ELSEVIER

Journal of Petroleum Science and Engineering 19 (1998) 171–176

JOURNAL OF PETROLEUM SCIENCE & ENGINEERING

A new laminar-to-turbulent transition criterion for yield-pseudoplastic fluids

Saad El-Din M. Desouky *, Musaed N. Al-Awad

King Saud University, P.O. Box 800, Riyadh 11421, Saudi Arabia

Received 27 November 1996; revised 19 August 1997; accepted 19 August 1997

Abstract

A new laminar-to-turbulent transition criterion for predicting the onset of turbulence for yield-pseudoplastic fluids was developed. This criterion is defined as the ratio of the laminar shear stress to the viscous shear stress. The value of the ratio is equal to unity in the viscous sublayer and should be greater than unity in the turbulent zone. The rheological characteristics of the fluids studied were described by a power-law yield-pseudoplastic model. Two equations relating the Metzner and Reed parameters (n' and k') to those of yield-pseudoplastic model (τ_y, μ_p, and n) were derived. The developed criterion can also be used to determine the onset of turbulent flow for Bingham and yield-dilatant fluids. © 1998 Elsevier Science B.V.

Keywords: rheology; yield-pseudoplastic; Reynolds number; viscous sublayer; turbulent flow

1. Introduction

A yield-pseudoplastic fluid is a fluid for which a finite shearing stress is required to initiate motion and for which there is a non-linear relationship between the shearing stress in excess of the initiating stress and the resulting velocity gradient. Fluids that behave as yield-pseudoplastic fluids include thickened hydrocarbon greases, certain asphalts and bitumens, some emulsions and waxy crude oils, and drilling fluids. The rheological properties of these fluids can be described by a power-law yield pseudoplastic model (Govier and Aziz, 1982). Design of a pipeline for handling such fluids depends on the accuracy of the equations used for determining the type of flow (whether it is laminar or turbulent) and frictional pressure loss.

The onset of turbulence for time-independent fluids can be set by three criteria: the values of critical Reynolds number, stability parameter, and generalized stability parameter. These three criteria determine the end of the laminar flow region. In the present work, a criterion is developed to determine the beginning of the turbulent pipe flow. The viscous interaction coefficient equation proposed by Zandi and Rust (1965), Murthy and Zandi (1969), Desouky and El-Emam (1990), and Desouky (1991) was used to derive the new criterion equation. It is a linear relationship between the viscous shear stress and laminar shear stress. The viscous shear stress and laminar shear stress were mathematically expressed in terms of the friction factor, Reynolds number, and

* Corresponding author. Fax: +966-1-4674422.

PII S0920-4105(97)00044-2

fluid properties. The validity of the developed criterion was verified by comparing the values of the critical Reynolds number calculated from the obtained equation and those determined from Hanks correlation. The evaluation emphasizes the validity of the criterion for determining if the type of flow is laminar or turbulent.

2. Literature review

The critical Reynolds number was investigated by Metzner and Reed (1955). They plotted the friction factors of power-law fluids against the generalized Reynolds number and concluded that the power-law fluids do not describe the region of laminar flow when the friction factor is less than or equal to 0.008. Dodge and Metzner (1959), Shaver and Merrill (1959), and Metzner and Park (1964) determined the critical Reynolds number by plotting the friction factor of power-law fluids against generalized Reynolds number. The values of the critical Reynolds number determined from their charts agreed with those obtained from the chart developed by Metzner and Reed (1955). Thus the critical Reynolds number criterion was limited to power-law fluids.

Ryan and Johnson (1959) did a stability analysis of laminar flow based upon the assumption of small perturbations being applied to the equation of motion. Their stability parameter criterion was expressed by the following equation,

$$Z = -\frac{Ru\rho}{\tau_w}\frac{du}{dr}. \tag{1}$$

The parameter Z has a value of zero at the pipe wall and at the pipe center. Its maximum value occurs at a radial position r_c, at which the end of the laminar region can be determined from the velocity profile equation. Since the formula for the Z-parameter changes as the laminar velocity profile changes, the stability parameter is not a generalized criterion.

