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جامعة الملك سعود
King Saud University

Ch. 31

Faraday's Law

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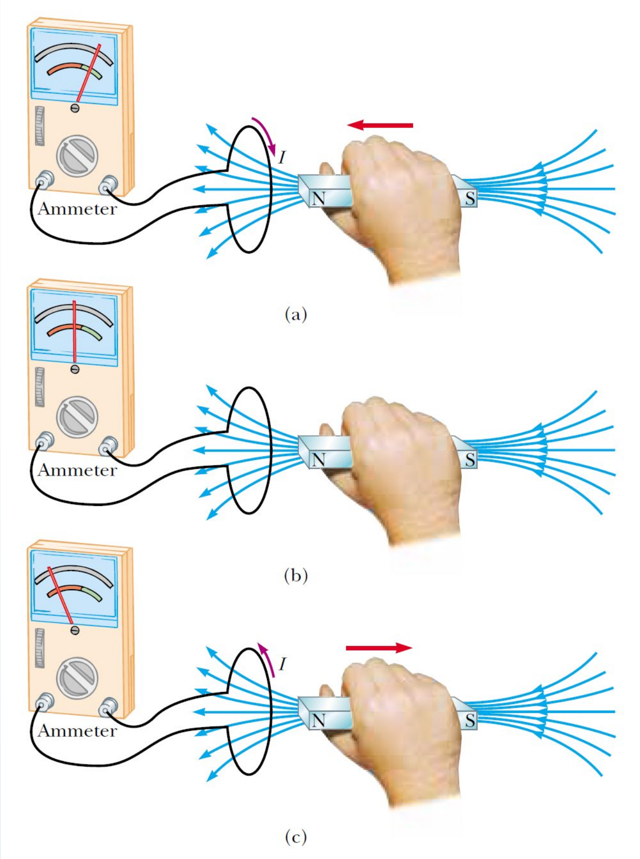
Faraday's Law of Induction

- The focus of our studies in electricity and magnetism so far has been the electric fields produced by stationary charges and the magnetic fields produced by moving charges.
- This chapter explores the effects produced by magnetic fields that vary in time.
- Experiments conducted by Michael Faraday in England in 1831 and independently by Joseph Henry in the United States that same year showed that an emf can be induced in a circuit by a changing magnetic field.
- The results of these experiments led to a very basic and important law of electromagnetism known as <u>Faraday's law of induction</u>.
- An emf (and therefore a current as well) can be induced in various processes that involve a change in a magnetic flux.

Faraday's Law of Induction

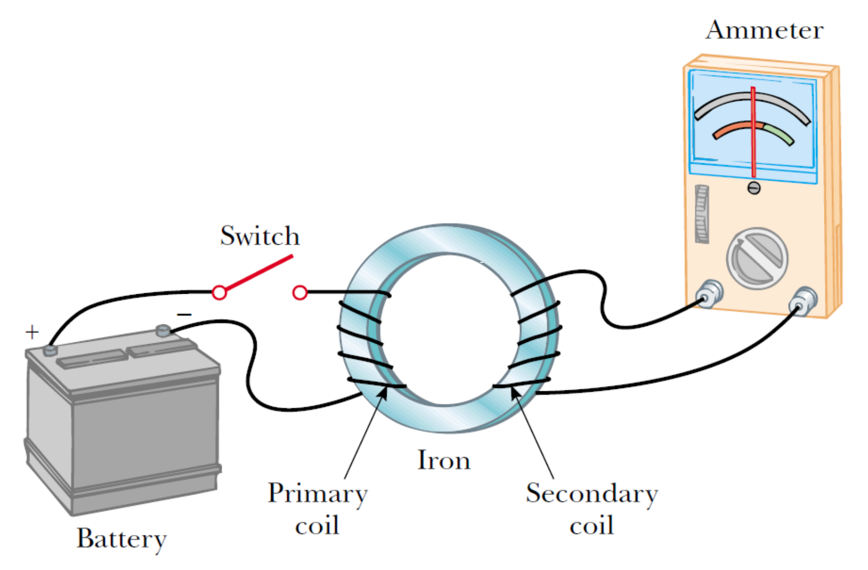
- To see how an emf can be induced by a changing magnetic field, consider a loop of wire connected to a sensitive ammeter.
    - When a magnet is moved toward the loop, the galvanometer needle deflects in one direction, arbitrarily shown to the right in Fig (a).
    - When the magnet is brought to rest and held stationary relative to the loop (Fig.(b)), no deflection is observed.
    - When the magnet is moved away from the loop, the needle deflects in the opposite direction, as shown in Fig.(c).
    - Finally, if the magnet is held stationary and the loop is moved either toward or away from it, the needle deflects.
- From these observations, we conclude that the loop detects that the magnet is moving relative to it and we relate this detection to a change in magnetic field.
- Thus, it seems that a relationship exists between current and changing magnetic field.
- These results are quite remarkable in view of the fact that **a current is set up even though no batteries are present in the circuit!** We call such a current an <u>induced current</u> and say that it is produced by an <u>induced emf</u>.



