PHYSICS 201
Solutions 2nd HOMEWORK
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Hand in: Tuesday 29th of October 2013

1. Use the method of augmented matrix to solve the following system:

$$\begin{aligned} 5x + 11y - 21z &= -22 \\ x + 2y - 4z &= -4 \\ 3x - 2y + 3z &= 11 . \end{aligned}$$

Solution:

Construct the augmented matrix of the system

$$\left(\begin{array}{ccc|c} 5 & 11 & -21 & -22 \\ 1 & 2 & -4 & -4 \\ 3 & -2 & 3 & 11 \end{array}\right)$$

and we interchange the second with the first line (or row). Then we do the following steps

$$\left(\begin{array}{ccc|c} 1 & 2 & -4 & -4 \\ 5 & 11 & -21 & -22 \\ 3 & -2 & 3 & 11 \end{array}\right)\begin{matrix} \\ (R_2 - 5R_1) \\ (R_3 - 3R_1) \end{matrix} \sim \left(\begin{array}{ccc|c} 1 & 2 & -4 & -4 \\ 0 & 1 & -1 & -2 \\ 0 & -8 & 15 & 23 \end{array}\right)\begin{matrix} \\ \\ (R_3 + 8R_2) \end{matrix}$$

$$\sim \left(\begin{array}{ccc|c} 1 & 2 & -4 & -4 \\ 0 & 1 & -1 & -2 \\ 0 & 0 & 7 & 7 \end{array}\right)\left(\frac{1}{7}R_3\right) \sim \left(\begin{array}{ccc|c} 1 & 2 & -4 & -4 \\ 0 & 1 & -1 & -2 \\ 0 & 0 & 1 & 1 \end{array}\right)\begin{matrix} (R_1 + 4R_3) \\ (R_2 + R_3) \\ \\ \end{matrix}$$

$$\sim \left(\begin{array}{ccc|c} 1 & 2 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \end{array}\right)\begin{matrix} (R_1 - 2R_2) \\ \\ \\ \end{matrix} \sim \left(\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \end{array}\right)$$

which gives the solution

$x = 2$
$y = -1$
$z = 1$

2. Solve the following system

$$3x + 2y = 7$$
$$-4x + 5y = -40$$

Solution:

$$D = \begin{vmatrix} 3 & 2 \\ -4 & 5 \end{vmatrix} = 15 - (-4) \cdot 2 = 15 + 8 = 23$$

$$D_x = \begin{vmatrix} 7 & 2 \\ -40 & 5 \end{vmatrix} = 7 \cdot 5 - (-40) \cdot 2 = 35 + 80 = 115$$

$$D_y = \begin{vmatrix} 3 & 7 \\ -4 & -40 \end{vmatrix} = 3 \cdot (-40) - 7 \cdot \left(-4\right) = -120 + 28 = -92$$

$$x = \frac{D_x}{D} = \frac{115}{23} = 5, \quad y = \frac{D_y}{D} = \frac{-92}{23} = -4$$

3. Show that the numbers a, b are roots of the equation

$$\begin{vmatrix} 1 & x & x^2 \\ 1 & a & a^2 \\ 1 & b & b^2 \end{vmatrix} = 0\,.$$

Solution:

$$\begin{vmatrix} 1 & x & x^2 \\ 1 & a & a^2 \\ 1 & b & b^2 \end{vmatrix} = 0 \Rightarrow 1 \cdot \begin{vmatrix} a & a^2 \\ b & b^2 \end{vmatrix} - 1 \cdot \begin{vmatrix} x & x^2 \\ b & b^2 \end{vmatrix} + 1 \cdot \begin{vmatrix} x & x^2 \\ a & a^2 \end{vmatrix} = 0 \Rightarrow$$
$$ab^2 - a^2 b - xb^2 + x^2 b + xa^2 - x^2 a = 0$$

Thus

$$ab^2 - a^2 b - xb^2 + x^2 b + xa^2 - x^2 a = 0 \Rightarrow x^2(b-a) - x(b^2 - a^2) + ab(b-a) = 0 \Rightarrow$$
$$x^2(b-a) - x(b-a)(b+a) + ab(b-a) = 0 \Rightarrow$$
$$(b-a)\left[x^2 - x(b+a) + ab\right] = 0 \Rightarrow x^2 - x(b+a) + ab = 0$$

The last equation has the following roots (assuming $b > a$)

$$x_1 = \frac{(b+a)+\sqrt{(b+a)^2-4ab}}{2} = \frac{(b+a)+\sqrt{(b-a)^2}}{2} = b$$

$$x_2 = \frac{(b+a)-\sqrt{(b-a)^2-4ab}}{2} = \frac{(b+a)-\sqrt{(b-a)^2}}{2} = a$$

4. If the following system has a unique solution then calculate *a*.

$$\begin{aligned} x+y+az&=6\\ 2x+3y+4z&=0\\ 3x+4y+5z&=1 \end{aligned}$$

$$D=\begin{vmatrix}1&1&a\\2&3&4\\3&4&5\end{vmatrix}=1\cdot\begin{vmatrix}3&4\\4&5\end{vmatrix}-1\cdot\begin{vmatrix}2&4\\3&5\end{vmatrix}+a\cdot\begin{vmatrix}2&3\\3&4\end{vmatrix}=-1+2-a=1-a$$

$$D_x=\begin{vmatrix}6&1&a\\0&3&4\\1&4&5\end{vmatrix}=6\cdot\begin{vmatrix}3&4\\4&5\end{vmatrix}-0\cdot\begin{vmatrix}3&4\\4&5\end{vmatrix}+1\cdot\begin{vmatrix}1&a\\3&4\end{vmatrix}=-6+4-3a=-2-3a$$

$$D_y=\begin{vmatrix}1&6&a\\2&0&4\\3&1&5\end{vmatrix}=1\cdot\begin{vmatrix}0&4\\1&5\end{vmatrix}-6\cdot\begin{vmatrix}2&4\\3&5\end{vmatrix}+a\cdot\begin{vmatrix}2&0\\3&1\end{vmatrix}=-4+12+2a=2a+8$$

$$D_z=\begin{vmatrix}1&1&6\\2&3&0\\3&4&1\end{vmatrix}=1\cdot\begin{vmatrix}3&4\\4&5\end{vmatrix}-1\cdot\begin{vmatrix}2&4\\3&5\end{vmatrix}+6\cdot\begin{vmatrix}2&3\\3&4\end{vmatrix}=-1+2-6=-5$$

To have a unique solution then $D\neq 0 \Rightarrow 1-a\neq 0 \Rightarrow a\neq 1$