Design of Packed Towers for Absorption

Figure 10.6-7. Location of operating lines: (a) for absorption of A from V to L stream, (b) for stripping of A from L to V stream.
**Operating-line derivation**

For the case of solute A diffusing through a stagnant gas and then into a stagnant fluid, an overall material balance on component A in the figure for a packed absorption tower is,

\[
L' \frac{x_2}{(1 - x_2)} + V' \frac{y_1}{(1 - y_1)} = L' \frac{x_1}{(1 - x_1)} + V' \frac{y_2}{(1 - y_2)}
\]

10.6-4

<table>
<thead>
<tr>
<th>Over-all</th>
<th>[ L' \frac{x_2}{(1 - x_2)} + V' \frac{y_1}{(1 - y_1)} = L' \frac{x_1}{(1 - x_1)} + V' \frac{y_2}{(1 - y_2)} ]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Local</td>
<td>[ L' \frac{x}{(1 - x)} + V' \frac{y_1}{(1 - y_1)} = L' \frac{x_1}{(1 - x_1)} + V' \frac{y}{(1 - y)} ]</td>
</tr>
</tbody>
</table>

- \( L' \) and \( V' \) are constant throughout the tower
- Total flows \( L \) and \( V \) are not constant
- Eqn 10.6-4 is OPERATING LINE EQUATION, which may be a curved line
- Operating line can also be written in terms of partial pressure of A
- For dilute L & V, \((1 - x)\) and \((1 - y)\) can be taken as 1, Eqn. (10.6-4) becomes straight
Limiting solvent flow, $L'_{\text{min}}$ and optimum $(L'/V')$ ratios

- Inlet gas conditions $V, y_1$ are known
- Exit concentration $y_2$ is set
- Concentration $x_2$ of the entering liquid is often fixed
- Entering liquid flow $L_2$ or $L'$ needs to be determined
- When the operating line has a minimum slope and touches the equilibrium line at point P, $L'$ is a minimum at $L'_{\text{min}}$. The value of $x_1$ is a maximum at $L'_{\text{min}}$.
- At point P, the driving forces are all zero.
- To determine $L'_{\text{min}}$, the following operation line equation can be used

$$L'_{\text{min}} \frac{x_2}{(1 - x_2)} + V' \frac{y_1}{(1 - y_1)} = L'_{\text{min}} \frac{x_{1\text{max}}}{(1 - x_{1\text{max}})} + V' \frac{y_2}{(1 - y_2)}$$

- If the equilibrium line is curved concavely downward, the minimum value of L is reached by the operating line becoming tangent to the equilibrium line instead of intersecting it.
- The choice of the optimum ratio $(L/V)$ depends the economics. In absorption, too high a value requires a large liquid flow, and hence a large-diameter tower. The cost of recovering the solute from the liquid by distillation will be high. A small liquid flow results in a high tower, which is costly. As an approximation, the optimum liquid flow is obtained by using a value of about 1.5 for the ratio of the average slope of the operating line to that of the equilibrium line for absorption. This factor can vary depending on the value of the solute and tower type.
Design Equations

Defining \( a \) as interfacial area in m\(^2\) per m\(^3\) volume of packed section, the volume of packing in a height \( dz \) m (Fig. 10.6-6) is \( Sdz \). Therefore,

\[
dA = aS \, dz
\]

where, \( S \) is cross-sectional area of tower. The volumetric film and overall mass transfer coefficients are \( k'_x a, k'_y a, K'_x a, K'_y a \). Since the mass exchange of solute takes place between \( L (=\text{kg mol total liquid/s}) \) and \( V (=\text{kg mol total gas/s}) \) phases, one can therefore write for \( dz \),

\[
N_A dA = d(Vy) = d(Lx)
\]

Since,

\[
N_A = \frac{k'_y}{(1 - y_A)_{iM}}(y_{AG} - y_{Ai}) = \frac{k'_x}{(1 - x_A)_{iM}}(x_{Ai} - x_{AL})
\]

\[
N_A dA = \frac{k'_y a}{(1 - y_A)_{iM}}(y_{AG} - y_{Ai})S \, dz = \frac{k'_x a}{(1 - x_A)_{iM}}(x_{Ai} - x_{AL})S \, dz
\]

\[
d(Vy_{AG}) = \frac{k'_y a}{(1 - y_A)_{iM}}(y_{AG} - y_{Ai})S \, dz
\]

\[
d(Lx_{AL}) = \frac{k'_x a}{(1 - x_A)_{iM}}(x_{Ai} - x_{AL})S \, dz
\]

