



(a) The magnetic field lines of a bar magnet form closed loops. Note that the net magnetic flux through a closed surface surrounding one of the poles (or any other closed surface) is zero. (The dashed line represents the intersection of the surface with the page). (b) The electric field lines surrounding an electric dipole begin on the positive charge and terminate on the negative charge. The electric flux through a closed surface surrounding one of the charges is not zero.