Quiz 1. Numerical Methods (Math-254) Section: 82442 Date: 2nd April 2023

Name: ID: Time Allowed: 35 minutes

Q1. Apply four iterations of Bisection Method to find the root of:

$e^{-x}(3.2sinx - 0.5cosx) = 0$ in the interval [3,4]. [3 Marks]

Q2. Determine the point of intersection of $y = 3x^2$ and $y = e^x$ accurate within 10^{-2} by using fixed point method. [3 Marks]

Q3. a) Using Newton's Method, find a solution accurate to 10^{-3} for:

$x^2 - 2xe^{-x} + e^{-2x} = 0$ Take $x_0 = 0.5$ [2+2 Marks]

b) Use the 2nd Modified Newton's Method to find the root of equation given in part (a).

Q.1

$f(3) = 0.047$ $f(4) = -0.038$

n.	a	b	c	f(c)	update
1.	3	4	3.5	-0.01976	b = c
2	3	3.5	3.25	0.005848	a = c
3	3.25	3.5	3.375	-0.00868	b = c
4	3.25	3.375	3.3125	-0.00188	b = c.

$f(x) = e^x - 3x^2 = 0$

$f(0) = 1$ (+ve) ; $f(1) = e - 3$ (-ve)

Q.2

$3x^2 = e^x \Rightarrow x^2 = \frac{e^x}{3} \Rightarrow x = \sqrt{\frac{e^x}{3}}$

$g(x) = \sqrt{\frac{e^x}{3}}$; $g'(x) = \frac{1}{2}\frac{e^x}{3}\left(\frac{1}{3}e^x\right) = \frac{e^{2x}}{18}$

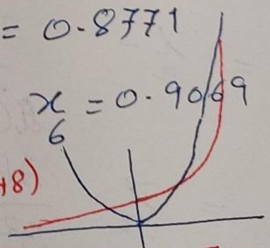
$|g'(x)| < 1$ in $[0,1]$

take $x_0 = 0.5$

$x_1 = 0.7413$; $x_2 = 0.8364$, $x_3 = 0.8771$

$x_4 = 0.8952$; $x_5 = 0.9033$; $x_6 = 0.9069$.

pt. of intersection ; (0.91, 2.48)



-0.459 , 0.910 , 3.733

[-1,0] ; [0,1] , [3,4]

Q.3 a) $f(x) = x^2 - 2xe^{-x} + e^{-2x} = (x-e^{-x})^2$.

$(x-e^{-x})(x-e^{-x}) = 0$ Double root.

$f_1(x) = x - e^{-x}$ $f_1(0) = -1$ -ve.

$[0,1]$; $f_1'(x) = 1+e^{-x}$ $f_1(1) = 1 - \frac{1}{e}$ +ve.

take $x_0 = 0.5$

$$x_{n+1} = x_n - \frac{f_1(x_n)}{f'(x_n)}$$

$$= x_n - \frac{(x_n - e^{-x_n})}{1+e^{-x_n}}.$$

$x_1 = 0.5665$

$x_2 = 0.5671$

$x_3 = 0.5671$

$f(x) = (x-e^{-x})^2$

b). $f'(x) = 2(x-e^{-x})(1+e^{-x})$.

$f''(x) = 2[(1+e^{-x})^2 - e^{-x}(x-e^{-x})]$

$$x_{n+1} = x_n - \frac{f(x_n)f'(x_n)}{[f'(x_n)]^2 - f(x_n)f''(x_n)}.$$

$$= x_n - \frac{2(x-e^{-x})^3(1+e^{-x})}{4(x-e^{-x})^2(1+e^{-x})^2 - 2(1+e^{-x})^2(x-e^{-x})^2 + 2e^{-x}(x-e^{-x})^3}$$

$$= x_n - \frac{2(x-e^{-x})(1+e^{-x})}{2(1+e^{-x})^2 + 2e^{-x}(x-e^{-x})}$$

$A = x_n - e^{-x_n}$

$B = 1+e^{-x_n}$

$$= x_n - \frac{AB}{B^2 + Ae^{-x_n}}$$

$x_0 = 0.5$

$x_1 = 0.566$

$x_2 = 0.567$