\[
d(Vy_{AG}) = d\left(\frac{V'}{(1 - y_{AG})y_{AG}}\right) = V' d\left(\frac{y_{AG}}{(1 - y_{AG})}\right) = \frac{V'dy_{AG}}{(1 - y_{AG})^2}
\]

\[
= \frac{Vd y_{AG}}{(1 - y_{AG})}
\]
\[
\frac{V d y_{AG}}{(1 - y_{AG})} = \frac{k'_y a}{(1 - y_A)_{iM}} (y_{AG} - y_{Ai}) S \, dz \quad 10.6-15
\]

\[
\frac{L d x_{AL}}{(1 - x_{AL})} = \frac{k'_x a}{(1 - x_A)_{iM}} (x_{Ai} - x_{AL}) S \, dz \quad 10.6-16
\]
Simplified Design Methods for Absorption of Dilute Gas Mixtures in Packed Towers

For solute A concentration in L & V streams less than 10%, the flows will vary by less than 10% and the mass-transfer coefficients by considerably less than this.

As a result, the average values of the flows V and L and the mass-transfer coefficients at the top and bottom of the tower can be taken outside the integral.

Likewise, the following terms can be taken outside, and average values of the values at the top and bottom of the tower used.

\[
\frac{(1 - y_{AG})}{(1 - y_A)_{IM}} \cdot \frac{(1 - y_{AG})}{(1 - y_A)_{*M}} \cdot \frac{(1 - x_{AL})}{(1 - x_A)_{IM}} \cdot \frac{(1 - x_{AL})}{(1 - x_A)_{*M}}
\]

\[
z = \int_{y_2}^{y_1} \frac{V}{k'y} d\frac{y_{AG}}{aS} \left(1 - y_{AG}\right) \left(y_{AG} - y_{AI}\right) = \int_{y_2}^{y_1} \frac{V}{K'y} aS \left(1 - y_{AG}\right) \left(y_{AG} - y_{*}\right)
\]

\[
z = \left[ \frac{V}{k'y} aS \frac{(1 - y_A)_{IM}}{(1 - y_{AG})} \right]_{av} \int_{y_2}^{y_1} \frac{dy_{AG}}{(y - y_{AI})} = \left[ \frac{V}{K'y} aS \frac{(1 - y_A)_{*M}}{(1 - y_{AG})} \right]_{av} \int_{y_2}^{y_1} \frac{dy_{AG}}{(y - y_{*})}
\]

For dilute soln.,

\[
\frac{(1 - y_A)_{IM}}{(1 - y_{AG})} \cong \frac{(1 - y_A)_{*M}}{(1 - y_{AG})} \cong 1
\]

gives,

\[
z = \left[ \frac{V}{k'y} aS \left(1 - y_{AI}\right) \right]_{av} \int_{y_2}^{y_1} \frac{dy}{(y - y_i)} = \left[ \frac{V}{K'y} aS \left(1 - y_{*}\right) \right]_{av} \int_{y_2}^{y_1} \frac{dy}{(y - y_{*})}
\]

Approximation of integration

\[
z = \left[ \frac{V}{k'y} aS \right]_{av} \frac{(y_1 - y_2)}{(y - y_i)_{M}} \cong \left[ \frac{V}{K'y} aS \right]_{av} \frac{(y_1 - y_2)}{(y - y_{*})_{M}}
\]
where,

\[
(y - y_i)_M = \frac{(y_1 - y_{i1}) - (y_2 - y_{i2})}{\ln[(y_1 - y_{i1})/(y_2 - y_{i2})]}
\]

\[
(y - y^*)_M = \frac{(y_1 - y_{1}^*) - (y_2 - y_{2}^*)}{\ln[(y_1 - y_{1}^*)/(y_2 - y_{2}^*)]}
\]

\[
\frac{V}{S}(y_1 - y_2) = k'_y az (y - y_i)_M = K'_y az (y - y^*)_M
\]

\[
\frac{L}{S}(x_1 - x_2) = k'_x az (x_i - x)_M = K'_x az (x^* - x)_M
\]
Design Procedure for Dilute Solutions

- Determine compositions of stream $x_1, y_1, x_2, y_2$ and draw the operating line. Material balance equation may be needed.
- Use thermodynamic equilibrium data to draw the equilibrium line.
- Obtain mass transfer coefficients either from experimental values or empirical correlations, $k'_x a, k'_y a, K'_x a, K'_y a$
- From point $P_1$, draw line $P_1M_1$ with slope

$$\frac{k_x}{k_y} = -\left[\frac{k'_x a}{(1-x)_{iM}}\right] - \left[\frac{k'_y a}{(1-y)_{iM}}\right]$$

