جامعة الملك سعود
King Saud University

# Summary of Math-106

(All material only in 12 pages)


Prepared by:

Lecturer: Fawaz bin Saud Al-Otaibi

Mathematics department-KSU


$$\int x^n \, dx$$

$$\int 1\,dx = x + C$$

$$\int x^r\,dx = \frac{x^{r+1}}{r+1} + C \quad ; r \neq -1$$

$$\int \cos x\,dx = \sin x + C$$

$$\int \sin x\,dx = -\cos x + C$$

$$\int \sec^2 x\,dx = \tan x + C$$

$$\int \csc^2 x\,dx = -\cot x + C$$

$$\int \sec x \tan x\,dx = \sec x + C$$

$$\int \csc x \cdot \cot x \cdot dx = -\csc x + C$$

$$\int \frac{d}{dx}(f(x))\,dx = f(x) + C$$

$$\frac{d}{dx}\left(\int f(x)\,dx\right) = f(x)$$

$$\int c\,f(x)\,dx = c\int f(x)\,dx$$

$$\int [f(x) \pm g(x)]\,dx = \int f(x)\,dx \pm \int g(x)\,dx$$

$$\sum_{k=1}^{n} c = cn$$

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$$

$$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{k=1}^{n} k^3 = \left[\frac{n(n+1)}{2}\right]^2$$

$$R_p = \sum_{k=1}^{n} f(w_k)\,\Delta x_k$$

$$\int_a^b f(x)\,dx = \lim_{n\to\infty} R_p$$

أو

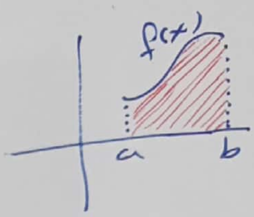
$$\|P\| \to 0$$

# $\boxed{\text{ch4}}$

$$\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$$

$$\int_a^a f(x)\,dx = 0$$

$$\text{Area} = \int_a^b f(x)\,dx$$




$$\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$$

$$f(x) \geq g(x) \text{ on } [a,b] \Rightarrow \int_a^b f(x)\,dx \geq \int_a^b g(x)\,dx$$

Average value of $f$ ($f_{av}$):

$$f_{av} = \frac{\int_a^b f(x)\,dx}{b-a}$$

Trapezoidal rule:

$$I \approx \frac{\Delta x}{2}[\text{sum}] \quad ; \quad \Delta x = \frac{b-a}{n}$$

Simpson rule:

$$I \approx \frac{\Delta x}{3}[\text{sum}] \quad ; \quad \Delta x = \frac{b-a}{n}$$

# Ch 6 (P1)

$\ln(pq) = \ln p + \ln(q)$

$\ln\left(\frac{p}{q}\right) = \ln p - \ln(q)$

$\ln p^r = r \ln p \quad (r \in \mathbb{Q})$

$\ln(e) = 1$

$\ln e^x = x$

$e^{\ln x} = x$

$e^p \cdot e^q = e^{p+q}$

$e^p / e^q = e^{p-q}$

$(e^p)^r = e^{pr}$

قواعد:

$a^x = e^{x \ln a} \quad , (a>0)$

$y = \log_a x \iff a^y = x$

~~متطابقات~~ قو

خصائص وقوانين

خصائص ln

$y = [f(x)]^{g(x)}$ الطريقة $\ln y = g(x) \ln f(x)$

$y' = ?$ $\quad \frac{y'}{y} = \dots \Rightarrow y' = ?$

$\ln(x)$

## المشتقات:

(u أي دالة مناسبة)

$(\ln u)' = \frac{u'}{u}$

$(e^u)' = e^u \cdot u'$

$(a^u)' = a^u \cdot u' \cdot \ln a \quad ; (a>0)$ عدد

$(\log_a u)' = \frac{u'}{u} \cdot \frac{1}{\ln a} \quad ; (a>0)$ عدد

## التكاملات:

(u أي دالة مناسبة)