(a) When a magnet is moved toward a loop of wire connected to a sensitive ammeter, the ammeter deflects as shown, indicating that a current is induced in the loop. (b) When the magnet is held stationary, there is no induced current in the loop, even when the magnet is inside the loop. (c) When the magnet is moved away from the loop, the ammeter deflects in the opposite direction, indicating that the induced current is opposite that shown in part (a). Changing the direction of the magnet's motion changes the direction of the current induced by that motion.

Faraday's Law of Induction

- Now let us describe an experiment conducted by Faraday.
  - A primary coil is connected to a switch and a battery.
  - The coil is wrapped around an iron ring, and a current in the coil produces a magnetic field when the switch is closed.
  - A secondary coil also is wrapped around the ring and is connected to a sensitive ammeter.
  - No battery is present in the secondary circuit, and the secondary coil is not electrically connected to the primary coil.
  - Any current detected in the secondary circuit must be induced by some external agent.
- At the instant the switch is closed, the galvanometer needle deflects in one direction and then returns to zero.
- At the instant the switch is opened, the needle deflects in the opposite direction and again returns to zero.
- Finally, the galvanometer reads zero when there is either a steady current or no current in the primary circuit.
- When the switch is closed, the current in the primary circuit produces a magnetic field that penetrates the secondary circuit which changes from zero to some value over some finite time, and this changing field induces a current in the secondary circuit.



Faraday's experiment. When the switch in the primary circuit is closed, the ammeter in the secondary circuit deflects momentarily. The emf induced in the secondary circuit is caused by the changing magnetic field through the secondary coil.

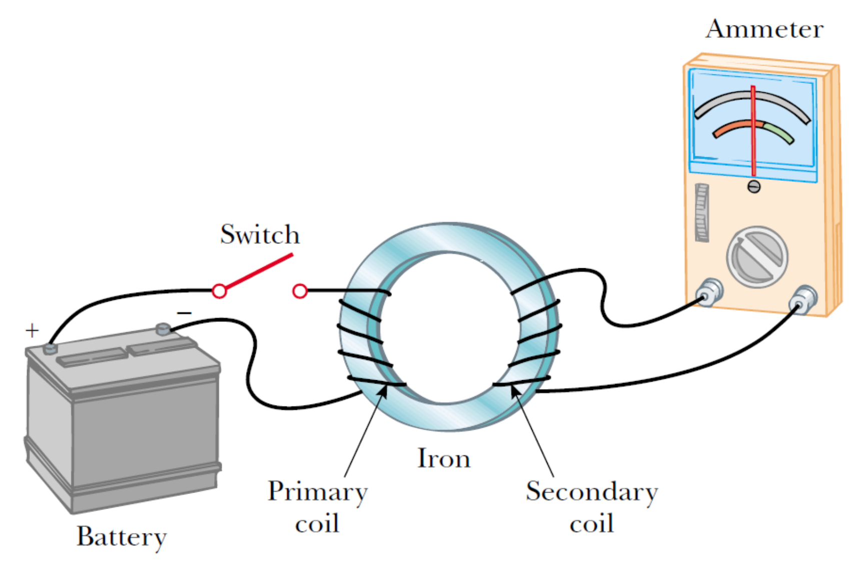
Faraday's Law of Induction

- As a result of these observations, Faraday concluded that **an electric current can be induced in a circuit (the secondary circuit in our setup) by a changing magnetic field.**
- The induced current exists for only a short time while the magnetic field through the secondary coil is changing.
- Once the magnetic field reaches a steady value, the current in the secondary coil disappears.
- In effect, the secondary circuit behaves as though a source of emf were connected to it for a short time.
- It is customary to say that **an induced emf is produced in the secondary circuit by the changing magnetic field.**
- In both experiments, an emf is induced in the circuit when the magnetic flux through the circuit changes with time. In general,

  The emf induced in a circuit is directly proportional to the time rate of change of the magnetic flux through the circuit.
- This statement, known as **Faraday's law of induction**, can be written:

$$\mathcal{E} = -\frac{d\Phi_B}{dt}$$

- Where $\Phi_B = \int \vec{B} \cdot d\vec{A}$ is the magnetic flux through the circuit. (See Section 30.5.)