Since interface concentration are unknown, use following for dilute solution for slope

$$= -\left[\frac{k'_x a}{(1-x_1)}\right] - \left[\frac{k'_y a}{(1-y_1)}\right]$$

And determine the $M_1(x_{1i}, y_{1i})$ on the equilibrium line.
• Similarly, draw line $P_2M_2$ with slope

$$-\left[\frac{k'_{x}a}{(1-x_2)}\right]/\left[\frac{k'_{y}a}{(1-y_2)}\right]$$

And determine the $M_2(x_{2i}, y_{2i})$ on the equilibrium line

![Diagram showing operating line and interface compositions in a packed tower for absorption of dilute gases.](image)

- Compute average values for V and L streams, and

$$\frac{(y - y_i)}{M} = \frac{(y_1 - y_{i1}) - (y_2 - y_{i2})}{\ln[(y_1 - y_{i1})/(y_2 - y_{i2})]}$$

- Compute height of the absorption column using,

$$\frac{V_{av}}{S}(y_1 - y_2) = k'_{y}a(z)(y - y_i)_M$$
**EXAMPLE 10.6-4. Absorption of Acetone in a Packed Tower**

Acetone is being absorbed by water in a packed tower having a cross-sectional area of 0.186 m$^2$ at 293 K and 101.32 kPa (1 atm). The inlet air contains $y_1 = 2.6 \text{ mol}\%$ acetone and outlet $y_2 = 0.5 \text{ mol}\%$. The gas flow is $V' = 13.65 \text{ kg mol inert air/h}$. The pure water ($x_2 = 0 \text{ mol}\%$) inlet flow is $L' = 45.36 \text{ kg mol water/h}$. Film coefficients for the given flows in the tower are:

$$k_y'a = 3.78 \times 10^{-2} \text{ kg mol/s} \cdot \text{m}^3 \cdot \text{mol frac}$$

$$k_x'a = 6.16 \times 10^{-2} \text{ kg mol/s} \cdot \text{m}^3 \cdot \text{mol frac}$$

Equilibrium data are given in Appendix A.3, which can be represented as $y = 1.186 \times x$.

(a) Calculate the tower height using $k_y'a$.
(b) Repeat using $k_x'a$.
(c) Calculate $K_y'a$ and the tower height.
Given:

\[ x_2 = 0.00; y_2 = 0.005; x_1 = ??; y_1 = 0.026; \]

Over-all MB:

\[
L' \frac{x_2}{(1 - x_2)} + V' \frac{y_1}{(1 - y_1)} = L' \frac{x_1}{(1 - x_1)} + V' \frac{y_2}{(1 - y_2)}
\]

10.6-3

gives,

\[ x_2 = 0.00; y_2 = 0.005; x_1 = 0.00648; y_1 = 0.026; \]

\[
- \frac{k_x}{k_y} = - \left[ \frac{k'_x a}{(1 - x_1)} \right] / \left[ \frac{k'_y a}{(1 - y_1)} \right]
\]

\[ = - \left[ \frac{6.16 \times 10^{-2}}{(1 - 0.00648)} \right] / \left[ \frac{3.78 \times 10^{-2}}{(1 - 0.026)} \right] = -1.60 \]

\[
- \frac{k_x}{k_y} = - \left[ \frac{k'_x a}{(1 - x_2)} \right] / \left[ \frac{k'_y a}{(1 - y_2)} \right]
\]

\[ = \left[ \frac{6.16 \times 10^{-2}}{(1 - 0.00)} \right] / \left[ \frac{3.78 \times 10^{-2}}{(1 - 0.0050)} \right] = -1.62 \]

\[
(y - y_i)_M = \frac{(y_1 - y_{i1}) - (y_2 - y_{i2})}{\ln[(y_1 - y_{i1})/(y_2 - y_{i2})]}
\]

\[ = \frac{(0.026 - 0.154) - (0.005 - 0.002)}{\ln[(0.026 - 0.154)/(0.005 - 0.002)]}
\]

\[ = 0.00602 \]

\[
(x_i - x)_M
\]

\[ = \frac{(x_{i1} - x_1) - (x_{i2} - x_2)}{\ln[(x_{i1} - x_1)/(x_{i2} - x_2)]}
\]

\[ = 0.00368 \]

