104 PHYS

Ch. 32

Inductance

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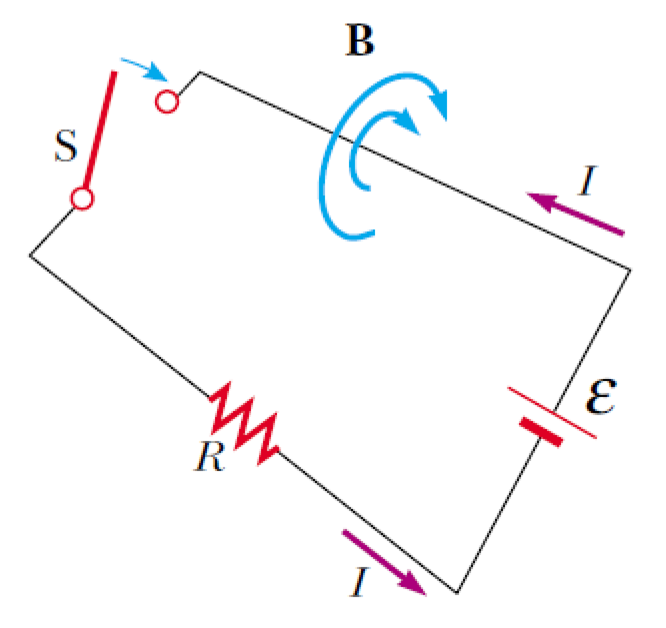
Self-Inductance

- In Chapter 31, we saw that an emf and a current are induced in a circuit when the magnetic flux through the area enclosed by the circuit changes with time.

First example:

- Consider a circuit consisting of a switch, a resistor, and a source of emf.
  - When the switch is thrown to its closed position, the current does not immediately jump from zero to its maximum value $\mathcal{E}/R$.
  - As the current increases with time, the magnetic flux through the circuit loop due to this current also increases with time. This increasing flux creates an induced emf in the circuit.
  - The direction of the induced emf is such that it would cause an induced current in the loop (if the loop did not already carry a current), which would establish a magnetic field opposing the change in the original magnetic field. Thus, the direction of the induced emf is opposite the direction of the emf of the battery.
  - This results in a gradual rather than instantaneous increase in the current to its final equilibrium value.
- This effect is called **self-induction** because the changing flux through the circuit and the resultant induced emf arise from the circuit itself. The emf $\mathcal{E}_L$ set up in this case is called a **self-induced emf**.
- Because of the direction of the induced emf, it is also called a <u>back emf</u>.

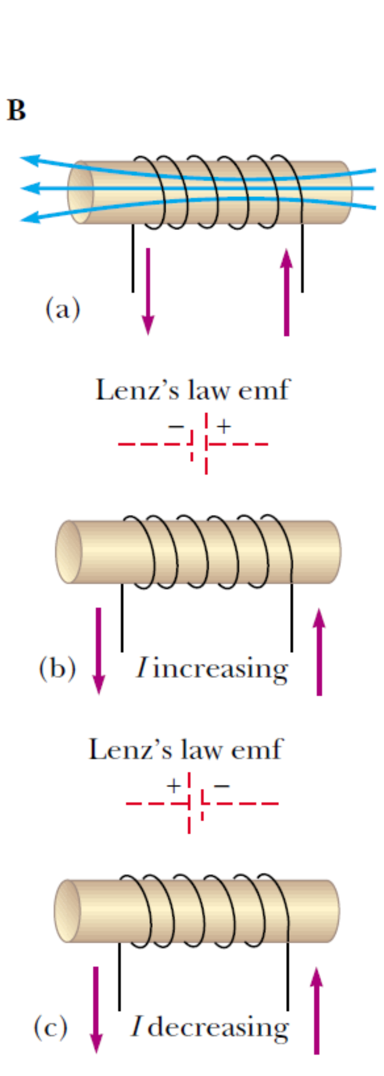


After the switch is closed, the current produces a magnetic flux through the area enclosed by the loop. As the current increases toward its equilibrium value, this magnetic flux changes in time and induces an emf in the loop.

Self-Inductance

Second example:

- Assume that the current in a coil wound on a cylindrical core either increases or decreases with time.
  - When the current is in the direction shown, a magnetic field directed from right to left is set up inside the coil, as seen in Fig. (a).
  - As the current changes with time, the magnetic flux through the coil also changes and induces an emf in the coil.
  - The polarity of this induced emf must be such that it opposes the change in the magnetic field from the current.
  - If the current is increasing, the polarity of the induced emf is as pictured in Fig. (b), and if the current is decreasing, the polarity of the induced emf is as shown in Fig. (c).



(a) A current in the coil produces a magnetic field directed to the left. (b) If the current increases, the increasing magnetic flux creates an induced emf in the coil having the polarity shown by the dashed battery. (c) The polarity of the induced emf reverses if the current decreases.