Faraday's experiment. When the switch in the primary circuit is closed, the ammeter in the secondary circuit deflects momentarily. The emf induced in the secondary circuit is caused by the changing magnetic field through the secondary coil.

Faraday's Law of Induction

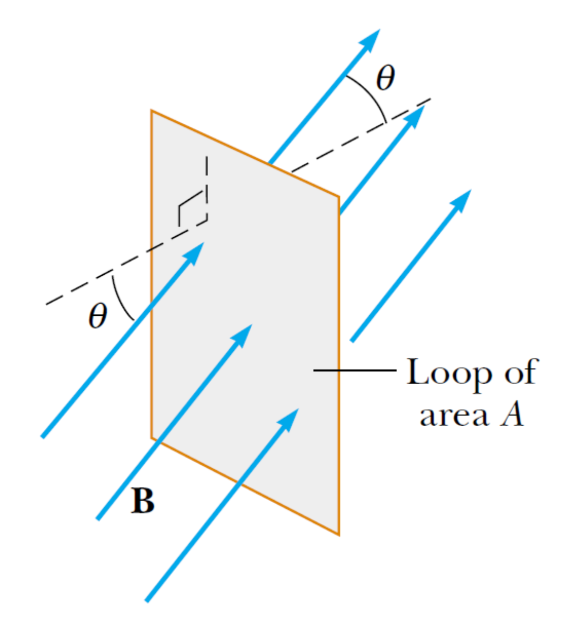
- If the circuit is a coil consisting of N loops all of the same area and if Φ_B is the magnetic flux through one loop, an emf is induced in every loop. Thus, the total induced emf in the coil is given by the expression:

$$\mathcal{E} = -N\frac{d\Phi_B}{dt}$$

- Suppose that a loop enclosing an area A lies in a uniform magnetic field $\vec{B}$. The magnetic flux through the loop is equal to $BA\cos\theta$; hence, the induced emf can be expressed as:

$$\mathcal{E} = -\frac{d}{dt}(BA\cos\theta)$$

- From this expression, we see that an emf can be induced in the circuit in several ways:
  - The magnitude of $\vec{B}$ can change with time.
  - The area enclosed by the loop can change with time.
  - The angle θ between $\vec{B}$ and the normal to the loop can change with time.
  - Any combination of the above can occur.