\[
V_1 = \frac{V'}{(1 - y_1)} = \frac{13.65/3600}{(1 - 0.026)}
\]

\[ = 3.893 \times 10^{-3} \frac{kg \text{ mol}}{s} \]

\[
V_2 = \frac{V'}{(1 - y_2)} = \frac{13.65/3600}{(1 - 0.005)}
\]

\[ = 3.811 \times 10^{-3} \frac{kg \text{ mol}}{s} \]
\( V_{av} = \frac{V_1 + V_2}{2} = 3.852 \times 10^{-3} \frac{kg\ mol}{s} \)

\[
L_{av} \approx L_1 \approx L_2 \approx L' = 1.260 \times 10^{-2} \frac{kg\ mol}{s}
\]

\[
\frac{V_{av}}{S} (y_1 - y_2) = k'_{y} az (y - y_i)_{M}
\]

\[
L_{av} \frac{1}{S} (x_1 - x_2) = k'_{x} az (x_i - x)_{M}
\]

\[
\frac{3.852 \times 10^{-3}}{0.186} (0.026 - 0.005) = 3.78 \times 10^{-2} z \times 0.00602
\]

\[
\frac{1.260 \times 10^{-2}}{0.186} (0.00648 - 0.0) = 6.16 \times 10^{-2} z \times 0.00368
\]

\( z = 1.911 m \)

\( z = 1.936 m \)
Using overall gas mass transfer coefficient

<table>
<thead>
<tr>
<th>Equation</th>
<th>Value</th>
</tr>
</thead>
</table>
| \[
\frac{1}{K_y a} = \frac{1}{k_y a} + \frac{m'}{k_x a} \]                |             |
| \[
\frac{1}{K_y a} = \frac{1}{k_y a} + \frac{m'}{x_{Ai} - x_{AL}} \]   |             |
| \[
(1 - x_A)_{iM} = \frac{(1 - x_{Ai}) - (1 - x_A)}{\ln[(1 - x_{Ai})/(1 - x_A)]} \] | 0.993      |
| \[
(1 - y_A)_{iM} = \frac{(1 - y_{Ai}) - (1 - y_A)}{\ln[(1 - y_{Ai})/(1 - y_A)]} \] | 0.979      |
| \[
(1 - y_A)_{iM} = \frac{(1 - y_A^*) - (1 - y_{iM})}{\ln[(1 - y_A^*)/(1 - y_A)]} \] | 0.983      |
| \[
m' = \frac{y_{Ai} - y_A^*}{x_{Ai} - x_A} \]                          | 1.186      |
| \[
K_y a \]                                                           | 2.183 × 10^{-2} |
| \[
(y - y^*)_M = \frac{(y_1 - y_1^*) - (y_2 - y_2^*)}{\ln[(y_1 - y_1^*)/(y_2 - y_2^*)]} \] | \((0.026 - 0.0077) - (0.005 - 0.00)\) \ln\((0.026 - 0.0077)/(0.005 - 0.00)\) = 0.01025 |
| \[
\frac{V_{av}}{S} (y_1 - y_2) = K_y a z (y - y^*)_M \]       |            |
| \[
\frac{3.852 \times 10^{-3}}{0.186} (0.026 - 0.005) = 2.183 \times 10^{-2} z \times 0.01025; \] | Z = 1.944 m  |
### Bottom Conditions of Absorption Column

<table>
<thead>
<tr>
<th>(X_{AI} )</th>
<th>(Y_{AG} )</th>
<th>(X_{AI} )</th>
<th>(Y_{AI} )</th>
<th>(Y_{A^*} )</th>
<th>(X_{A^*} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0065</td>
<td>0.0260</td>
<td><strong>0.0130</strong></td>
<td>0.0155</td>
<td>0.008</td>
<td>0.021922</td>
</tr>
<tr>
<td>(1-X_{AI} )</td>
<td>((1-Y_{AG}))</td>
<td>(1-X_{AI} )</td>
<td>((1-Y_{AI}))</td>
<td>((1-Y_{A^*}))</td>
<td>((1-X_{A^*}))</td>
</tr>
<tr>
<td>0.99</td>
<td>0.97</td>
<td>0.99</td>
<td>0.98</td>
<td>0.99</td>
<td>0.98</td>
</tr>
<tr>
<td>((1-X_{AI})_M)</td>
<td>((1-Y_{AG})_M)</td>
<td>((1-X_{AI})_M)</td>
<td>((1-Y_{AI})_M)</td>
<td>((1-Y_{A^*})_M)</td>
<td>(m')</td>
</tr>
<tr>
<td>(k^{'}_x)</td>
<td>(k^{'}_y)</td>
<td>(k_x)</td>
<td>(k_y)</td>
<td>(K_x)</td>
<td>(K_y)</td>
</tr>
<tr>
<td>0.06160</td>
<td>0.03780</td>
<td>0.06221</td>
<td>0.03860</td>
<td>0.02637</td>
<td>0.02224</td>
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### Upper Conditions of Absorption Column

