Q [1]: 4.11 (Text Book)

4.11 Consider the system of Figure 4.2, where the input continuous-time signal $x_a(t)$ has a band-limited spectrum $X_a(j\Omega)$, as sketched in Figure P4.1(a), and is being sampled at the Nyquist rate. The discrete-time processor is an ideal lowpass filter with a frequency response $H(e^{j\omega})$, as shown in Figure P4.1(b), and has a cutoff frequency $\omega_c = \Omega_m T/3$, where T is the sampling period. Sketch as accurately as possible the spectrum $Y_a(j\Omega)$ of the output continuous-time signal $y_a(t)$.

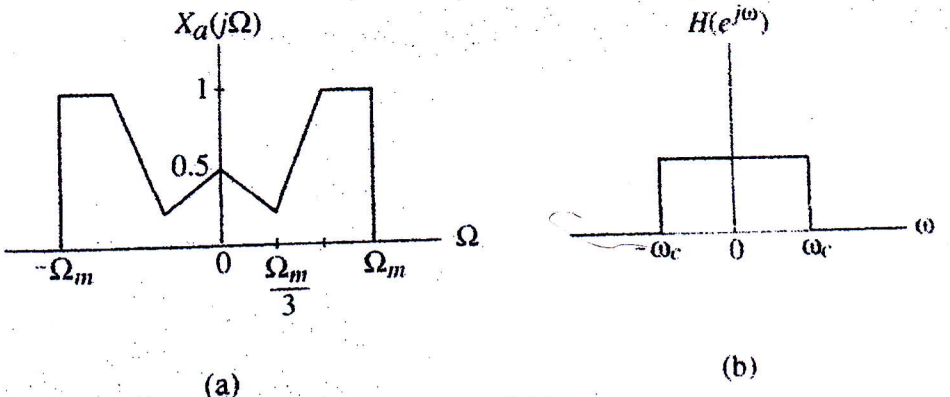

Figure P4.1

Nyquist rate → $\Omega_T = 2\Omega_m$

$\Rightarrow \Omega_T = 2\pi f_s$

$$f_s = \frac{\Omega_T}{2\pi} = \frac{2\Omega_m}{2\pi} = \frac{\Omega_m}{\pi}$$

$$\rightarrow T = \frac{1}{f_s} = \frac{\pi}{\Omega_m}$$

$$\rightarrow \omega_c = \frac{\Omega_m}{3}(T) = \frac{\Omega_m}{3}\left(\frac{\pi}{3}\right) = \frac{\pi}{3}$$

$$\rightarrow \Omega_c = \frac{\pi}{3}\left(\frac{1}{T}\right) = \frac{\pi}{3}\left(\frac{\Omega_m}{\pi}\right) = \frac{\Omega_m}{3}$$