

Integral Calculus (M-106), S. 3

Exercise 1:

Evaluate the integral.

$$\begin{array}{lll} 1) \int_1^4 (x^2 - 4x - 3) dx & 2) \int_4^9 \frac{t-3}{\sqrt{t}} dt & 3) \int_1^0 s^2(\sqrt[3]{s} - \sqrt{s}) ds \\ 4) \int_{-3}^6 |x-4| dx & 5) \int_{-2}^{-1} (x - \frac{1}{x})^2 dx & 6) \int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} (x + \sin 5x) dx \end{array}$$

Exercise 2:

a) Find a number z that satisfies the conclusion of the mean value theorem for the given interval $\int_a^b f(x)dx$, and b) Find the average value of f on $[a, b]$:

$$1) \int_0^4 \frac{x}{\sqrt{x^2+9}} dx \quad 2) \int_{-2}^0 \sqrt[3]{x+1} dx$$

Exercise 3:

a) Find the derivatives without integrating:

$$\begin{array}{ll} 1) \frac{d}{dx} \int_0^3 \sqrt{x^2+16} dx & 2) \frac{d}{dx} (x \int_0^3 \sqrt{x^2+16} dx) \\ 3) \frac{d}{dx} \int_0^x \frac{1}{\sqrt{1-t^2}} dt, |x| < 1 & 4) \frac{d}{dx} \int_{-x}^x \frac{1}{t+1} dt \end{array}$$

b) Find the derivatives

$$1) \frac{d}{dx} \int_2^{x^4} \frac{t}{\sqrt{t^3+2}} dt$$

$$2) \frac{d}{dx} \int_{\frac{1}{x}}^{\sqrt{x}} \sqrt{t^4 + t^2 + 4} dt$$

$$1) \frac{d}{dx} (\sin(x) \int_2^{x^4} \frac{t}{\sqrt{t^3+2}} dt) \quad 2) \frac{d}{dx} (\tan(x) \int_{\frac{1}{x}}^{\sqrt{x}} \sqrt{t^4 + t^2 + 4} dt)$$

Exercise 4:

Use Simpson's and Trapezoidal rule, with $n = 4$, to approximate the average of f on the given interval.

1) $f(x) = \frac{1}{x^4+1}$; $[0, 4]$

2) $f(x) = \sqrt{\cos x}$; $[-1, 1]$.