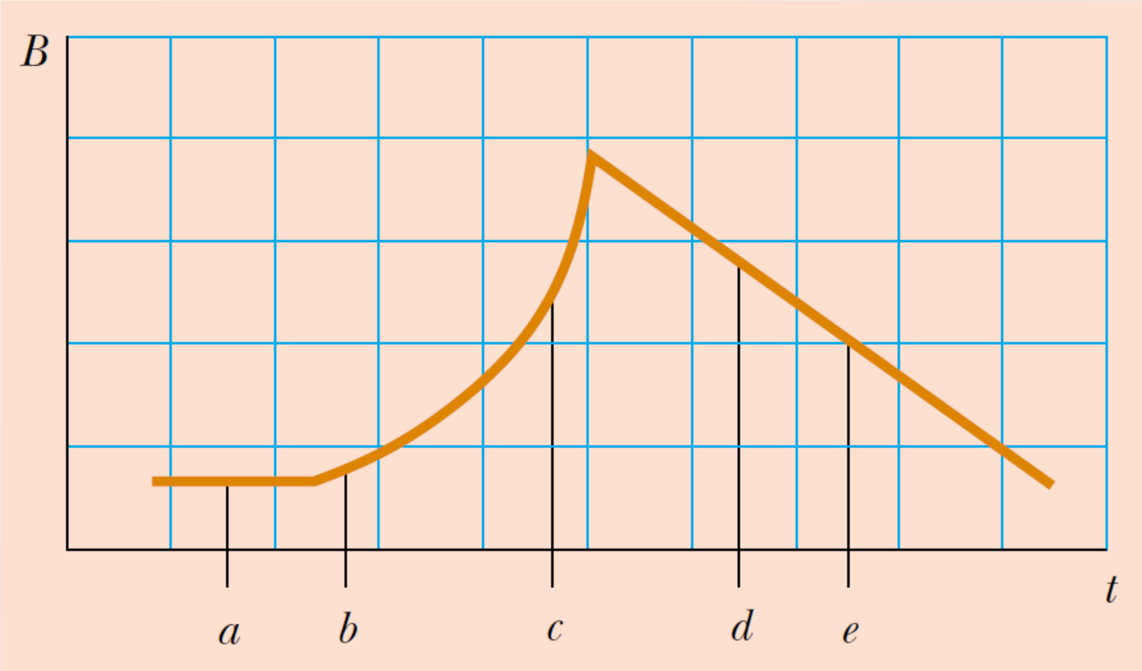


A conducting loop that encloses an area A in the presence of a uniform magnetic field $\vec{B}$. The angle between $\vec{B}$ and the normal to the loop is θ.

Faraday's Law of Induction

Quick Quiz 31.2 The following figure shows a graphical representation of the field magnitude versus time for a magnetic field that passes through a fixed loop and is oriented perpendicular to the plane of the loop. The magnitude of the magnetic field at any time is uniform over the area of the loop. Rank the magnitudes of the emf generated in the loop at the five instants indicated, from largest to smallest.

$c, d = e, b, a$



The time behavior of a magnetic field through a loop.

Faraday's Law of Induction

Example 31.01

A coil consists of 200 turns of wire. Each turn is a square of side 18 cm, and a uniform magnetic field directed perpendicular to the plane of the coil is turned on. If the field changes linearly from 0 to 0.50 T in 0.80 s, what is the magnitude of the induced emf in the coil while the field is changing?

The area of one turn of the coil is $4.(0.18\text{ m})^2 = 0.0324\text{ m}^2$. The magnetic flux through the coil at $t = 0$ is zero because $B = 0$ at that time. At $t = 0.80\text{ s}$, the magnetic flux through one turn is $\Phi_B = BA = (0.50\text{ T})(0.0324\text{ m}^2) = 0.0162\text{ T}\cdot\text{m}^2$. Therefore, the magnitude of the induced emf is:

$$|\mathcal{E}| = N\frac{\Delta\Phi_B}{\Delta t} = 200 \times \frac{0.0162 - 0}{0.80} = 4.1\ \text{T}\cdot\text{m}^2/\text{s} = 4.1\text{ V}$$

You should be able to show that $1\ \text{T}\cdot\text{m}^2/\text{s} = 1\text{ V}$.

What If? What if you were asked to find the magnitude of the induced current in the coil while the field is changing? Can you answer this question?

Answer: If the ends of the coil are not connected to a circuit, the answer to this question is easy—the current is zero! (Charges will move within the wire of the coil, but they cannot move into or out of the ends of the coil.) In order for a steady current to exist, the ends of the coil must be connected to an external circuit. Let us assume that the coil is connected to a circuit and that the total resistance of the coil and the circuit is 2.0 Ω. Then, the current in the coil is:

$$I = \frac{\mathcal{E}}{R} = \frac{4.1}{2.0} = 2.0\text{ A}$$

Problem 31.05

A strong electromagnet produces a uniform magnetic field of 1.60 T over a cross-sectional area of 0.200 cm^2. We place a coil having 200 turns and a total resistance of 20.0 Ω around the electromagnet. We then smoothly reduce the current in the electromagnet until it reaches zero in 20.0 ms. What is the current induced in the coil?