Several investigators (Hanks and Christiansen, 1962; Hanks, 1963; Hanks, 1968; Hanks and Ricks, 1975) have taken a different approach to the laminar stability problem and proposed a generalized stability parameter that is applicable to laminar flow in any geometry. They defined a parameter S as the ratio of the magnitude of the acceleration force term to the viscous force term in the fundamental equation of motion,

$$S = \frac{|\rho\nu\varepsilon|}{|\nabla\cdot\tau|g_c}. \tag{2}$$

The parameter S, like Ryan and Johnson's (1959) parameter Z, is zero at the pipe wall, at an intermediate maximum point, and at a line of symmetry in the velocity field. Eq. (2) was applied to Bingham plastics, and the following relations were developed:

$$\mathrm{Re}_{BC} = \frac{\mathrm{He}}{8X}\left(1 - \frac{4}{3}X + \frac{1}{3}X^4\right), \tag{3}$$

and

$$\frac{X}{(1-X)^3} = \frac{\mathrm{He}}{16800}. \tag{4}$$

3. Criterion development

The critical Reynolds number, stability parameter, and generalized stability parameter criteria were developed to determine the boundary of the laminar region. The present criterion determines the beginning of the turbulent region for fluids with a yield stress. The rheological behavior of these fluids was described by the following power-law yield-pseudoplastic model (Govier and Aziz, 1982):

$$\tau = \tau_y + K\left(\frac{du}{dr}\right)^n. \tag{5}$$

A linear relationship between laminar shear stress and viscous shear stress in turbulent flow is given by Zandi and Rust (1965), Murthy and Zandi (1969), Desouky and El-Emam (1990) and Desouky (1991) as:

$$C = \tau^v/\tau. \tag{6}$$

The parameter C is the viscous interaction coefficient which signifies the concept that as a result of chaotic motion in turbulence, the viscous shear stress τ^v is amplified and yields much higher values than the laminar shear stress τ. The value of C is equal to unity in the viscous sublayer and should be greater than unity in the turbulent zone. The laminar shear stress can be expressed in terms of the generalized Reynolds number (Re_{MR}) defined by Metzner and Reed (1955) as:

$$\tau = \frac{\rho v d}{Re_{MR}} \frac{du}{dr}. \quad (7)$$

The viscous shear stress was defined by Knudson and Katz (1958) as:

$$\tau^{v} = 0.2\left(1 - \frac{r}{R}\right)\rho V D\sqrt{\frac{f}{2}}\frac{du}{dr}. \quad (8)$$

Substituting Eqs. (7) and (8) into Eq. (6), the following equation is obtained.

$$C = 0.2 Re_{MR}\left(1 - \frac{r}{R}\right)\sqrt{\frac{f}{2}}, \quad (9)$$

where

$$Re_{MR} = \frac{D^{n'} V^{2-n'} \rho}{K 8^{n'-1}}. \quad (10)$$

The friction factor can be obtained from the equation of Torrance (1963) using the Clapp correlation (Clapp, 1961),

$$\frac{1}{\sqrt{f}} = \frac{4.53}{n}\left[\log(1-x) + \log Re_{PLC} f^{1-n/2}\right] + 0.45 - 2.75/n, \quad (11)$$

where

$$x = \tau_y / \tau_w, \quad (12)$$

and

$$Re_{PLC} = \frac{D^{n} \rho V^{2-n}}{K 8^{n-1}}. \quad (13)$$

The relative thickness of the viscous sublayer ($1 - r/R$) can be evaluated from Bingham fluid data, since the yield-pseudoplastic fluid data in the transition zone between the viscous sublayer and the turbulent zone are not available in the literature (Govier and Aziz, 1982; Desouky and El-Emam, 1990; Hemeida, 1993).