<table>
<thead>
<tr>
<th>(X_{AI} )</th>
<th>(Y_{AG} )</th>
<th>(X_{AI} )</th>
<th>(Y_{AI} )</th>
<th>(Y_{A^*} )</th>
<th>(X_{A^*} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0000</td>
<td>0.0050</td>
<td><strong>0.00178</strong></td>
<td>0.0021</td>
<td>0.000</td>
<td>0.00411</td>
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<tr>
<td>(1-X_{AI} )</td>
<td>((1-Y_{AG}))</td>
<td>(1-X_{AI} )</td>
<td>((1-Y_{AI}))</td>
<td>((1-Y_{A^*}))</td>
<td>((1-X_{A^*}))</td>
</tr>
<tr>
<td>1.00</td>
<td>0.9950</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>((1-X_{AI})_M)</td>
<td>((1-Y_{AG})_M)</td>
<td>((1-X_{AI})_M)</td>
<td>((1-Y_{AI})_M)</td>
<td>((1-Y_{A^*})_M)</td>
<td>(m')</td>
</tr>
<tr>
<td>(k^{'}_x)</td>
<td>(k^{'}_y)</td>
<td>(k_x)</td>
<td>(k_y)</td>
<td>(K_x)</td>
<td>(K_y)</td>
</tr>
<tr>
<td>0.06160</td>
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<td>0.06165</td>
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<table>
<thead>
<tr>
<th>Slope</th>
<th>(\text{Slope(Cal.)})</th>
<th>(\text{Slp-Slp})</th>
<th>(N_A = k_y(Y_{AG}-Y_{A^*}))</th>
<th>(N_A = k_y(Y_{AG}-Y_{AI}))</th>
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<tr>
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<td>-1.612</td>
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<td>4.072E-04</td>
<td>4.072E-04</td>
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### Upper Conditions of Absorption Column

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<tr>
<th>((X_i-X)_M)</th>
<th>((Y-Y_i)_M)</th>
<th>((Y-Y^*)_M)</th>
<th>(S)</th>
</tr>
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<tr>
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<td>0.0059</td>
<td>0.0103</td>
<td>0.186</td>
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</table>

<table>
<thead>
<tr>
<th>(V')</th>
<th>(V_1)</th>
<th>(V_2)</th>
<th>(V_{av})</th>
<th>(\text{Mat. Bal.})</th>
<th>(k^{'}_y)</th>
<th>((Y-Y_i)_M)</th>
<th>(Z)</th>
</tr>
</thead>
<tbody>
<tr>
<td>13.65</td>
<td>3.89E-03</td>
<td>3.81E-03</td>
<td>3.85E-03</td>
<td>0.000435</td>
<td>0.03780</td>
<td>0.00592</td>
<td><strong>1.9448</strong></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>(V')</th>
<th>(V_1)</th>
<th>(V_2)</th>
<th>(V_{av})</th>
<th>(\text{Mat. Bal.})</th>
<th>(k^{'}_y)</th>
<th>((Y-Y^*)_M)</th>
<th>(Z)</th>
</tr>
</thead>
<tbody>
<tr>
<td>13.65</td>
<td>3.89E-03</td>
<td>3.81E-03</td>
<td>3.85E-03</td>
<td>0.000435</td>
<td>0.02156</td>
<td>0.01025</td>
<td><strong>1.9667</strong></td>
</tr>
</tbody>
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<table>
<thead>
<tr>
<th>(L')</th>
<th>(L_1)</th>
<th>(L_2)</th>
<th>(L_{av})</th>
<th>(\text{Mat. Bal.})</th>
<th>(k^{'}_x)</th>
<th>((X_i-X)_M)</th>
<th>(Z)</th>
</tr>
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<tr>
<td>45.36</td>
<td>1.27E-02</td>
<td>1.26E-02</td>
<td>1.26E-02</td>
<td>0.000440</td>
<td>0.06160</td>
<td>0.00366</td>
<td><strong>1.9536</strong></td>
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</tbody>
</table>
Design of Packed Towers Using Transfer Units

General eqn.

