PHYS 404
HANDOUT 3- *Associated Legendre Functions*

1. Express the polynomials $P_1^1(x), P_2^1(x), P_2^2(x)$ and also find their forms if $x=\cos\theta$.

2. Verify that $P_3^2(x)$ is a solution of the associated Legendre equation.

3. Prove the recurrence relation:
$$\left(2n+1\right)\left(1-x^2\right)^{1/2} P_n^m(x) = P_{n+1}^{m+1}(x) - P_{n-1}^{m+1}(x)$$

4. Show the parity relation: $P_n^m(-x) = \left(-1\right)^{n+m} P_n^m(x)$.

5. Expand the function $v_0\left(1-x^2\right)$ in a series of the form $\sum_{n=0}^{\infty} A_n P_n^m(x)$ where m=2 and v_0 a constant.

6. Show that $\sin\theta P_n^{'}(x) = P_n^1(\cos\theta)$.

7. Use the generating function for the associated Legendre functions to show that:
$$P_{2n}^0(0) = 0, \quad P_{2n+1}^0(0) = \left(-1\right)^n \frac{\left(2n+1\right)!}{\left(2^n n!\right)^2}.$$