Question 1 [2] Prove that $x2^x = 1$ for some $x \in (0,1)$.

Question 2 [2] Let $f(x) = \begin{cases} x^2 \sin(\frac{1}{x}), & if \ \text{x} \neq 0 \\ \\ 0, & if \ x = 0 \end{cases}$

Use the definition of the derivative to prove that f is differentiable at 0.

 Question 3 [3]

Let $f: (0,2) \to R$. Assume that that $\lim_{x \to 1} \ f(x) = \frac{1}{2}$. Prove that there exists δ >0 such that if $x \in (0,2)$ and $|x - 1| < \delta$ then $f(x) > 0$.

Question 4 [3]

Prove that $g(x) = \frac{1}{1+|x|}$ is uniformly continuous on R.