$z = \int_{y_2}^{y_1} \frac{V dy_{AG}}{k_y a S (1 - y_A)(y_{AG} - y_{AI})}$

For dilute soln, $x_A, y_A < 10\%$

$z = \frac{V}{k_y a S} \int_{y_2}^{y_1} \frac{(1 - y)_{IM} dy}{(1 - y)(y - y_i)} = \frac{L}{K_x a S} \int_{x_2}^{x_1} \frac{(1 - x)_{*M} dx}{(1 - x)(x^* - x)}$

Define,

$H_G = \frac{V}{k_y a S}, (m); N_G = \int_{y_2}^{y_1} \frac{(1 - y)_{IM} dy}{(1 - y)(y - y_i)}, (\text{m})$

$H_{OL} = \frac{L}{K_x a S}, (m); N_{OL} = \int_{x_2}^{x_1} \frac{(1 - x)_{*M} dx}{(1 - x)(x^* - x)}, (\text{m})$

$z = H_G N_G = H_{OL} N_{OL} = H_L N_L = H_{OG} N_{OG};$

$z = H_G \left[ \frac{(1 - y)_{IM}}{1 - y} \right]_{av} \int_{y_2}^{y_1} \frac{dy}{y - y_i} = H_{OL} \left[ \frac{(1 - x)_{*M}}{1 - x} \right]_{av} \int_{x_2}^{x_1} \frac{dx}{x^* - x}$

$z = H_G \left[ \frac{(1 - y)_{IM}}{1 - y} \right]_{av} \frac{(y_1 - y_2)}{(y - y_i)_{IM}} = H_L \left[ \frac{(1 - x)_{IM}}{1 - x} \right]_{av} \frac{(x_1 - x_2)}{(x_i - x)_{IM}}$

$z = H_{OG} \left[ \frac{(1 - y)_{*M}}{1 - y} \right]_{av} \frac{(y_1 - y_2)}{(y - y^*)_{M}} = H_{OL} \left[ \frac{(1 - x)_{*M}}{1 - x} \right]_{av} \frac{(x_1 - x_2)}{(x^* - x)_{M}}$

- Major resistance to mass tranfer in the gas phase (Acetone absorption from air by water) , Use $N_{OG}$ or $N_G$
- Major resistance to mass tranfer in the liquid phase (CO2/O2 absorption from air by water) , Use $N_{OL}$ or $N_L$
Analytical equations for countercurrent absorber design

For **ABSORPTION** (transfer of solute A from V to L)

**Number of transfer units**

\[
N_{OG} = \frac{1}{(1 - 1/A)} \ln \left[ (1 - 1/A) \frac{y_1 - mx_2 + 1}{y_2 - mx_2 + 1} \right]
\]

**Number of theoretical stages**

\[
N = \ln \left[ \frac{y_{N+1} - mx_0}{y_1 - mx_0} \left( 1 - \frac{1}{A} \right) + \frac{1}{A} \right] / \ln A
\]

\[
N_{OG} = \frac{\ln A}{(1 - 1/A)^N}
\]

\[
A = \frac{L}{mV}; \ y = mx
\]
For **STRIPPING** (transfer of solute A from L to V)

### Number of transfer units

\[ N_{OL} = \frac{1}{1-A} \ln \left[ (1-A) \frac{x_2 - y_1/m}{y_2 - y_1/m} + A \right] \]

### Number of theoretical stages

\[ N = \log \left[ \frac{x_0 - (y_{N+1}/m)}{x_N - (y_{N+1}/m)} \left( 1 - \frac{1}{S} \right) + \frac{1}{S} \right] / \log S \]

\[ A = \frac{1}{S} = \frac{L}{mV}; y = mx \]
**EXAMPLE 10.6-2. Absorption of Acetone in a Packed Tower**

Acetone is being absorbed by water in a packed tower having a cross-sectional area of 0.186 m$^2$ at 293 K and 101.32 kPa (1 atm). The inlet air contains $y_1 = 2.6 \text{ mol}\%$ acetone and outlet $y_2 = 0.5 \text{ mol}\%$. The gas flow is $V' = 13.65 \text{ kg mol inert air/h}$. The pure water ($x_2 = 0 \text{ mol}\%$) inlet flow is $L' = 45.36 \text{ kg mol water/h}$. Film coefficients for the given flows in the tower are:

$$k_y'a = 3.78 \times 10^{-2} \text{ kg mol/s} \cdot \text{ m}^3 \cdot \text{ mol frac}$$

$$k_x'a = 6.16 \times 10^{-2} \text{ kg mol/s} \cdot \text{ m}^3 \cdot \text{ mol frac}$$

Equilibrium data are given in Appendix A.3, which can be represented as $y = 1.186x$.