The following definitions are used to derive a mathematical expression for ($1 - r/R$):

$$f = \frac{16}{Re_B(1 - 4/3x + x^4/3)}$$

(Bingham friction factor) (14)

$$n = 1 \text{ (Bingham model)} \quad (15)$$

$$Re_{MR} = Re_B \text{ (Bingham Reynolds number)} \quad (16)$$

$$Re_B = \frac{2100}{(1-x)^3}\left(1 - \frac{4}{3}x + \frac{x^4}{3}\right)$$

(Critical Bingham Reynolds number) (17)

$$C = 1 \text{ (Value of } C \text{ at laminar sublayer)} \quad (18)$$

Substituting Eqs. (14)–(18) into Eq. (9), one gets,

$$\left(1 - \frac{r}{R}\right) = 0.0386(1-x)^{3/2}. \quad (19)$$

Substituting Eq. (19) into Eq. (9), the following equation is obtained:

$$C = 5.46 \times 10^{-3} Re_{MR}\sqrt{f}(1-x)^{3/2}. \quad (20)$$

Eq. (20) expresses the new laminar–turbulent-transition criterion by which the onset of turbulence can be predicted. If $C > 1.0$ the type of flow is turbulent; if $C \leq 1.0$ laminar flow is present.

3.1. Relationship between (n', K') and (τ_y, n, k)

Two equations relating the Metzner and Reed (1955) parameters (n', K') to yield-pseudoplastic model parameters (τ_y, n, k) were derived as follows:

$$\frac{1}{n'} = \frac{1}{n} + \frac{\lambda_1}{\lambda_2}, \quad (21)$$

and

$$K' = \tau_w / \lambda_3^{n'}, \quad (22)$$

where

$$\lambda_1 = x(1-x)^{n_3-1} + (1-x)^{n_2-1}\left[2x^2 - 2x(1-x)/n_2\right] + (1-x)^{n_1-1}\left[x^3 - 2x^2(1-x)/n_1\right], \quad (23)$$

$$\lambda_2 = (1-x)^{n_3}/n_3 + 2x(1-x)^{n_2}/n_2 + x^2(1-x)^{n_1}/n_1, \quad (24)$$

$$\lambda_3 = \left[4(\tau_w/K)^{1/n'}\lambda_2\right]^{n'}, \quad (25)$$

$$n_1 = (1+n)/n, \quad (26)$$

$$n_2 = (1+2n)/n, \quad (27)$$

and

$$n_3 = (1 + 3n)/n \tag{28}$$

Eqs. (21)–(28) are used to determine the values of n' and K' from the constants of Eq. (5), The values of n' and K' are needed for calculating the Metzer and Reed Reynolds number (Re_{MR}) from Eq. (10).

4. Calculations procedure

To calculate the value of C from Eq. (20), the following procedure is proposed.

(1) Correlate the rheological characteristics of the yield fluid with Eq. (5) and determine the values of τ_y, K, and n.

(2) Calculate the value of τ_w from the following flow shear equation of power-law yield-pseudoplastic model:

$$\frac{8V}{D} = 4\left(\frac{1}{K}\right)^{1/n}(\tau_w - \tau_y)^{1+n/n}\left[(\tau_w - \tau_y)^2 \Big/ \left(\frac{1+3n}{n}\right)\right] + \left[2\tau_y(\tau_w - \tau_y) \Big/ \left(\frac{1+2n}{n}\right) + \tau_y^2 \Big/ \left(\frac{1+n}{n}\right)\right] \Big/ \tau_w^3 \tag{29}$$

(3) Determine the value of x, using Eq. (12).

(4) Determine the friction factor from Eq. (11).

(5) Calculate the value of Re_{MR} from Eq. (10).

(6) Calculate the value of C from Eq. (20). If $C > 1.0$, the flow is turbulent. If $C \leq 1.0$, the flow is laminar.