$\int \frac{du}{u} = \ln|u| + c$

$\int e^u du = e^u + c$

$\int \sec u \, du = \ln|\sec u + \tan u| + c$

$\int \csc u \, du = \ln|\csc u - \cot u| + c$

$\int a^u \cdot du = \frac{a^u}{\ln a} + c \quad ; (a>0)$ عدد

مهم: $\int e^{mx} dx = \frac{e^{mx}}{m} + c$ $\quad m$ عدد

# Ch 6 (P2) Inverse and hyperbolic

## المشتقات:

(u أي دالة شاملة)

$(\sin^{-1}u)' = \dfrac{u'}{\sqrt{1-u^2}}$

$(\cos^{-1}u)' = \dfrac{-u'}{\sqrt{1-u^2}}$

$(\tan^{-1}u)' = \dfrac{u'}{1+u^2}$

$(\cot^{-1}u)' = \dfrac{-u'}{1+u^2}$

$(\sec^{-1}u)' = \dfrac{u'}{u\sqrt{u^2-1}}$

$(\csc^{-1}u)' = \dfrac{-u'}{u\sqrt{u^2-1}}$

$(\sinh u)' = \cosh(u) \cdot u'$

$(\cosh u)' = \sinh(u) \cdot u'$

$(\tanh u)' = \operatorname{sech}^2(u) \cdot u'$

$(\coth u)' = -\operatorname{csch}^2(u) \cdot u'$

$(\operatorname{sech} u)' = -\operatorname{sech} u \cdot \tanh u \cdot u'$

$(\operatorname{csch} u)' = -\operatorname{csch} u \cdot \coth u \cdot u'$

$(\sinh^{-1}u)' = \dfrac{u'}{\sqrt{u^2+1}}$

$(\cosh^{-1}u)' = \dfrac{u'}{\sqrt{u^2-1}}$

$(\tanh^{-1}u)' = \dfrac{u'}{1-u^2}$ ; $|u|<1$

$(\operatorname{sech}^{-1}u)' = \dfrac{-u'}{u\sqrt{1-u^2}}$

$(\operatorname{csch}^{-1}u)' = \dfrac{-u'}{u\sqrt{1+u^2}}$

## التكاملات:

$\int \dfrac{du}{\sqrt{a^2-u^2}} = \sin^{-1}\left(\dfrac{u}{a}\right) + c$

$\int \dfrac{du}{a^2+u^2} = \dfrac{1}{a}\tan^{-1}\left(\dfrac{u}{a}\right) + c$

$\int \dfrac{du}{u\sqrt{u^2-a^2}} = \dfrac{1}{a}\sec^{-1}\left(\dfrac{u}{a}\right) + c$

$\int \sinh u \, du = \cosh(u) + c$

$\int \cosh u \, du = \sinh(u) + c$

$\int \operatorname{sech}^2 u \, du = \tanh(u) + c$

$\int \operatorname{csch}^2(u) \, du = -\coth(u) + c$

$\int \operatorname{sech}(u) \cdot \tanh(u) \, du = -\operatorname{sech}(u) + c$

$\int \operatorname{csch}(u) \coth(u) \, du = -\operatorname{csch}(u) + c$

$\int \dfrac{du}{\sqrt{a^2+u^2}} = \sinh^{-1}\left(\dfrac{u}{a}\right) + c$

$\int \dfrac{du}{\sqrt{u^2-a^2}} = \cosh^{-1}\left(\dfrac{u}{a}\right) + c$

$\int \dfrac{du}{a^2-u^2} = \dfrac{1}{a}\tanh^{-1}\left(\dfrac{u}{a}\right) + c$ , $|u|<a$

$\int \dfrac{du}{u\sqrt{a^2-u^2}} = -\dfrac{1}{a}\operatorname{sech}^{-1}\left(\dfrac{u}{a}\right) + c$

$\int \dfrac{du}{u\sqrt{a^2+u^2}} = -\dfrac{1}{a}\operatorname{csch}^{-1}\left(\dfrac{u}{a}\right) + c$