(a) Calculate the tower height using $H_G$ and $N_G$.

(b) Calculate the tower height using $H_{OG}$ and $N_{OG}$.

**SOLUTION**

Given:

$x_2 = 0.00; y_2 = 0.005; x_1 = ?; y_1 = 0.026$;

Over-all MB:

$$L' \frac{x_2}{(1-x_2)} + V' \frac{y_1}{(1-y_1)} = L' \frac{x_1}{(1-x_1)} + V' \frac{y_2}{(1-y_2)}$$

10.6-3

Gives,

$x_2 = 0.00; y_2 = 0.005; x_1 = 0.00648; y_1 = 0.026$;

<table>
<thead>
<tr>
<th>$\frac{-k_x}{k_y} = -\left[ \frac{k_x' \cdot a}{(1-x_1)} \right] \frac{k_y' \cdot a}{(1-y_1)}$</th>
<th>$x_{1i} = 0.0130; y_{1i} = 0.0154$</th>
<th>$(y-y_i)<em>M = \frac{(y_1-y</em>{1i})-(y_2-y_{2i})}{\ln[(y_1-y_{1i})/(y_2-y_{2i})]}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-1.60$</td>
<td>$-1.60$</td>
<td>$0.00602$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$\frac{-k_x}{k_y} = -\left[ \frac{k_x' \cdot a}{(1-x_2)} \right] \frac{k_y' \cdot a}{(1-y_2)}$</th>
<th>$x_{2i} = 0.0018; y_{2i} = 0.002$</th>
<th>$(x_i-x)<em>M = \frac{(x</em>{i1}-x_1)-(x_{i2}-x_2)}{\ln[(x_{i1}-x_1)/(x_{i2}-x_2)]}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-1.62$</td>
<td>$-1.62$</td>
<td>$0.00368$</td>
</tr>
</tbody>
</table>

$$\frac{(1-y_1)_{IM}}{(1-y_1)} = \frac{0.979}{1-0.0260} = 1.005$$

$y_1 = 0.0260; y_{1i} = 0.0154$

$\frac{(1-y_1)_{IM}}{(1-y_1)} = \frac{(1-y_{1i})-(1-y_1)}{\ln[(1-y_{1i})/(1-y_1)]}$

$= 0.979$
\[
\frac{(1 - y_2)_{iM}}{(1 - y_2)} = \frac{0.997}{1 - 0.005} = 1.002
\]

\[
y_2 = 0.005; \quad y_{2i} = 0.002
\]

\[
(1 - y_2)_{iM} = \frac{(1 - y_{1i}) - (1 - y_1)}{\ln[(1 - y_{1i})/(1 - y_1)]}
\]

\[
N_G = \left[\frac{(1 - y)_{iM}}{(1 - y)}\right]_{av} \frac{(y_1 - y_2)}{(y - y_{iM})} = \frac{1.005 + 1.002}{2} \frac{0.0260 - 0.005}{0.00602} = 3.5
\]

\[
H_G = \frac{V}{k'_{y}aS} = \frac{3.852 \times 10^{-3}}{3.78 \times 10^{-2} \times 0.186} = 0.548 \text{ m}
\]

\[
z = 0.548 \times 3.5 = 1.918 \text{ m}
\]

\[
\frac{1}{K_ya} = \frac{1}{k_ya} + \frac{m'}{k_xa}
\]

\[
m' = \frac{y_{Ai} - y_A^*}{x_{Ai} - x_A}
\]

\[
\frac{1}{K'_{y}a/(1 - y)_{*M}} = \frac{1}{k'_{y}a/(1 - y)_{iM}} + \frac{m'}{k_xa/(1 - x)_{iM}}
\]

\[
(1 - x_A)_{iM} = \frac{(1 - x_{Ai}) - (1 - x_A)}{\ln[(1 - x_{Ai})/(1 - xy_A)]} = 0.993
\]

\[
(1 - y_A)_{iM} = \frac{(1 - y_{Ai}) - (1 - y_A)}{\ln[(1 - y_{Ai})/(1 - y_A)]} = 0.979
\]

\[
(1 - y_A)_{*M} = \frac{(1 - y_A^*) - (1 - y_A)}{\ln[(1 - y_A^*)/(1 - y_A)]} = 0.983
\]