4.1. Comparison with published data

To verify the validity of the proposed laminar turbulent-transition criterion, a comparison was made between the critical Reynolds numbers calculated from Eq. (20) and those estimated by Hanks (1963) and Turian and Yuan (1977). The data were used to calculate the viscous interaction coefficient (C) from Eq. (20). The results are plotted in Fig. 1. It can be observed that the value of C increases with increasing Reynolds number. The critical Reynolds numbers are determined from this figure at $C = 1$. The same data were used to calculate the critical Reynolds number given by Eqs. (3) and (4). The results obtained are plotted in Fig. 2, and a 45° straight line is drawn on the same plot. This figure reveals the closeness of the plotted data to the 45° straight line. This ensures that Eq. (20) can be used to determine the type of flow as efficiently as Eqs. (3) and (4).

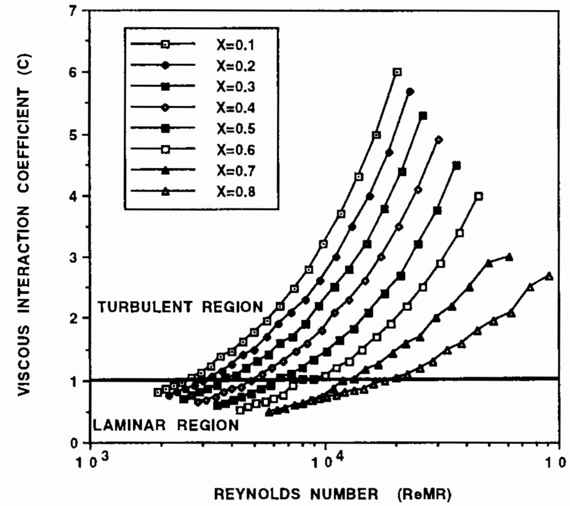

Fig. 1. Predicting the onset of turbulence for different yield-pseudoplastic fluids.

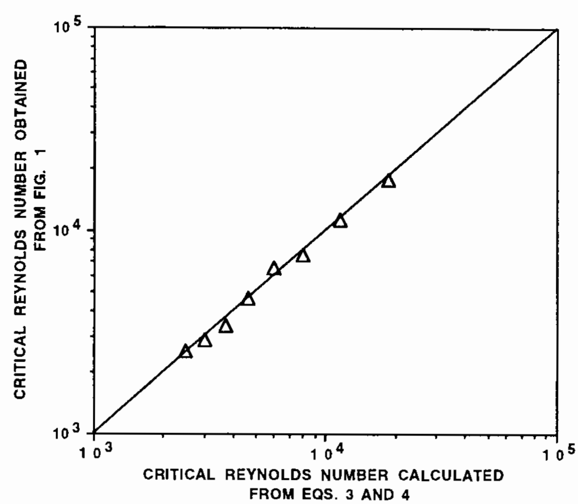

Fig. 2. Crossplot for the critical Reynolds number obtained from Fig. 1 and those calculated from Eqs. (3) and (4).

5. Conclusions

The developed criterion given by Eq. (20) can be used to predict adequately the type of flow, whether it is laminar or turbulent, for yield-pseudoplastic, yield-dilatant, and Bingham fluids. A comparison between the critical Reynolds number calculated from the proposed criterion Eq. (20) and Hank's correlations (Eqs. (3) and (4)) ensures the validity of the developed criterion. Two equations that relate the Metzner and Reed parameters (n', K') to the power-law yield-pseudoplastic model parameters (τ_y, K, n) were derived.

6. Nomenclature

C viscous	interaction coefficient
D	inside pipe diameter, m
f	Fanning friction factor
g_c	gravity coefficient, m/s^2
He	Hedstrom number
K'	Metzner and Reed parameter, Pa · s
k	parameter in Eq. (5), Pa · s
n	parameter in Eq. (5)
n'	Metzner and Reed parameter
R	Pipe radius, m
Re_B	Reynolds number of Bingham fluids
Re_{BC}	critical Reynolds number of Bingham fluids
Re_{MR}	Reynolds number defined by Metzner and Reed
Re_{PLC}	Reynolds number defined by Clapp
r	radial position, m
S	generalized stability parameter defined by Hanks
u	point velocity, m/s
V	average velocity
X	yield stress/wall shear stress ratio
Z	stability parameter defined by Ryan and Johnson