Noting unit conversions from $\vec{F} = q\vec{v} \times \vec{B}$ and $U = qV$, the induced voltage is:

$$\mathcal{E} = -N\frac{d\Phi_B}{dt} = -N\frac{d\left(\vec{B}\cdot\vec{A}\right)}{dt} = -N\frac{0 - B_i A\cos\theta}{\Delta t} = 200 \times \frac{1.60 \times 0.200 \times 10^{-4} \times \cos 0}{20.0 \times 10^{-3}} = 0.32\ \text{V}$$

$$I = \frac{\mathcal{E}}{R} = \frac{0.32}{20.0} = 0.016\ \text{A} = 16\ \text{mA}$$

Faraday's Law of Induction

Problem 31.13

Additional problem

A long solenoid has $n = 400$ turns per meter and carries a current given by $I = (30\text{ A})\left(1 - e^{-1.6t}\right)$. Inside the solenoid and coaxial with it is a coil that has a radius of 6 cm and consists of a total of N = 250 turns of fine wire (see the figure below). What emf is induced in the coil by the changing current?

$$B = \mu_0 nI = \mu_0 n \times 30 \times \left(1 - e^{-1.6t}\right) = 30\mu_0 n\left(1 - e^{-1.6t}\right)$$

$$\Phi_B = BA = 30\mu_0 n\left(1 - e^{-1.6t}\right) \times \left(\pi R^2\right) = 30\pi\mu_0 nR^2\left(1 - e^{-1.6t}\right)$$

$$\mathcal{E} = -N\frac{d\Phi_B}{dt}$$

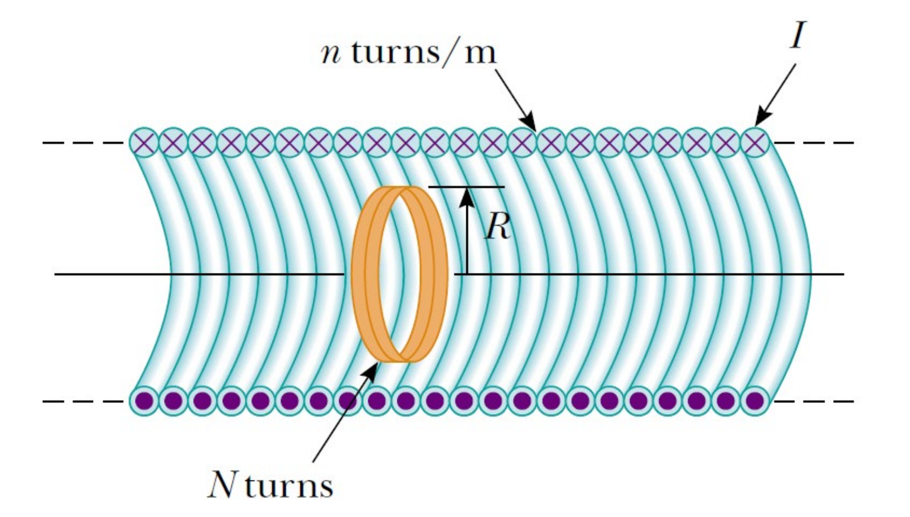
$$\mathcal{E} = -N\frac{d}{dt}\left(30\pi\mu_0 nR^2\left(1 - e^{-1.6t}\right)\right)$$

$$\mathcal{E} = -30\pi\mu_0 nR^2 N\frac{d}{dt}\left(1 - e^{-1.6t}\right) = -30\pi\mu_0 nR^2 N\left(1.6\, e^{-1.6t}\right)$$

$$\mathcal{E} = -48\pi\mu_0 nR^2 N\, e^{-1.6t}$$

$$\mathcal{E} = -48 \times \pi \times 4\pi \times 10^{-7} \times 400 \times \left(6 \times 10^{-2}\right)^2 \times 250 \times e^{-1.6t}$$

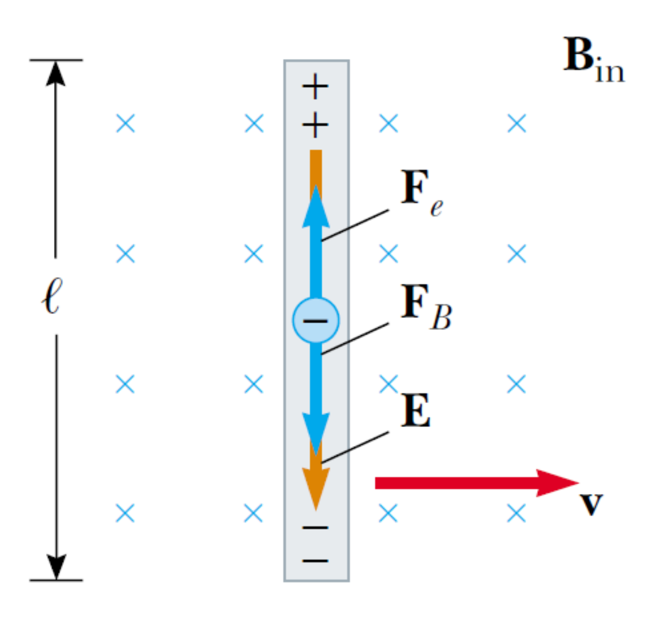
$$\mathcal{E} = \left(-6.82 \times 10^{-2} \times e^{-1.6t}\right)\text{ V} = \left(-68.2\, e^{-1.6t}\right)\text{ mV}$$