**ملاحظة مهمة جدا: في التكاملات نفرض u بالدالة اللي تحت التربيع**

$x^5 = (x^{\frac{5}{2}})^2 \to u = x^{\frac{5}{2}}$

$x^3 = (x^{\frac{3}{2}})^2 \to u = x^{\frac{3}{2}}$

$x = (x^{\frac{1}{2}})^2 \to u = x^{\frac{1}{2}}$

$e^{2x} = (e^x)^2 \to u = e^x$

$x^4 = (x^2)^2 \to u = x^2$

$x^6 = (x^3)^2 \to u = x^3$

$\cos^4 x = (\cos^2 x)^2 \to u = \cos^2 x$

# $\boxed{\text{ch } 6}$ (P3)

علاقات أساسية:

$$\sinh(x) = \frac{e^{x} - e^{-x}}{2}$$

$$\cosh(x) = \frac{e^{x} + e^{-x}}{2}$$

$$\cosh^2(x) - \sinh^2(x) = 1$$

$$\tanh(x) = \frac{\sinh(x)}{\cosh(x)}$$

$$\coth(x) = \frac{\cosh(x)}{\sinh(x)}$$

$$\operatorname{sech}(x) = \frac{1}{\cosh(x)}$$

$$\operatorname{csch}(x) = \frac{1}{\sinh(x)}$$

$$1 - \tanh^2(x) = \operatorname{sech}^2(x)$$

$$\coth^2(x) - 1 = \operatorname{csch}^2(x)$$

$$\sinh^{-1}(x) = \ln\left(x + \sqrt{x^2+1}\right)$$

$$\cosh^{-1}(x) = \ln\left(x + \sqrt{x^2-1}\right) \; ; \; x \geq 1$$

$$\tanh^{-1}(x) = \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right) \; ; \; |x| < 1$$

$$\operatorname{sech}^{-1}(x) = \ln\left(\frac{1+\sqrt{1-x^2}}{x}\right) \; ; \; 0 < x \leq 1$$

## l'Hôpital's Rule:

فقط حالتين

$$\left\{ \frac{0}{0} \text{ or } \frac{\infty}{\infty} \right. \quad \lim_{x \to c} \frac{f(x)}{g(x)} = \lim_{x \to c} \frac{f'(x)}{g'(x)}$$

$x \to c$

$x \to \infty$

$x \to -\infty$

إذا ظهرت صيغة أخرى نحاول تغيير شكل الدالة لنحصل على احدى الصيغتين $\frac{\infty}{\infty}$ or $\frac{0}{0}$

صعبة جداً $[f(x)]^{g(x)}$

Put $y = [f(x)]^{g(x)} \to \ln y = g(x) \ln f(x)$

ثم نأخذ ln الطرفين

# طرق [Ch7] لحساب التكاملات

**By Parts:** $\int u\,dv = uv - \int v\,du$

في حالة ضرب دالتين لنسبة إحداها مشتقة للأخرى

$\boxed{u:}$ ~~LIMA~~ LIATE

لوغاريتمية ، جبرية ، عكسية (معكوس الدالة) ، مثلثية ، أسية

$dv$ ما تبقى (بعد اختيار $u$)

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$\sqrt{a^2-x^2} \longrightarrow x = a\sin(\theta)$

$\hookrightarrow dx = ---$

$\sqrt{a^2+x^2} \longrightarrow x = a\tan(\theta)$

$\hookrightarrow dx = ---$

$\sqrt{x^2-a^2} \longrightarrow x = a\sec(\theta)$

$\hookrightarrow dx = ---$

لتحويل الجواب من $\theta$ إلى $x$


نرسم

$\sin(2\theta) = 2\sin\theta\cos\theta$

$\int \sin^2(\theta)\,d\theta \to \sin^2\theta = \frac{1-\cos(2\theta)}{2}$

$\int \cos^2(\theta)\,d\theta \to \cos^2(\theta) = \frac{1+\cos(2\theta)}{2}$

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$\int \frac{\text{كثيرة حدود}}{\text{كثيرة حدود}}\,dx$

- درجة البسط أكبر أو تساوي درجة المقام ← قسمة مطولة ← خارج القسمة + الباقي / المقسوم عليه
- درجة البسط أقل ← Partial fractions (نحلل المقام ونكتب الثوابت ونوجدها ونكامل) ← تعتمد على شكل المقام فقط

$ax^2+bx+c$ (أحياناً نستخدم إكمال المربع)