Self-Inductance

- To obtain a quantitative description of self-induction, we recall from Faraday's law that the induced emf is equal to the negative of the time rate of change of the magnetic flux.
- The magnetic flux is proportional to the magnetic field due to the current, which in turn is proportional to the current in the circuit.
- Therefore, **a self-induced emf is always proportional to the time rate of change of the current**. For any coil, we find that:

$$\mathcal{E}_L = -L\frac{dI}{dt}$$

- Where L is a proportionality constant—called the **inductance** of the coil.
- Combining this expression with Faraday's law, $\mathcal{E}_L = -N\, d\Phi_B/dt$, we see that the inductance of a closely spaced coil of N turns carrying a current I and containing N turns is:

$$L = \frac{N\Phi_B}{I}$$

- Where it is assumed that the same magnetic flux passes through each turn. We can also write the inductance as the ratio:

$$L = -\frac{\mathcal{E}_L}{dI/dt}$$

- The inductance is a measure of the opposition to a <u>change</u> in current.
- The SI unit of inductance is the **henry** (H), which is 1 volt-second per ampere:

$$1\ \mathrm{H} = 1\ \frac{\mathrm{V \cdot s}}{\mathrm{A}}$$

Self-Inductance

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Example 32.01

Find the inductance of a uniformly wound solenoid having N turns and length l. Assume that l is much longer than the radius of the windings and that the core of the solenoid is air.

We can assume that the interior magnetic field due to the current is uniform and given by:

$$B = \mu_0 n I = \mu_0 \frac{N}{l} I$$

where $n = N/l$ is the number of turns per unit length. The magnetic flux through each turn is:

$$\Phi_B = BA = \mu_0 \frac{NA}{l} I$$

where A is the cross-sectional area of the solenoid. We find that the inductance of the solenoid is:

$$L = \frac{N\Phi_B}{I} = \mu_0 \frac{N^2}{l} A$$

This result shows that L depends on geometry and is proportional to the square of the number of turns. Because $N = nl$, we can also express the result in the form:

$$L = \mu_0 \frac{(nl)^2}{l} A = \mu_0 n^2 A l = \mu_0 n^2 \mathcal{V}$$

where $\mathcal{V} = Al$ is the interior volume of the solenoid.

Self-Inductance

Example 32.01

Find the inductance of a uniformly wound solenoid having N turns and length l. Assume that l is much longer than the radius of the windings and that the core of the solenoid is air.

What If? What would happen to the inductance if you inserted a ferromagnetic material inside the solenoid?

Answer The inductance would increase. For a given current, the magnetic flux in the solenoid is much greater because of the increase in the magnetic field originating from the magnetization of the ferromagnetic material. For example, if the material has a magnetic permeability of $500\mu_0$, the inductance increases by a factor of 500.

Self-Inductance

Example 32.02

(a) Calculate the inductance of an air-core solenoid containing 300 turns if the length of the solenoid is 25.0 cm and its cross-sectional area is 4.00 cm^2.
(b) Calculate the self-induced emf in the solenoid if the current it carries is decreasing at the rate of 50.0 A/s.

(a) $$L = \mu_0 \frac{N^2}{l} A = 4\pi \times 10^{-7} \times \frac{(300)^2}{25.0 \times 10^{-2}} \times 4.00 \times 10^{-4} = 1.81 \times 10^{-4}\ \text{T} \cdot \text{m}^2/\text{A} = 0.181\ \text{mH}$$

(b) Using the equation of the self-induced emf and given that $dI/dt = 50.0\ \text{A/s}$, we obtain:

$$\mathcal{E}_L = -L\frac{dI}{dt} = -1.81 \times 10^{-4} \times (-50.0) = 9.05 \times 10^{-3}\ \text{V} = 9.05\ \text{mV}$$

Self-Inductance

Problem 32.06

An emf of 24.0 mV is induced in a 500 – turn coil at an instant when the current is 4.00 A and is changing at the rate of 10.0 A/s. What is the magnetic flux through each turn of the coil?

$$|\mathcal{E}_L| = L\frac{\Delta I}{\Delta t}$$

$$L = \frac{|\mathcal{E}_L|}{\Delta I/\Delta t} = \frac{24.0 \times 10^{-3}}{10.0} = 2.40 \times 10^{-3}\ \mathrm{H} = 2.40\ \mathrm{mH}$$

$$L = \frac{N\Phi_B}{I}$$

$$\Phi_B = \frac{IL}{N} = \frac{4.00 \times 2.40 \times 10^{-3}}{500} = 1.92 \times 10^{-5}\ \mathrm{T \cdot m^2} = 19.2\ \mathrm{\mu T \cdot m^2}$$

Self-Inductance

Problem 32.09

A $40.0-\text{mA}$ current is carried by a uniformly wound air-core solenoid with 450 turns, a $15.0-\text{mm}$ diameter, and $12.0-\text{cm}$ length. Compute (a) the magnetic field inside the solenoid, (b) the magnetic flux through each turn, and (c) the inductance of the solenoid. (d) **What If?** If the current were different, which of these quantities would change?

