Tutorial Session ②

* pb 1.3.2 p.24 Textbook

$$pr(X=x) = \binom{n}{x} p^x q^{n-x}$$

$X \sim Bin(10, 0.05)$

a) $pr(X=1) = \binom{10}{1}(0.05)^1(0.95)^9$

$= 0.31512$

b) $pr(X \leq 1) = pr(X=0) + pr(X=1)$

$= \binom{10}{0}(0.05)^0(0.95)^{10} + 0.31512$

$= (0.95)^{10} + 0.31512$

$\therefore pr(X \leq 1) \simeq 0.9139$

* pb 1.3.4 p.25 Textbook

$$pr(X=x) = \frac{e^{-\lambda}\lambda^x}{x!}$$

where, $X \sim poisson(2)$

a) $pr(X=2) = \frac{e^{-2}2^2}{2!} = 2e^{-2}$

$\simeq 0.2707$

b) $pr(X \leq 2) = pr(X=0) + pr(X=1) + pr(X=2)$

$= e^{-2}\left[\frac{2^0}{0!} + \frac{2^1}{1!} + \frac{2^2}{2!}\right]$

$= e^{-2}(1+2+2)$

$= 5e^{-2}$

$\therefore pr(X \leq 2) \simeq 0.6767$

لا يكتب في هذا الهامش

pb 1.4.1 p. 32 Textbook

$$T \sim \exp(2), \quad \lambda = 2$$

$\Rightarrow$ the pdf is $f_T(t) = \lambda e^{-\lambda t}$

and cdf is $F_T(t) = 1 - e^{-\lambda t}$

$\Rightarrow F_T(t) = 1 - e^{-2t}$

$= pr(T \le t)$

a) $pr(T > 1.5) = 1 - [1 - e^{-2(1.5)}]$

$= e^{-3}$

$\simeq 0.0498$

Another solution

OR Reliability fn (survival fn) is given by

$R(t) = pr(T > t)$

$R(t) = e^{-\lambda t} = e^{-2t}, \quad \lambda = 2$

$R(1.5) = e^{-2(1.5)} = e^{-3} \simeq 0.0498$

b) $pr(T = 1.5) = 0$

Note that:

For continuous r.v T, $pr(T = t) = 0 \quad \forall t \in \mathbb{R}$

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For exponential distn

$$E(X) = \frac{1}{\lambda} \quad \text{inch}$$

$$= \frac{2.54}{\lambda} \quad \text{Cm}$$

المتوسط للأطوال
Mean of lengths of Cotton fibers

, where 1 inch = 2.54 Cm

$$\Rightarrow X \sim \exp\left(\frac{\lambda}{1}\right) = \exp(\lambda) \text{ in mill system}$$

but, $X \sim \exp\left(\frac{\lambda}{2.54}\right)$ in metric system

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We have $X_K \sim \text{uniform}(0,1)$, $K = 1, 2, \dots, 12$

$$E(X_K) = \frac{1}{2}(a+b) = \frac{1}{2}(0+1) = \frac{1}{2},$$

$$Var(X_K) = \frac{1}{12}(b-a)^2 = \frac{1}{12}(1-0)^2 = \frac{1}{12}$$

for $Z = X_1 + X_2 + \dots + X_{12} - 6$,

Mean $E(Z) = \cancel{12}^{6}\left(\frac{1}{\cancel{2}}\right) - 6 = 0$

where $E(X_1) = E(X_2) = \dots = E(X_{12}) = \frac{1}{2}$, $E(6) = 6$

and $Var(Z) = \cancel{12}\left(\frac{1}{\cancel{12}}\right) - 0 = 1$

Variance

where $Var(X_1) = Var(X_2) = \dots = Var(X_{12}) = \frac{1}{12}$

, $Var(6) = 0$