- لازم الثنين موجودان

عدد

$\int \sin(mx)\,dx = -\frac{\cos(mx)}{m}$

$\int \cos(mx)\,dx = \frac{\sin(mx)}{m}$

بشكل أعم

$\int \sin(ax+b)\,dx = -\frac{\cos(ax+b)}{a}$

$\int \cos(ax+b) = \frac{\sin(ax+b)}{a}$

---

$\sqrt[n]{f(x)} \longrightarrow u^n = [f(x)]$

---

$\int \frac{\sin(x)}{\cos(x)+\tan(x)}\,dx$ (تحويلة نصف الزاوية)

$\to \sin(x) = \frac{2u}{1+u^2}$

$\cos(x) = \frac{1-u^2}{1+u^2}$

$dx = \frac{2\,du}{1+u^2}$

وفي الناتج النهائي نعوض $u = \tan\left(\frac{x}{2}\right)$ بـ

طريقة إكمال المربع: معامل $x$ بدون إشارة $\div 2 = \xrightarrow{\text{نضع}} \bigcirc^2 \xrightarrow{\text{نربع}} \Delta$ $+\Delta - \Delta$

لا بد أن يكون معامل $x^2 = 1$ قبل البدء بإكمال المربع

---

Improper integral $\lim_{t\to\pm\infty} \int dx$

$= l$ عدد حقيقي converges to $l$

عدا ذلك ($\pm\infty$ غير موجود) diverges ch 5 :

# $\boxed{\text{Area}}$ :

١- نرسم المنطقة

٢- نختار إما شريحة رأسية : **dx**

أو شريحة أفقية : **dy**

٣- $\text{Area} = \int (\ -\ )\,dx$ أو $\int (\ -\ )\,dy$

طرفي الشريحة

طرفي الشريحة

---

لا بد من تسمية طرفي الشريحة

$(x, y_1)$ , $(x_1, y)$

$(x, y_2)$ $(x_2, y)$

# $\boxed{\text{Volume}}$

بإحدى طريقتين (بعد رسم المنطقة)

**Disks (Washer):** نأخذ شريحة ⊥ محور الدوران في المنطقة

$$V = \int \pi (\text{radius})^2 \cdot \underbrace{\text{thickness}}_{dx \text{ أو } dy}$$

السمك

أو $= \int \pi (y_1^2 - y_2^2)\,dx$

$= \int \pi (x_1^2 - x_2^2)\,dy$

**Cylindrical shells:** نأخذ شريحة // محور الدوران في المنطقة

$$V = \int 2\pi (\text{radius}) \cdot \text{altitude} \cdot \underbrace{\text{thickness}}_{dx \text{ أو } dy}$$

الارتفاع

---

# $\boxed{\text{Arc length}}$ :

$$L = \int_a^b \sqrt{1 + [f'(x)]^2}\,dx \quad \text{or} \quad L = \int_c^d \sqrt{1 + [g'(y)]^2}\,dy$$

---

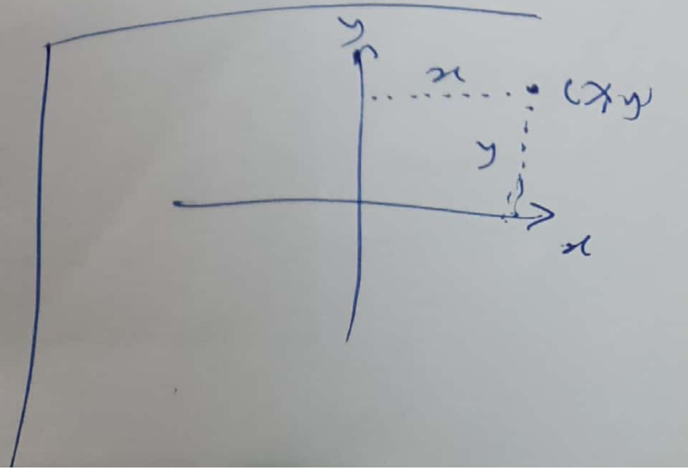
# $\boxed{\text{Surface area}}$ :

$$S = \int_a^b 2\pi f(x) \sqrt{1 + [f'(x)]^2}\,dx \quad (\text{around } x\text{-axis})$$

or $\boxed{f(x) = y}$

$$S = \int_c^d 2\pi g(y) \sqrt{1 + [g'(y)]^2}\,dy \quad (\text{around } y\text{-axis})$$

