Exercise (Tut. Session (3))

Q(1) The joint probability density function of the two random variables X and Y is $f(x,y)=8xy$, $0 \le x \le y \le 1$. Find $f_{Y|X}(y|\frac{1}{3})$

Answer

$$f_{Y|X}(y|x) = \frac{f_{X,Y}(x,y)}{f_X(x)}$$

$$f_{X,Y}(x,y) = 8xy\,,\ 0 \le x \le y \le 1$$

$$\because\ f_X(x) = \int_{-\infty}^{\infty} f(x,y)dy$$

$$= \int_{x}^{1} 8xydy$$

$$= 8x\left[\frac{y^2}{2}\right]_x^1$$

$$\therefore\ f_X(x) = 4x(1\text{-}x^2), \qquad 0 \le x \le 1$$

$$\therefore\ f_{Y|X}(y|x) = \frac{8xy}{4x(1\text{-}x^2)}$$

$$= \frac{2y}{1\text{-}x^2}, \qquad x \le y \le 1$$

$$\therefore\ f_{Y|X}(y|\tfrac{1}{3}) = \tfrac{9}{4}y, \qquad \tfrac{1}{3} \le y \le 1$$

Q(2) Given the joint probability mass functions of two random variables X and Y as in the following table.

Y / X	1	2	3
0	1/8	0	0
1	0	1/4	1/8
2	0	1/4	1/8
3	1/8	0	0

i) Find ρ (X,Y)

ii) Determine whether X and Y are two independent random variables or not? Justify your answer.

Answer

Y / X	1	2	3	$P_X(x)$
0	1/8	0	0	1/8
1	0	1/4	1/8	3/8
2	0	1/4	1/8	3/8
3	1/8	0	0	1/8
$P_Y(y)$	2/8	4/8	2/8	Sum=1

$E(X)=\frac{3}{2}$, $E(X^2)=3$, $Var(X)=\frac{3}{4}$

$E(Y)=2$, $E(Y^2)=\frac{9}{2}$, $Var(Y)=\frac{1}{2}$

$E(XY)=3$

$Cov(X,Y)=E(XY)-E(X)E(Y)=0$

$$\rho(X,Y)=\frac{Cov(X,Y)}{\sigma_X\sigma_Y}=0$$

$\Rightarrow$ X and Y are not correlated

$\because$ for example, $P(X=1,Y=1)=0$, but $P(X=1)P(Y=1)=\frac{3}{8}(\frac{2}{8})=\frac{3}{32}$

$\Rightarrow P(X=1,Y=1)\neq P(X=1)P(Y=1)$

$\therefore$ X and Y are not independent r.vs

Q(3) Pb 1.5.7 p.42

Given $V = (X_1, X_2, ..., X_n)$ is a multivariate random variable where $X_1, X_2, ..., X_n$ be independent random variables that are exponentially distributed with respective parameters $\lambda_1, \lambda_2, ..., \lambda_n$.

Identify the distribution of V such that min $V = \min\{X_1, X_2, ..., X_n\}$.

Answer:

Given $X_1 \sim \exp(\lambda_1), X_2 \sim \exp(\lambda_2), \dots, X_n \sim \exp(\lambda_n)$ are independent r,vs

To get distribution of V s.t min V=min $(X_1, X_2, X_3, \dots, X_n)$

Let min V=v, $v \in \mathbb{R}$

$\therefore$ $pr(V>v)=pr(X_1>v)pr(X_2>v) \dots pr(X_n>v)$

$$=e^{-\lambda_1 v} e^{-\lambda_2 v} \dots e^{-\lambda_n v}$$

$\therefore$ $pr(V>v)=e^{-(\sum_{i=1}^{n}\lambda_i)v}$

$\therefore$ $V \sim \exp(\sum_i \lambda_i)$

$\therefore$ V is exponentially distributed with parameter $\sum_{i=1}^{n}\lambda_i$