(a) $B=\mu_0\frac{N}{l}I=4\pi\times10^{-7}\times\frac{450}{12.0\times10^{-2}}\times40.0\times10^{-3}=1.88\times10^{-4}\ \text{T}=188\ \mu\text{T}$

(b) $\Phi_B=BA=1.88\times10^{-4}\times\pi\times\left(7.50\times10^{-3}\right)^2=3.33\times10^{-8}\ \text{T}\cdot\text{m}^2$

(c) $L=\frac{N\Phi_B}{I}=\frac{450\times3.33\times10^{-8}}{40.0\times10^{-3}}=3.75\times10^{-4}\ \text{H}=0.375\ \text{mH}$

(d) B and Φ_B are proportional to current; L is independent of current

Energy in a Magnetic Field

- Because the emf induced in an inductor prevents a battery from establishing an instantaneous current, the battery must provide more energy than in a circuit without the inductor.
- Part of the energy supplied by the battery appears as internal energy in the resistor, while the remaining energy is stored in the magnetic field of the inductor.

$$I\mathcal{E} = I^2R + LI\frac{dI}{dt}$$

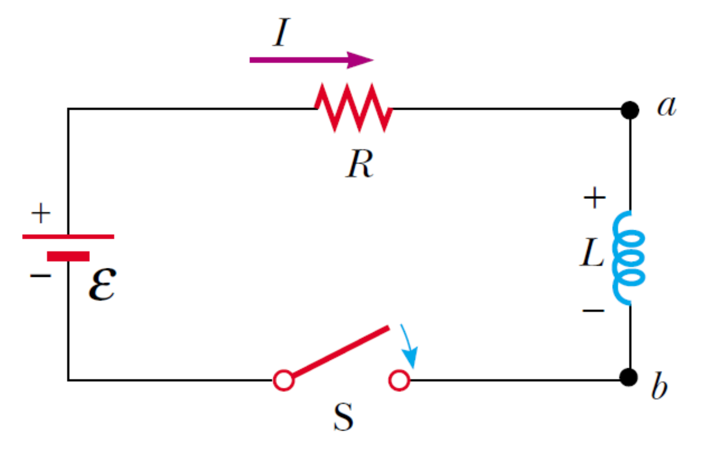
- Recognizing $I\mathcal{E}$ as the rate at which energy is supplied by the battery and I^2R as the rate at which energy is delivered to the resistor, we see that $LI(dI/dt)$ must represent the rate at which energy is being stored in the inductor.
- If we let U denote the energy stored in the inductor at any time, then we can write the rate dU/dt at which energy is stored as:

$$\frac{dU}{dt} = LI\frac{dI}{dt}$$

- To find the total energy stored in the inductor, we can rewrite this expression as $dU = LIdI$ and integrate:

$$U = \int dU = \int_0^I LIdI = L\int_0^I IdI = \frac{1}{2}LI^2$$

- Where L is constant and has been removed from the integral.



A series RL circuit. As the current increases toward its maximum value, an emf that opposes the increasing current is induced in the inductor.

Energy in a Magnetic Field

- This expression represents the energy stored in the magnetic field of the inductor when the current is I.
- We can also determine the energy density of a magnetic field. For simplicity, consider a solenoid whose inductance is given by:

$$L = \mu_0 n^2 A l$$

- The magnetic field of a solenoid is given by:

$$B = \mu_0 n I$$

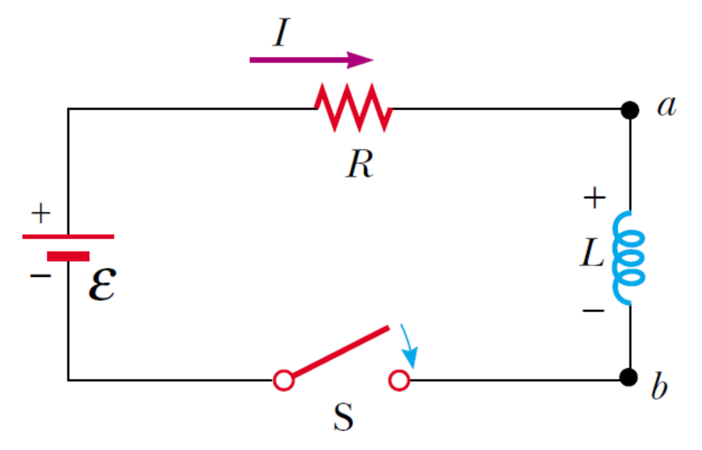
- Substituting the expression for L and $I = B/\mu_0 n$ into $U = \frac{1}{2} L I^2$ gives:

$$U = \frac{1}{2} L I^2 = \frac{1}{2} \mu_0 n^2 A l \left(\frac{B}{\mu_0 n}\right)^2 = \frac{1}{2} \frac{B^2}{\mu_0} A l$$

- Because Al is the volume of the solenoid, the magnetic energy density, or the energy stored per unit volume in the magnetic field of the inductor is:

$$u_B = \frac{U}{Al} = \frac{1}{2} \frac{B^2}{\mu_0}$$

- The energy density is proportional to the square of the field magnitude.