$\boxed{g(y) = x}$




ε

# ch 9 :

$x = f(t)$ , $y = g(t)$

Slope $\frac{dy}{dx}$ at p is:

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} \quad \text{at } t = ?$$

Length of a curve C:

$C: x = f(t), y = g(t)$

$$L = \int_a^b \sqrt{[x'(t)]^2 + [y'(t)]^2}\, dt$$

Surface area:

$$S = \int_a^b 2\pi |y(t)| \sqrt{[x'(t)]^2 + [y'(t)]^2}\, dt \quad \text{(about x-axis)}$$

~~$S = \cancel{2\pi}\, |x(t)|$~~ (about y-axis)

قانونان

## Polar Coordinates: $(r, \theta)$

$x = r\cos\theta$ , $y = r\sin\theta$

$x^2 + y^2 = r^2$ , $\tan(\theta) = \frac{y}{x}$

Slope of tangent line: $m = \dfrac{\frac{dr}{d\theta}\sin\theta + r\cos\theta}{\frac{dr}{d\theta}\cos\theta - r\sin\theta}$

Area: $A = \int_\alpha^\beta \frac{1}{2} r^2 \, d\theta$ $\xrightarrow{\text{شكلين}}$ $\int_\alpha^\beta \frac{1}{2}[r_1^2 - r_2^2]\, d\theta$

Arc length: $L = \int_\alpha^\beta \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta$ $\cancel{\ \ }$

Surface area:
- about the Polar axis: $S = \int_\alpha^\beta 2\pi r \sin\theta \, ds$
- about the line $\theta = \frac{\pi}{2}$: $S = \int_\alpha^\beta 2\pi r\cos\theta . ds$

