

Integral Calculus (M-106), S. 4

Exercise 1:

Find $f'(x)$ if $f(x)$ is the given expression.

- | | | |
|------------------------|-----------------------------------|--|
| 1) $x \ln x$ | 2) $\ln(\ln x)$ | 3) $\frac{1}{\ln x} + \ln \frac{1}{x}$ |
| 4) $\ln 5x^2 - 1 ^3$ | 5) $\ln(\sqrt{4x - 5}(3x + 8)^2)$ | 6) $\ln \frac{\sqrt{x^2 + 1}}{(9x - 4)^2}$ |
| 7) $\ln(\ln(\sec 2x))$ | 8) $\cos(\ln 2x)$ | 9) $\ln \csc x - \cot x $ |

Exercise 2:

a) Use implicit differentiation to find $y'(x)$.

1) $x \ln y(x) - y(x) \ln x = 0$ 2) $xy^3(x) + x^3 \ln y(x) = 5x + 3$

b) Use logarithmic differentiation to find $\frac{dy(x)}{dx}$.

1) $y(x) = \sqrt{4x + 7}(x - 5)^3$ 2) $y(x) = (x + 1)^2(x + 2)^3(x + 3)^4$ 3) $y(x) = \frac{(x^2 + 3)^5}{\sqrt{3x + 1}}$

Exercise 3:

a) Find $f'(x)$ if $f(x)$ is the given expression.

- | | | |
|----------------------|--|-------------------------|
| 1) $e^{\sqrt{x+1}}$ | 2) $e^{\sqrt{x}} + \sqrt{e^x}$ | 3) $e^{\sin 5x}$ |
| 4) $e^{-3x} \cos 3x$ | 5) $\frac{e^x - e^{-x}}{e^x + e^{-x}}$ | 6) $e^{-x} \tan^2 x$ |
| 7) $xe^{\cot x}$ | 8) $\ln(\csc e^{3x})$ | 9) $\frac{x}{e^{-x^2}}$ |

b) Use implicit differentiation to find $y'(x)$.

1) $e^x \cos y = xe^y$ 2) $xe^y + 2x - \ln(y + 1) = 3$

Exercise 4:

Evaluate the integral.

1)

$$a) \int \frac{1}{4-5x} dx \quad b) \int_{-1}^0 \frac{1}{4-5x} dx$$

2)

$$a) \int \frac{3x}{x^2+4} dx \quad b) \int_1^2 \frac{3x}{x^2+4} dx$$

3)

$$a) \int x^2 e^{3x^3} dx \quad b) \int_1^2 x^2 e^{3x^3} dx$$

4)

$$a) \int \cot\left(\frac{1}{3}x\right) dx \quad b) \int_{\frac{3\pi}{2}}^{\frac{3\pi}{4}} \cot\left(\frac{1}{3}x\right) dx$$

5)

$$a) \int (\csc x - 1)^2 dx \quad b) \int e^{2x} \sec e^{2x} dx \quad c) \int (\tan 3x + \sec 3x) dx \quad d) \int \frac{\cot \sqrt[3]{x}}{\sqrt[3]{x^2}} dx$$

Exercise 5:

Solve the differential equation subject to the given conditions.

1)

$$y'(x) = 4e^{2x} + 3e^{-2x}; \quad y = 4 \text{ if } x = 0$$

2)

$$y'(x) = 3e^{4x} - 8e^{-2x}; y = -2 \text{ if } x = 0$$

Exercise 6:

Find $f'(x)$ if $f(x)$ is given expression.

$$\begin{array}{lll} 1) 7^x & 2) 5^{-x} & 3) 8^{x^2+1} \\ 4) (10^x + 10^{-x})^{10} & 5) \log_5 \left| \frac{1-x^2}{2-5x^3} \right| & 6) x^{\tan x} \\ 7) (\cos 2x)^{2x} & 8) (x\pi)^x & 9) (\log_2 |x| \pi)^x \end{array}$$

Exercise 7:

Evaluate the integral.

1)

$$\begin{array}{ll} a) \int 7^x dx & aa) \int_{-2}^1 7^x dx \\ b) \int 2^{3x-1} dx & bb) \int_{-1}^1 2^{3x-1} dx \\ c) \int 6^{-6x} dx & cc) \int_1^2 6^{-6x} dx \end{array}$$

2)

$$\begin{array}{ll} a) \int x 3^{-x^2} dx & b) \int \frac{(2^x + 1)^2}{2^x} dx \\ c) \int \frac{2^x}{2^x + 1} dx & d) \int 3^{\cos x} \sin x dx \\ e) \int e^\pi dx & f) \int \frac{10^{\sqrt{x}}}{\sqrt{x}} dx \end{array}$$