\[
m' = \frac{y_{Ai} - y_A^*}{x_{Ai} - x_A} = 1.186
\]

\[
K'_{y}a = 2.183 \times 10^{-2}
\]

\[
(y - y^*)_{M} = \frac{(y_1 - y_{1i}^*) - (y_2 - y_{2i}^*)}{\ln[(y_1 - y_{1i}^*)/(y_2 - y_{2i}^*)]} = \frac{(0.026 - 0.0077) - (0.005 - 0.00)}{\ln[(0.026 - 0.0077)/(0.005 - 0.00)]} = 0.01025
\]

\[
N_{OG} = \left[\frac{(1 - y)_{*M}}{(1 - y)}\right]_{av} \frac{(y_1 - y_2)}{(y - y^*)_{M}} = \left[1\right] \frac{0.0260 - 0.005}{0.0125} = 2.05
\]

\[
H_{OG} = \frac{V}{K'_{y}aS} = \frac{3.852 \times 10^{-3}}{2.183 \times 10^{-2} \times 0.186} = 0.949 \text{ m}
\]

\[
z = H_{OG}N_{OG} = 0.949 \times 2.05 = 1.945 \text{ m}
\]
**Analytical approach:**

\[ y = mx = 1.186x \]

\[ A = \frac{L}{mV} = \frac{1.260 \times 10^{-2}}{1.186 \times 3.852 \times 10^{-3}} = 2.758 \]

\[ N_{OG} = \frac{1}{(1 - 1/A)} \ln \left( \frac{(1 - 1/A) \frac{y_1 - mx_2}{y_2 - mx_2} + \frac{1}{A}}{1 - \frac{1}{2.758}} \right) \]

\[ = \frac{1}{(1 - 1/2.758)} \ln \left( \frac{0.0260 - 1.186 \times 0 + \frac{1}{2.758}}{0.005 - 1.186 \times 0 + \frac{1}{2.758}} \right) \]

\[ = 2.043 \]

**2.04 m (analytical) \cong 2.05 (graphical)**

Note that, since

\[ \frac{1}{K_y' a} = \frac{1}{k_y' a} + \frac{m}{k_x' a} ; H_G = \frac{V}{k_y' aS} ; H_L = \frac{L}{k_x' aS} ; H_{OG} = \frac{V}{K_y' aS} \]

Therefore,

\[ H_{OG} = H_G + (mV/L)H_L = H_G + H_L/A \]

Similarly,

\[ H_{OL} = H_L + (L/mV)H_G = H_L + AH_G \]
HETP (Height Equivalent to a Theoretical Plate)

Instead of a tray (plate) column, a packed column can be used for continuous or batch distillation, or gas absorption. With a tray column, the gas/vapor leaving an ideal plate will be richer in the more volatile component than the gas/vapour entering the plate. Similarly, when packings are used instead of trays, the same enrichment of the vapour will occur over a certain height of packings. This height is termed as **height equivalent to a theoretical plate** (HETP). As all sections of the packings are physically the same, it is assumed that one equilibrium (theoretical) plate is represented by a given height of packings. Thus the required height of packings for any desired separation is given by (HETP \times \text{No. of ideal trays required}).

HETP values are in fact complex functions of temperature, pressure, composition, density, viscosity, diffusivity, pressure drop, vapour and/or liquid flowrates, packing characteristics, etc. In industrial practice, the HETP concept is used to convert empirically the number of theoretical trays to packing height. In the above example,

\[
\text{HETP} = H_{\text{og}} \frac{\ln(1/A)}{(1 - A)/A} = 0.949 \frac{\ln(1/2.758)}{(1 - 2.758)/2.758} = 1.510 \text{ m}
\]

\[
N = \ln \left[ \frac{y_{N+1} - mx_0}{y_1 - mx_0} \left(1 - \frac{1}{A}\right) + \frac{1}{A} \right] / \ln A = \ln \left[ \frac{y_1 - mx_2}{y_2 - mx_2} \left(1 - \frac{1}{A}\right) + \frac{1}{A} \right] / \ln A
\]

\[
= \ln \left[ \frac{0.026 - 1.186 \times 0}{0.005 - 1.186 \times 0} (1 - 1/2.758) + \frac{1}{2.758} \right] / \ln 2.758 = 1.283
\]

Therefore,

\[
z = N \times \text{HETP} = 1.283 \times 1.1510 \text{ m} = 1.938 \text{ m}
\]