where $ds = \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta$ y-axis

رسمات المنحنيات القطبية : أنظر الصفحة التالية

: Polar graphs

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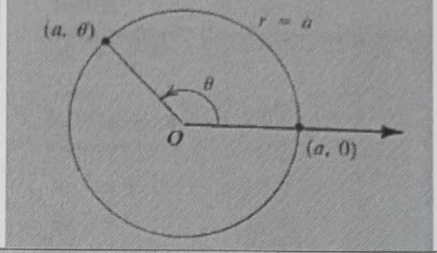
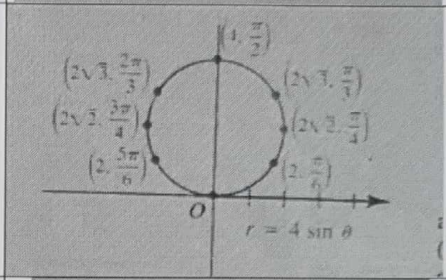
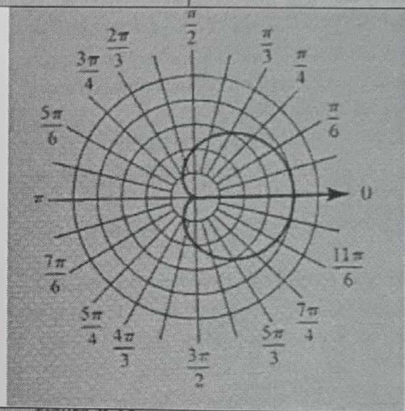
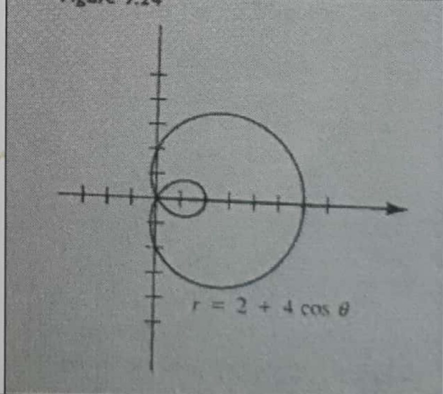
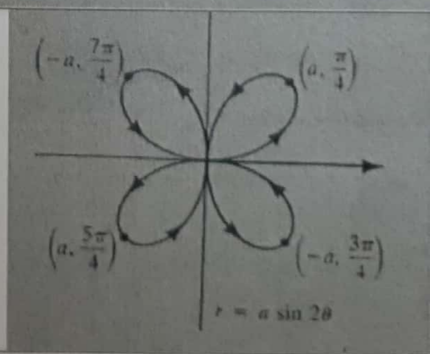
<tr><td>r=a</td><td>Figure 9.20<br><br></td></tr>
<tr><td>$r = 4\sin\theta.$</td><td><br></td></tr>
<tr><td>$r = 2 + 2\cos\theta$</td><td></td></tr>
<tr><td>r= $2 + 4\cos\theta.$</td><td><br></td></tr>
<tr><td>E ▪ 4 Sketch the graph of the polar equation $r = a\sin 2\theta$</td><td><br></td></tr>
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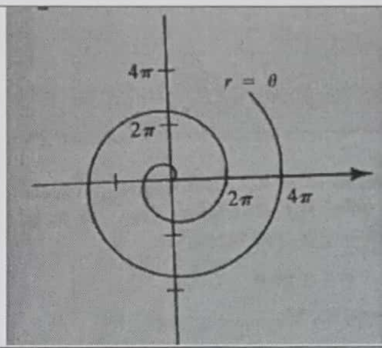
<tr><td>E ▪ 5 Sketch the graph of the polar equation $r = \theta$ for</td><td>
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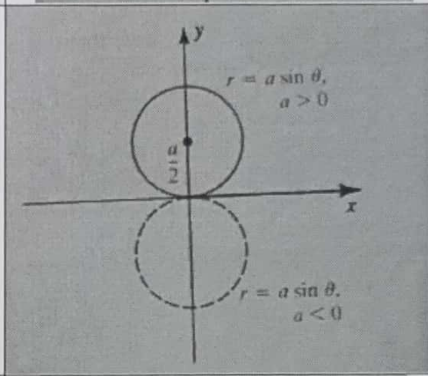
<tr><td></td><td>
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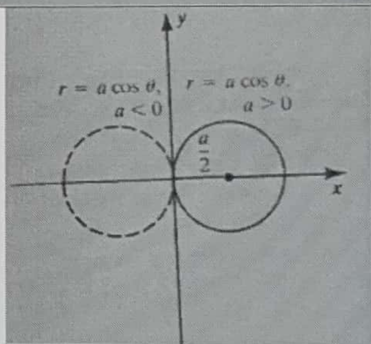
<tr><td></td><td>
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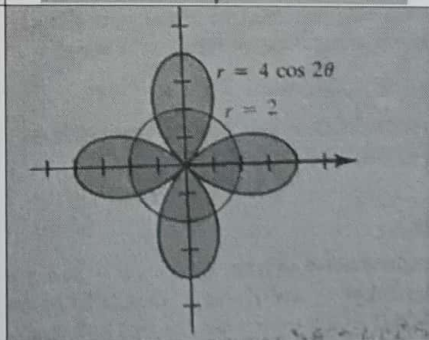
<tr><td></td><td>
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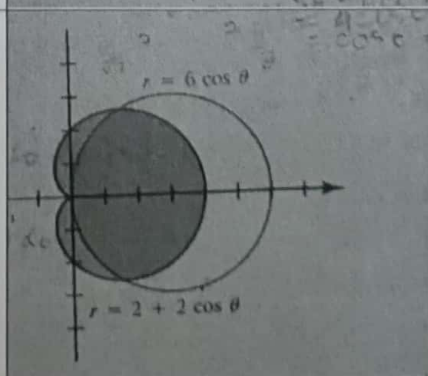
<tr><td></td><td>
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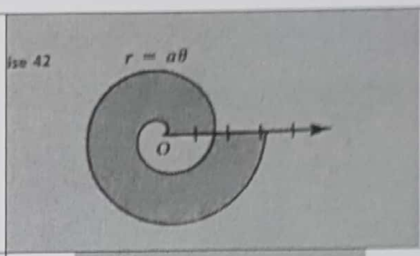
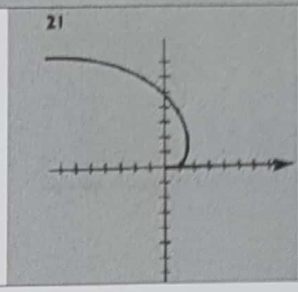
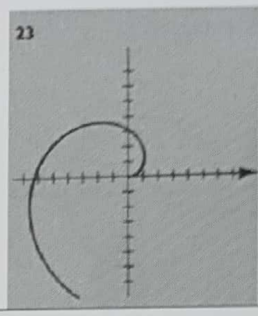
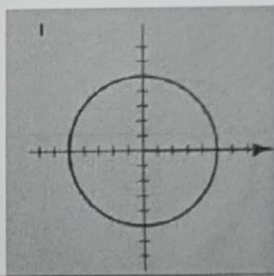
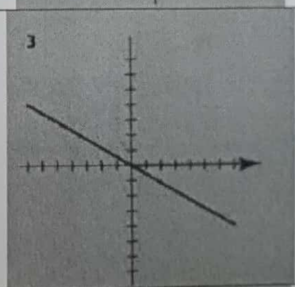
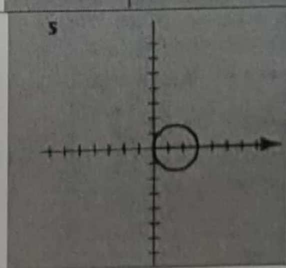
| |   |
| 21 $r = e^{\theta}$, $\theta \geq 0$ (logarithmic spiral) |   |
| 22 $r = 6\sin^2(\theta/2)$ |   |
| 1 $r = 5$ |   |
| 3 $\theta = -\pi/6$ |   |
| 5 $r = 3\cos\theta$ |   |