Motional emf

- In the previous section, we considered cases in which an emf is induced in a stationary circuit placed in a magnetic field when the field changes with time.
- In this section we describe what is called **motional emf**, which is the emf induced in a conductor moving through a constant magnetic field.
- The straight conductor of length l is moving through a uniform magnetic field directed into the page.
- For simplicity, we assume that the conductor is moving in a direction perpendicular to the field with constant velocity under the influence of some external agent.
- The electrons in the conductor experience a force $\vec{F}_B = q\vec{v} \times \vec{B}$ that is directed along the length l, perpendicular to both $\vec{v}$ and $\vec{B}$.
- Under the influence of this force, the electrons move to the lower end of the conductor and accumulate there, leaving a net positive charge at the upper end.
- As a result of this charge separation, an electric field $\vec{E}$ is produced inside the conductor.
- The charges accumulate at both ends until the downward magnetic force qvB on charges remaining in the conductor is balanced by the upward electric force qE.
- At this point, electrons move only with random thermal motion. The condition for equilibrium requires that:

$$qE = qvB \qquad \text{or} \qquad E = vB$$



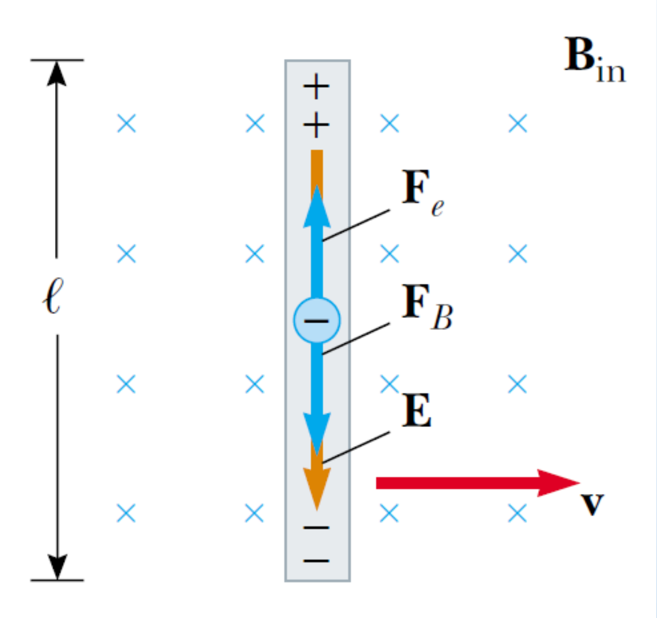
A straight electrical conductor of length l moving with a velocity $\vec{v}$ through a uniform magnetic field $\vec{B}$ directed perpendicular to $\vec{v}$. Due to the magnetic force on electrons, the ends of the conductor become oppositely charged. This establishes an electric field in the conductor. In steady state, the electric and magnetic forces on an electron in the wire are balanced.

Motional emf

- The electric field produced in the conductor is related to the potential difference across the ends of the conductor according to the relationship $\Delta V = El$. Thus, for the equilibrium condition:

$$\Delta V = El = Blv$$

- Where the upper end of the conductor is at a higher electric potential than the lower end.
- Thus, **a potential difference is maintained between the ends of the conductor as long as the conductor continues to move through the uniform magnetic field.**
- If the direction of the motion is reversed, the polarity of the potential difference is also reversed.



A straight electrical conductor of length l moving with a velocity $\vec{v}$ through a uniform magnetic field $\vec{B}$ directed perpendicular to $\vec{v}$. Due to the magnetic force on electrons, the ends of the conductor become oppositely charged. This establishes an electric field in the conductor. In steady state, the electric and magnetic forces on an electron in the wire are balanced.

Motional emf

- A more interesting situation occurs when the moving conductor is part of a closed conducting path. This situation is particularly useful for illustrating **how a changing magnetic flux causes an induced current in a closed circuit**.

