1

pb 4.1.3 p.173

$$P = \begin{array}{c} \\ 0 \\ 1 \\ 2 \end{array} \begin{array}{c} \begin{array}{ccc} 0 & 1 & 2 \end{array} \\ \begin{bmatrix} 0.1 & 0.1 & 0.8 \\ 0.2 & 0.2 & 0.6 \\ 0.3 & 0.3 & 0.4 \end{bmatrix} \\ \downarrow \quad \downarrow \quad \downarrow \\ \pi_0 \quad \pi_1 \quad \pi_2 \end{array}$$

what fraction of time in the long run does the process spend in state 1 ? π_1 ??

Ans:

$$\pi_j = \sum_{k=0}^{2} \pi_k P_{kj}$$

at $j=0 \Rightarrow \pi_0 = 0.1\pi_0 + 0.2\pi_1 + 0.3\pi_2$

$0.9\pi_0 = 0.2\pi_1 + 0.3\pi_2$ (×10)

$\therefore \boxed{9\pi_0 - 2\pi_1 - 3\pi_2 = 0}$ ①

at $j=1 \Rightarrow \pi_1 = 0.1\pi_0 + 0.2\pi_1 + 0.3\pi_2$

$0.1\pi_0 - 0.8\pi_1 + 0.3\pi_2 = 0$ (×10)

$\therefore \boxed{\pi_0 - 8\pi_1 + 3\pi_2 = 0}$ ②

$\therefore \boxed{\pi_0 + \pi_1 + \pi_2 = 1}$ ③

Solving ①, ② and ③

Add ①, ② $\Rightarrow 10\pi_0 - 10\pi_1 = 0 \Rightarrow \boxed{\pi_0 = \pi_1}$

$\therefore \boxed{\pi_1 = \pi_0}$

① + ③ × 3 $\Rightarrow 12\pi_0 + \pi_1 = 3$

$\therefore 12\pi_1 + \pi_1 = 3 \Rightarrow 13\pi_1 = 3$ $\boxed{\therefore \pi_1 = \frac{3}{13}}$ ##

Ch4: pb 4.2.5 p.189

$$P = \begin{array}{c} \\ A \\ B \\ C \end{array} \begin{array}{c} \begin{array}{ccc} A & B & C \end{array} \\ \begin{bmatrix} 0.6 & 0.2 & 0.2 \\ 0.1 & 0.7 & 0.2 \\ 0.1 & 0.1 & 0.8 \end{bmatrix} \\ \begin{array}{ccc} \downarrow & \downarrow & \downarrow \\ \pi_0 & \pi_1 & \pi_2 \end{array} \end{array}$$

$\pi_0 = \pi_A$??

Ans:

The limitting distn is $\pi = (\pi_0, \pi_1, \pi_2)$

$$\pi_0 = 0.6\pi_0 + 0.1\pi_1 + 0.1\pi_2$$

$$0.4\pi_0 - 0.1\pi_1 - 0.1\pi_2 = 0 \quad (\times 10)$$

$$\boxed{4\pi_0 - \pi_1 - \pi_2 = 0} \quad (1)$$

&

$$\pi_1 = 0.2\pi_0 + 0.7\pi_1 + 0.1\pi_2$$

$$0.2\pi_0 - 0.3\pi_1 + 0.1\pi_2 = 0 \quad (\times 10)$$

$$\boxed{2\pi_0 - 3\pi_1 + \pi_2 = 0} \quad (2)$$

&

$$\therefore \boxed{\pi_0 + \pi_1 + \pi_2 = 1} \quad (3)$$

Solving (1), (2) and (3) by using Cramer rule

$$\Delta = \begin{vmatrix} 4 & -1 & -1 \\ 2 & -3 & 1 \\ 1 & 1 & 1 \end{vmatrix} = -4 - 6 - 10 = -20$$

$$\Delta_0 = \begin{vmatrix} 0 & -1 & -1 \\ 0 & -3 & 1 \\ 1 & 1 & 1 \end{vmatrix} = -4 \quad , \quad \pi_0 = \frac{\Delta_0}{\Delta} = \frac{-4}{-20} = \frac{1}{5}$$

The fraction of time that the customer purchases brand A is $\pi_A = \pi_0 = \frac{1}{5} = 20\%$ #