Superscripts

v	viscous stress

Subscripts

y	yield stress
w	pipe wall

Greek Symbols

ρ	mass density, kg/m^3
ε	the vorticity ($\varepsilon = \nabla \times \nu$)
τ	shear stress, Pa
λ_1	parameter in Eq. (23)
λ_2	parameter in Eq. (24)
λ_3	parameter in Eq. (25)
∇	operator

References

Clapp, R.M., 1961. Developments in heat transfer, Int. Conf., Univ. Colorado, Boulder, CO, pp. 652–661.

Desouky, S.E.M., 1991. A new laminar–turbulent-transition criterion for pseudoplastic fluids. J. Pet. Sci. Eng. 5, 285–291.

Desouky, S.E.M., El-Emam, N.A., 1990. A generalized pipeline design correlation for pseudoplastic fluids. J. Can. Pet. Tech. 29 (5), 48–54.

Dodge, D.W., Metzner, A.B., 1959. Turbulent flow of non-Newtonian system. Am. Inst. Chem. Eng. J. 5 (2), 189–204.

Govier, G.W., Aziz, K., 1982. The flow of complex mixtures in pipes, Van Nostrand Reinhood, New York, 792 pp.

Hanks, R.W., 1963. The laminar–turbulent-transition for fluids with a yield stress. Am. Inst. Chem. Eng. J. 9 (3), 306–309.

Hanks, R.W., 1968. On the theoretical calculation of friction factors for laminar, transitional and turbulent flow of Newtonian fluids in pipes and between parallel plane walls. Am. Inst. Chem. Eng. J. 14 (5), 691–695.

Hanks, R.W., Christiansen, E.B., 1962. The laminar–turbulent-transition in non-isothermal flow of pseudoplastic fluids in tubes. Am. Inst. Chem. Eng. J. 8 (4), 467–471.

Hanks, R.W., Ricks, B.L., 1975. Transitional and turbulent pipe flow of pseudoplastic fluids. J. Hydrom. 9 (1), 39–44.

Hemeida, A.M., 1993. Friction factor for yieldless fluids in turbulent pipe flow. J. Can. Pet. Tech. 32 (1), 32–35.

Knudson, J.G., Katz, D.L., 1958. Fluid dynamics and heat transfer, McGraw-Hill Book Company, New York, 576 pp.

Metzner, A.B., Park, M.G., 1964. Turbulent flow characteristics of viscoelastic fluids. J. Fluid Mech. 20 (2), 291–303.

Metzner, A.B., Reed, J.C., 1955. Flow of non-Newtonian fluids-correlation of the laminar, transition and turbulent flow regions. Am. Inst. Chem. Eng. J. 14 (5), 434–440.

Murthy, V.R.K., Zandi, I., 1969. Turbulent flow of non-Newtonian suspension in pipe. J. Eng. Mech. Div. Proc. Am. Soc. Civ. Eng. 95(EMI), 1–17.

Ryan, N.W., Johnson, M.M., 1959. Transition from laminar to turbulent flow in pipes. Am. Inst. Chem. Eng. J. 5 (4), 433–435.

Shaver, R.G., Merrill, E.W., 1959. Turbulent flow of pseudoplastic polymer solutions in straight cylindrical tubes. Am. Inst. Chem. Eng. J. 5 (2), 181–188.

Torrance, B.Mck., 1963. Turbulent flow in smooth tubes of yield-pseudoplastic fluids. S. Afr. Mech. Eng. 13, 89–95.

Turian, R.M., Yuan, T.F., 1977. Flow of slurries in pipelines. J. Inst. Chem. Eng. 23 (3), 232–243.

Zandi, I.M.A., Rust, R.H., 1965. Turbulent non-Newtonian velocity profiles in pipes. J. Hydrom. Div. 91 (6), 37–55.