PHYSICS 201
3rd HOMEWORK
Dr. V. Lempesis

Hand in: Tuesday 26th of November 2013

Student Name : ____

Student ID: ______________________________

1. Simplify $(\mathbf{u}+\mathbf{v})\times(\mathbf{u}-\mathbf{v})$.

Solution:

$$(\mathbf{u}+\mathbf{v})\times(\mathbf{u}-\mathbf{v}) = \underbrace{\mathbf{u}\times\mathbf{u}}_{=0} - \mathbf{u}\times\mathbf{v} + \mathbf{v}\times\mathbf{u} - \underbrace{\mathbf{v}\times\mathbf{v}}_{=0} = -\mathbf{u}\times\mathbf{v} + \mathbf{v}\times\mathbf{u}$$
$$= \mathbf{v}\times\mathbf{u} + \mathbf{v}\times\mathbf{u} = 2(\mathbf{v}\times\mathbf{u})$$

2. Verify Cauchy-Schwartz inequality in the following case:
$\mathbf{u}$ = (-3, 1, 0), $\mathbf{v}$ = (2, -1, 3)

Solution:

$$\|\mathbf{u}\| = \sqrt{(-3)^2 + 1^2 + 0^2} = \sqrt{10}, \qquad \|\mathbf{v}\| = \sqrt{(2)^2 + (-1)^2 + 3^2} = \sqrt{14}$$

$$\mathbf{u}\cdot\mathbf{v} = (-3)\cdot 2 + 1\cdot(-1) + 0\cdot 3 = -6 - 1 = -7$$
$$|\mathbf{u}\cdot\mathbf{v}| = 7$$

$$|\mathbf{u}\cdot\mathbf{v}| = 7$$
$$\|\mathbf{u}\|\cdot\|\mathbf{v}\| = \sqrt{10}\sqrt{14} = \sqrt{140}$$
$$|\mathbf{u}\cdot\mathbf{v}| < \|\mathbf{u}\|\cdot\|\mathbf{v}\|$$

3. Find a unit vector in the opposite direction of the vector $\mathbf{v}$ = (-12, -5).

Solution: *A vector in the opposite direction is the* -$\mathbf{v}$ = (12, 5). *A unit vector in the direction of this vector is given by:*

$$\mathbf{u}_{-\mathbf{v}} = -\mathbf{v}/\|-\mathbf{v}\| = (12,\ 5)/\sqrt{12^2+5^2} =$$
$$(12,\ 5)/\sqrt{169} = (12,\ 5)/13 = \left(\frac{12}{13},\ \frac{5}{13}\right)$$

4. Prove that for two vectors $\mathbf{v}=\left(v_1,\ v_2,\ ...,v_N\right)$ and $\mathbf{w}=\left(w_1,\ w_2,\ ...,w_N\right)$ we have: $\mathbf{v}+\mathbf{w}=\mathbf{w}+\mathbf{v}$.

Solution:

$$\mathbf{v}+\mathbf{w}=\left(v_1,\ v_2,\ ...,v_N\right)+\left(w_1,\ w_2,\ ...,w_N\right)=\left(v_1+w_1,\ v_2+w_2,\ ...,v_N+w_N\right)=$$
$$\left(w_1+v_1,\ w_2+v_2,\ ...,w_N+v_N\right)=\mathbf{w}+\mathbf{v}$$

5. Which of the following vectors of R^6 is parallel to vector $\mathbf{v}$ = (-2, 1, 0, 3, 5, 1):

a) (0, 0, 0, 0, 0, 0) b) (0, 1, 2, 3, 10, 1) c) (-4, 2, 0, 6, 10, 2)

Solution: *Correct answer is c) since* (-4, 2, 0, 6, 10, 2) = 2(-2, 1, 0, 3, 5, 1)

6. Calculate the product $\mathbf{u}\cdot\left(\mathbf{v}\times\mathbf{w}\right)$ for the vectors:

$$\mathbf{u}=3\mathbf{i}-2\mathbf{j}-5\mathbf{k},\ \ \mathbf{v}=\mathbf{i}+4\mathbf{j}-4\mathbf{k},\ \ \mathbf{w}=3\mathbf{j}+2\mathbf{k}$$

$$\mathbf{v}\times\mathbf{w}=\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ 1 & 4 & -4\\ 0 & 3 & 2\end{vmatrix}=\mathbf{i}\begin{vmatrix}4 & -4\\ 3 & 2\end{vmatrix}-\mathbf{j}\begin{vmatrix}1 & -4\\ 0 & 2\end{vmatrix}+\mathbf{k}\begin{vmatrix}1 & 4\\ 0 & 3\end{vmatrix}=$$
$$\mathbf{i}\left(8+12\right)-\mathbf{j}\left(2-0\right)+\mathbf{k}\left(3-0\right)=20\mathbf{i}-2\mathbf{j}+3\mathbf{k}$$

$$\mathbf{u}\cdot\left(\mathbf{v}\times\mathbf{w}\right)=3\cdot 20+(-2)\cdot\left(-2\right)+\left(-5\right)\cdot 3=$$
$$60+4-15=49$$