PHYS 404
HANDOUT 2-*Legendre Functions*

1. Show that for $m \neq n$ we have $\int_{-1}^{1} P_m(x)P_n(x)dx = 0$.

2. Show that for we have $\int_{-1}^{1}\left[P_n(x)\right]^2 dx = 2/(2n+1)$.

3. If for $-1 < x < 1$ and $f(x) = \sum_{k=0}^{\infty} A_k P_k(x)$ show that:.

$$A_k = \frac{2k+1}{2}\int_{-1}^{1} P_k(x)f(x)dx.$$

4. Expand the function $f(x) = \begin{cases} 1 & 0 < x < 1 \\ 0 & -1 < x < 0 \end{cases}$ in a series of the form $f(x) = \sum_{k=0}^{\infty} A_k P_k(x)$.

5. Expand the function $f(x) = x^2$ in a series of the form $f(x) = \sum_{k=0}^{\infty} A_k P_k(x)$.

6. Expand the function $f(x) = x^4 - 3x^2 + x$ in a series of the form $f(x) = \sum_{k=0}^{\infty} A_k P_k(x)$.

7. Expand the function $f(x) = \begin{cases} 2x+1 & 0 < x \leq 1 \\ 0 & -1 \leq x < 0 \end{cases}$ in a series of the form $f(x) = \sum_{k=0}^{\infty} A_k P_k(x)$.

8. If $f(x) = \sum_{k=0}^{\infty} A_k P_k(x)$, then prove Parseval's identity:

$$\int_{-1}^{1}\left|f(x)\right|^2 dx = \sum_{k=0}^{\infty}\frac{2A_k^2}{2k+1}$$

9. Expand the Dirac delta function in a series of Legendre polynomials, using the interval $-1 \leq x \leq 1$.

10. Verify the Dirac delta function expansions:

$$\delta(1-x)=\sum_{n=0}^{\infty}\frac{2n+1}{2}P_n(x),\quad \delta(1+x)=\sum_{n=0}^{\infty}(-1)^n\frac{2n+1}{2}P_n(x).$$

11. Show that $\int_{-1}^{1} x^m P_n(x)\,dx=0$ if $m<n$.

12. Evaluate $\int_{-1}^{1} xP_n(x)\,dx$.