

Second Mid Term

Math 550

Exercise 1. Let $f : [-\frac{\pi}{2}, \pi] \rightarrow \mathbb{R}$ defined by

$$f(x) = \frac{x}{2} - \sin(x) + \frac{\pi}{6} - \frac{\sqrt{3}}{2}$$

1. Show that there exist two solutions $l^- < 0$ and $l^+ > 0$ of the equation $f(x) = 0$, $x \in [-\frac{\pi}{2}, \pi]$.
2. Is it possible to approximate these solutions by the Bisection method? Why? If it is possible, determine the number of iterations necessary to approximate the solution(s) with accuracy 10^{-10} after using an appropriate interval.
3. Give the Newton's method for the function f .
4. Deduce the order of the convergence of the method to approximate the two zeros.

Exercise 2.

Use three steps of the Newton's method to approximate the roots of the following system

$$\begin{cases} y + x^2 - x - 1 = 0, \\ x^2 - 2y^2 - y = 0 \end{cases}$$

by taking initial guess $x_0 = y_0 = 0$.

Exercise 3. Let $P(x) = x^3 + 6x^2 - 16$.

1. Find the Sturm sequence of P .
2. Determine the number of roots of P in the interval $[-6, 2]$.
3. Determine the number of positive roots of P .