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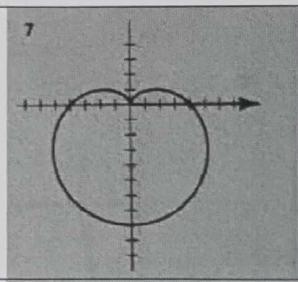
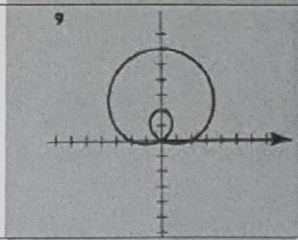
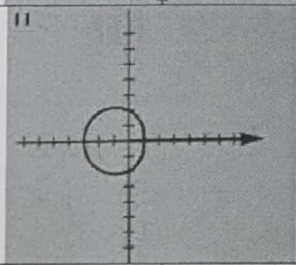
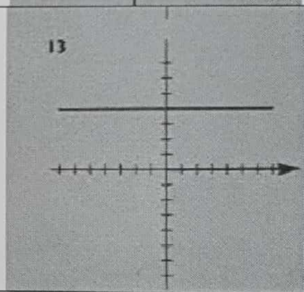
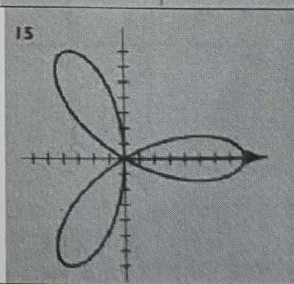
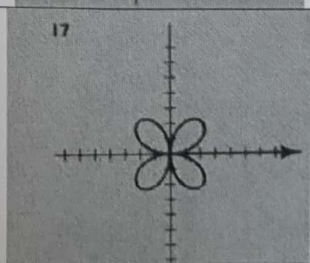
|---|---|
| 7 $r = 4 - 4\sin\theta$ |  |
| 9 $r = 2 + 4\sin\theta$ |  |
| 11 $r = 2 - \cos\theta$ |  |
| 13 $r = 4\csc\theta$ |  |
| 15 $r = 8\cos 3\theta$ |  |
| 17 $r = 3\sin 2\theta$ |  |

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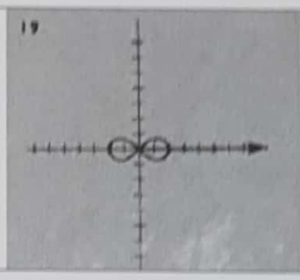
|---|---|
| 19 $r^2 = 4\cos 2\theta$ (lemniscate) |  |
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