A series RL circuit. As the current increases toward its maximum value, an emf that opposes the increasing current is induced in the inductor.

Energy in a Magnetic Field

Quick Quiz 32.5 You are performing an experiment that requires the highest possible energy density in the interior of a very long solenoid. Which of the following increases the energy density? (More than one choice may be correct.)

(a) increasing the number of turns per unit length on the solenoid

(b) increasing the cross-sectional area of the solenoid

(c) increasing only the length of the solenoid while keeping the number of turns per unit length fixed

(d) increasing the current in the solenoid.

Energy in a Magnetic Field

Problem 32.29

Calculate the energy associated with the magnetic field of a 200 – turn solenoid in which a current of 1.75 A produces a flux of 3.70×10^{-4} Wb in each turn.

$$L = \frac{N\Phi_B}{I} = \frac{200 \times 3.70 \times 10^{-4}}{1.75} = 4.22 \times 10^{-2}\ \text{H} = 42.2\ \text{mH}$$

$$U = \frac{1}{2}LI^2 = \frac{1}{2} \times 4.22 \times 10^{-2} \times (1.75)^2 = 0.0646\ \text{J}$$

Energy in a Magnetic Field

Problem 32.30

The magnetic field inside a superconducting solenoid is 4.50 T. The solenoid has an inner diameter of 6.20 cm and a length of 26.0 cm. Determine (a) the magnetic energy density in the field and (b) the energy stored in the magnetic field within the solenoid.

(a) The magnetic energy density is given by:

$$u_B = \frac{1}{2}\frac{B^2}{\mu_0} = \frac{1}{2} \times \frac{(4.50)^2}{4\pi \times 10^{-7}} = 8.06 \times 10^6 \ \text{J/m}^3$$

(b) The magnetic energy stored in the field equals u_B times the volume of the solenoid (the volume in which B is non-zero).

$$U = u_B \mathcal{V} = u_B l A = u_B l(\pi r^2) = 8.06 \times 10^6 \times 26.0 \times 10^{-2} \times \pi \times \left(3.10 \times 10^{-2}\right)^2 = 6.33 \times 10^3 \text{ J} = 6.33 \text{ kJ}$$

Energy in a Magnetic Field

Problem 32.31

An air-core solenoid with 68 turns is 8.00 cm long and has a diameter of 1.20 cm. How much energy is stored in its magnetic field when it carries a current of 0.770 A?

$$L = \mu_0 \frac{N^2}{l} A = \mu_0 \frac{N^2}{l} \left(\pi r^2\right) = 4\pi \times 10^{-7} \times \frac{(68)^2}{8.00 \times 10^{-2}} \times \pi \times \left(0.60 \times 10^{-2}\right)^2 = 8.21 \times 10^{-6}\ \mathrm{H} = 8.21\ \mu\mathrm{H}$$

$$U = \frac{1}{2} L I^2 = \frac{1}{2} \times 8.21 \times 10^{-6} \times (0.770)^2 = 2.43 \times 10^{-6}\ \mathrm{J} = 2.43\ \mu\mathrm{J}$$

Energy in a Magnetic Field

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Problem 32.37

A uniform electric field of magnitude 680 kV/m throughout a cylindrical volume results in a total energy of 3.40 μJ. What magnetic field over this same region stores the same total energy?

We have:

$$U = u_E Al = \frac{1}{2}\epsilon_0 E^2 Al \quad \text{and} \quad U = u_B Al = \frac{1}{2}\frac{B^2}{\mu_0} Al$$

$$\frac{1}{2}\epsilon_0 E^2 Al = \frac{1}{2}\frac{B^2}{\mu_0} Al$$

$$B^2 = \epsilon_0 \mu_0 E^2$$

$$B = \sqrt{\epsilon_0 \mu_0} E$$

$$B = \frac{E}{c_0}$$

where $c_0 = 1/\sqrt{\epsilon_0 \mu_0}$ is the speed of light in vacuum

$$B = \frac{680 \times 10^3}{3.00 \times 10^8}$$

$$B = 2.27 \times 10^{-3}\ \text{T} = 2.27\ \text{mT}$$