- Consider a circuit consisting of a conducting bar of length l sliding along two fixed parallel conducting rails, as shown in Fig.(a). For simplicity, we assume that the bar has zero resistance and that the stationary part of the circuit has a resistance R.

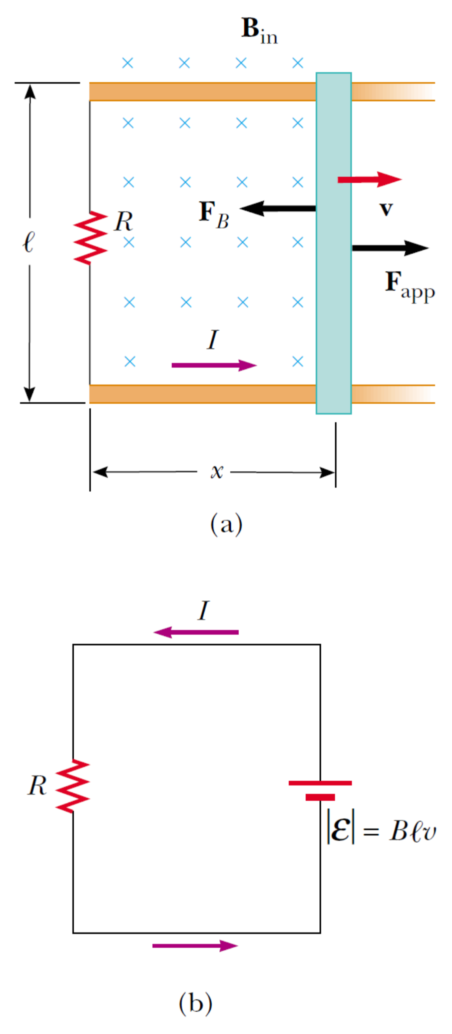
  - A uniform and constant magnetic field $\vec{B}$ is applied perpendicular to the plane of the circuit.

  - As the bar is pulled to the right with a velocity $\vec{v}$ under the influence of an applied force $\vec{F}_{app}$, free charges in the bar experience a magnetic force directed along the length of the bar.

  - This force sets up an induced current because the charges are free to move in the closed conducting path.

  - In this case, the rate of change of magnetic flux through the loop and the corresponding induced motional emf across the moving bar are proportional to the change in area of the loop.

  - If the bar is pulled to the right with a constant velocity, the work done by the applied force appears as internal energy in the resistor R.



(a) A conducting bar sliding with a velocity $\vec{v}$ along two conducting rails under the action of an applied force $\vec{F}_{app}$. The magnetic force $\vec{F}_B$ opposes the motion, and a counterclockwise current I is induced in the loop. (b) The equivalent circuit diagram for the setup shown in part (a).

Motional emf

- Because the area enclosed by the circuit at any instant is lx, where x is the position of the bar, the magnetic flux through that area is:

$$\Phi_B = Blx$$

- Using Faraday's law, and noting that x changes with time at a rate $dx/dt = v$, we find that the induced motional emf is:

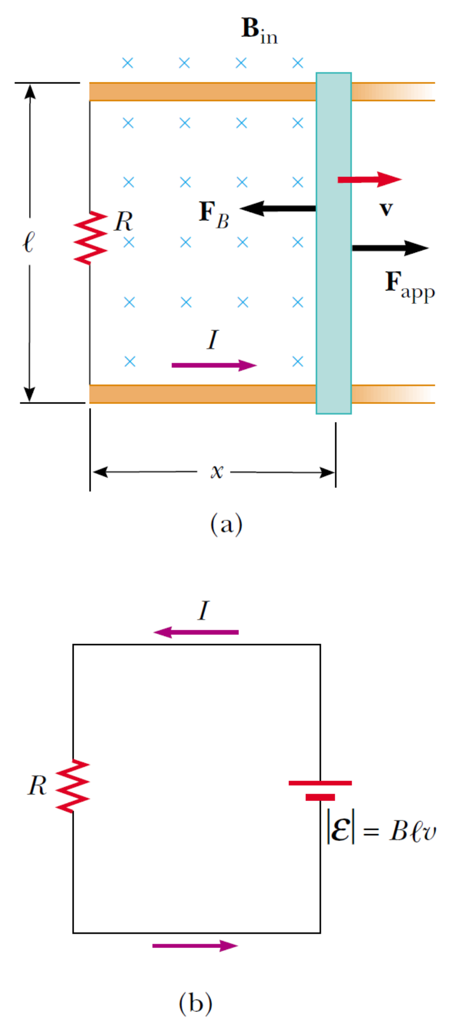
$$\mathcal{E} = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}(Blx) = -Bl\frac{dx}{dt} = -Blv$$

