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1/ Lecture 1 LP Simplex Method p.16

* Consider the toy-shop example.

Revise
- Toy-shop Example 8 LP. p.1 → p.5
- p.10 — p.12 Graph. Method of LP

$$\max\ 3x_1 + 2x_2$$
$$x_1 + x_2 \le 80$$
$$2x_1 + x_2 \le 100$$
$$x_1 \le 40$$
$$x_1, x_2 \ge 0$$

⇒ Canonical form by adding slack variables المتغيرات الكلية x_3, x_4 and x_5.

$$\max\ 3x_1 + 2x_2$$
$$x_1 + x_2 + x_3 = 80$$
$$2x_1 + x_2 + x_4 = 100$$
$$x_1 + x_5 = 40$$

1 * Starting feasible solution

Let $x_1 = x_2 = 0$

⇒ $x_3 = 80$, $x_4 = 100$, $x_5 = 40$ BFS basic feasible solution

In a pb with n variables and m constraints, a solution where at least $(n-m)$ variables are zero is a basic solution.

⇒ Basic solutions are precisely the corner points of the feasible region

Here x_1, x_2 are NBV non-basic variables, where x_3, x_4 and x_5 are BV basic variables (basis).

- Dictionary

A dictionary lists values of basic variables as a function of non-basic variables.

Now, $\max\ 3x_1 + 2x_2$

$$x_1 + x_2 + x_3 = 80$$
$$2x_1 + x_2 + x_4 = 100$$
$$x_1 + x_5 = 40$$
$$x_1, x_2, x_3, x_4, x_5 \ge 0$$

⇒

BV

$$x_3 = 80 - x_1 - x_2$$
$$x_4 = 100 - 2x_1 - x_2$$
$$x_5 = 40 - x_1$$
$$z = 0 + 3x_1 + 2x_2$$

Dictionary

- Note that: x_1, x_2 are indep. (non-basic) variables; x_3, x_4 and x_5 are dependent (basic) variables, i.e. $\{x_3, x_4, x_5\}$ is a basis.

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So, the initial basic feasible solution for our L.P is
$x_1 = x_2 = 0,\ x_3 = 80,\ x_4 = 100,\ x_5 = 40$
with value $z = 0$

[2] * Improving the solution
First by increasing x_1 (Consider x_1 as incoming Variable).
Note that x_1 has a +ve Coefficient in the z - eqn.

Ratio test

$x_3 : \frac{80}{1} = 80$, $x_4 : \frac{100}{2} = 50$, $x_5 : \frac{40}{1} = 40$ ✓

معامل x_1 معامل x_1 معامل x_1

Minimum achieved with x_5 ⇒ outgoing Variable
* Express x_1 from the eqn for x_5

$$x_5 = 40 - x_1 \Rightarrow x_1 = 40 - x_5$$

incoming Variable

* New Dictionary

$$\left.\begin{aligned} x_1 &= 40 - x_5 \\ x_3 &= 80 - (40 - x_5) - x_2 \\ x_4 &= 100 - 2(40 - x_5) - x_2 \\ z &= 0 + 3(40 - x_5) + 2x_2 \end{aligned}\right\} \Rightarrow \boxed{\begin{aligned} x_1 &= 40 \qquad\qquad - x_5 \\ x_3 &= 40 - x_2 \quad + x_5 \\ x_4 &= 20 - x_2 \quad + 2x_5 \\ z &= 120 + 2x_2 - 3x_5 \end{aligned}}$$

BV

• Note that: $\{x_1, x_3, x_4\}$ is a basis

So, the basic feasible solution for the L.P is
$x_1 = 40,\ x_2 = 0,\ x_3 = 40,\ x_4 = 20,\ x_5 = 0$
and $z = 120$

We pick the variable which having +ve Coefficient in the eqn of z.

[3] * Improving the solution
By increasing x_2 (Consider x_2 as incoming Variable)
Note that x_2 has a +ve Coefficient in the z - eqn.

Ratio test x_1 : doesn't contain x_2 (no constraint) , $x_3 : \frac{40}{1} = 40$, $x_4 : \frac{20}{1} = 20$ ✓

معامل x_2 معامل x_2

Minimum achieved for x_4 → outgoing variable
$$\Rightarrow x_2 = 20 - x_4 + 2x_5$$

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* New Dictionary

$$\begin{aligned} x_1 &= 40 \quad - x_5 \\ x_2 &= 20 \quad - x_4 + 2x_5 \\ x_3 &= 40 - (20 - x_4 + 2x_5) + x_5 \end{aligned} \Rightarrow \boxed{\begin{aligned} x_1 &= 40 \quad - x_5 \\ x_2 &= 20 \quad - x_4 + 2x_5 \\ x_3 &= 20 \quad + x_4 - x_5 \\ \hline z &= 160 - 2x_4 + x_5 \end{aligned}}$$

$$z = 120 + 2(20 - x_4 + 2x_5) - 3x_5$$

Improving the solution

4 * Consider x_5 be incoming variable (x_5 has a +ve coeff. in the eq$\underline{n}$ of z)

Ratio test $x_1 : \frac{40}{1} = 40$, x_2: Contains a +ve coeff. for x_5 (no Constraint), $x_3 : \frac{20}{1} = 20$ ✓

Minimum achieved for x_3 → outgoing variable

$$\Rightarrow x_5 = 20 - x_3 + x_4$$

* New Dictionary

$$\begin{aligned} x_1 &= 40 - (20 - x_3 + x_4) \\ x_2 &= 20 - x_4 + 2(20 - x_3 + x_4) \\ x_5 &= 20 - x_3 + x_4 \end{aligned} \Rightarrow \boxed{\begin{aligned} x_1 &= 20 + x_3 - x_4 \\ x_2 &= 60 - 2x_3 + x_4 \\ x_5 &= 20 - x_3 + x_4 \\ \hline z &= 180 - x_3 - x_4 \end{aligned}}$$

$$z = 160 - 2x_4 + (20 - x_3 + x_4)$$

If no variable appears with a positive coefficient in the eq$\underline{n}$ for z then stop and consider the current solution is the optimal solution.

∴ The optimal sol$\underline{n}$ is

$x_1 = 20$, $x_2 = 60$, $x_3 = 0$, $x_4 = 0$, $x_5 = 20$

where $z = 180$ (max. profit).

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