- Because the resistance of the circuit is R, the magnitude of the induced current is:

$$I = \frac{|\mathcal{E}|}{R} = \frac{Blv}{R}$$

- The applied force does work on the conducting bar, thereby moving charges through a magnetic field. Their movement through the field causes the charges to move along the bar with some average drift velocity, and hence a current is established.
- Because the bar moves with constant velocity, the applied force must be equal in magnitude and opposite in direction to the magnetic force.
- The power delivered by the applied force is:

$$\mathcal{P} = F_{app}v = (IlB)v = \frac{B^2l^2v^2}{R} = \frac{\mathcal{E}^2}{R}$$



(a) A conducting bar sliding with a velocity $\vec{v}$ along two conducting rails under the action of an applied force $\vec{F}_{app}$. The magnetic force $\vec{F}_B$ opposes the motion, and a counterclockwise current I is induced in the loop. (b) The equivalent circuit diagram for the setup shown in part (a).

Motional emf

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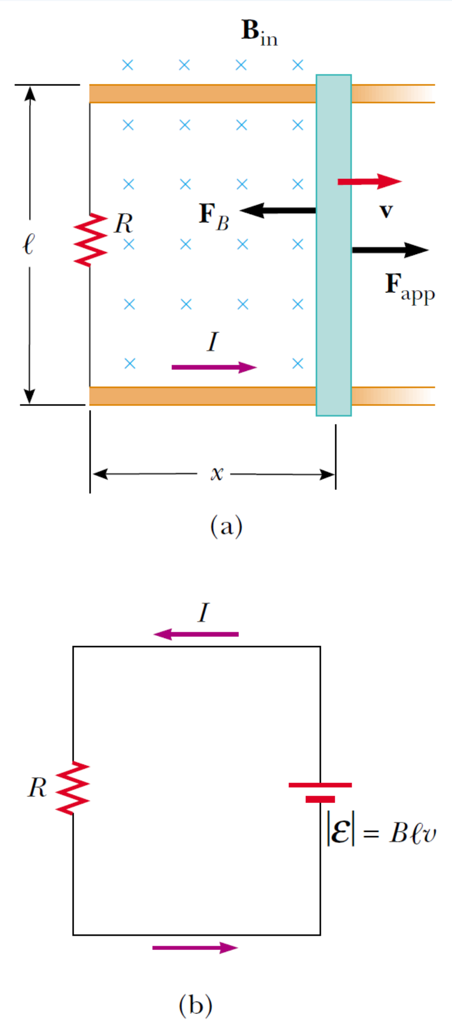
Quick Quiz 31.5 In Fig (a) given applied force of magnitude F_{app} results in a constant speed v and a power input $\mathcal{P}$. Imagine that the force is increased so that the constant speed of the bar is doubled to $2v$. Under these conditions, the new force and the new power input are:

(a) $2F$ and $2\mathcal{P}$ (b) $4F$ and $2\mathcal{P}$ (c) $2F$ and $4\mathcal{P}$ (d) $4F$ and $4\mathcal{P}$.



(a) A conducting bar sliding with a velocity $\vec{v}$ along two conducting rails under the action of an applied force $\vec{F}_{app}$. The magnetic force $\vec{F}_B$ opposes the motion, and a counterclockwise current I is induced in the loop. (b) The equivalent circuit diagram for the setup shown in part (a).

Motional emf

Problem 31.20

Consider the arrangement shown in the following figure. Assume that $R = 6.00\ \Omega$, $l = 1.20$ m, and a uniform 2.50 T magnetic field is directed into the page. At what speed should the bar be moved to produce a current of 0.500 A in the resistor?

$$I = \frac{|\mathcal{E}|}{R} = \frac{Blv}{R}$$

$$v = \frac{IR}{Bl} = \frac{0.500 \times 6.00}{2.50 \times 1.20} = 1.00\ \text{m/s}$$