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MATHEMATICAL AND COMPUTATIONAL MODELLING IN SCIENCE AND TECHNOLOGY

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Abstract Book

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Preface ICMCMST 2015

The International Conference "Mathematical and Computational Modelling in Science and Technology" ICMCMST'15, is organized by Izmir University, Izmir-Turkey, during the period August 02-07, 2015. This conference is aimed to bring experts, researchers and postgraduate students on Mathematical and Computational Modeling in several fields of Science, Technology and Engineering, such as theoretical and computational aspects in Mathematics, Informatics, Physics, Chemistry, Mechanics, Biology, Economics, and other sciences, from the entire world in order to discuss high level scientific questions, exchange solid knowledge of pure and applied sciences, and investigate diverse backgrounds, theoretically and practically.

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TOPICS

Partial Differential Equations: Theory and Applications
Fractional Operators and Their Applications
Inverse Problems: Modeling and Simulation
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PLENARY LECTURES

TALKS

POSTERS

PLENARY LECTURES

Heat Content Asymptotics - Theory and Practice

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Abstract

The heat content asymptotics are a short time measure of the total heat content of a solid in $\mathbb{R}^3$ which have been studied extensively in the mathematical literature. We propose a series of experiments to determine the extent to which the mathematical theories describe physical reality. This is joint work with B. Boggs and S. Espy.

Introduction Let $\Delta = -\partial^2_{x^1} - \cdots - \partial^2_{x^m}$ be the Laplacian on a solid $M \subset \mathbb{R}^3$. Consider the heat equation

$\partial_t u + \Delta u = 0$ (Evolution equation)

$\lim_{t\downarrow 0} u(\cdot;t) = \phi(\cdot)$ (Initial condition)

$\mathscr{B} u = 0$ (Boundary condition)

Here $\mathscr{B}$ is a suitable description of what happens near the boundary. Typical examples are

$\mathscr{B}_D u = u|_{\partial M}$ (Dirichlet boundary conditions)

$\mathscr{B}_R u = (\partial_\nu u + Su)|_{\partial M}$ for ν the inward unit normal. (Robin boundary conditions)

If (M, g) is a smooth bounded domain in $\mathbb{R}^3$, then Dirichlet boundary conditions correspond to dropping the body into ice-water. The boundary is instantaneously cooled to 0C. For Robin boundary conditions, the heat flow across the boundary is proportional to the temperature of the boundary; again the exterior is held at 0C. Neumann boundary conditions correspond to $S = 0$; there is no heat flow across the boundary - the boundary is perfectly insulated. If ρ is the specific heat, then the total heat energy content is $\beta(\phi,\rho,D,\mathscr{B})(t) := \int_M u(x;t)\rho(x)dx$. Assume the solid is in equilibrium at $t = 0$ so $\phi = \kappa$ is constant. For Dirichlet or Robin boundary conditions, $\beta \sim \beta_0 + \beta_1 t^{1/2} + \beta_2 t + \beta_3 t^{3/2} + \beta_4 t^2 + \ldots$ for $\beta_0 = \kappa \int_M \rho dx$ where the β_i are locally computable. Let L be the second fundamental form.

1. Dirichlet boundary conditions:
   (a) $\beta_1 = -\frac{2}{\sqrt{\pi}}\kappa \int_{\partial M} \rho dy$, (b) $\beta_2 = \kappa \int_{\partial M} \{\frac{1}{2} L_{aa}\rho - \rho_{;m}\} dy$.
   (c) $\beta_3 = \kappa \int_{\partial M} \{\frac{2}{3}\rho_{;mm} - \frac{2}{3} L_{aa}\rho_{;m} + \frac{1}{12} L_{aa}L_{bb} - \frac{1}{6} L_{ab}L_{ab}\}\rho dy$.
2. Robin boundary conditions:
   (a) $\beta_1 = 0$. (b) $\beta_2 = \kappa \int_{\partial M} S\rho dy$.
   (c) $\beta_3 = \frac{2}{3} \cdot \frac{2}{\sqrt{\pi}} \kappa \int_{\partial M} S(\partial_\nu + S)\rho dy$. (d) $\beta_4 = \kappa \int_{\partial M} \{-\frac{1}{2} S\Delta\rho + (\frac{1}{2} S + \frac{1}{4} L_{aa}) S(\partial_\nu + S)\rho\} dy$.

For Dirichlet boundary conditions, $\beta(t) \sim \beta_0 - \kappa \frac{2}{\sqrt{\pi}} t^{1/2} \mathrm{vol}(\partial M) + O(t)$ yields a power law in $t^{1/2}$ for the cooling. With Robin boundary conditions, $\beta(t) \sim \beta_0 + tS\kappa\,\mathrm{vol}(\partial M) + O(t^{3/2})$ yields a power law in t for the cooling (S will in general be negative).

A 19th centure measurement 10^{-3} sec

Keywords : *Heat content asymptotics, Dirichlet and Robin boundary conditions, interferometric measurements, CAMCOR focused ion beam.*
2010 Mathematics Subject Classification : **58J32; 58J35; 35K20**

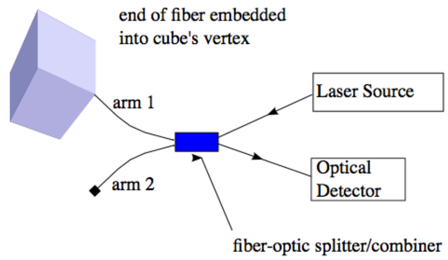
• Construct test shapes out of brass and aluminum in the shape of a sphere, a torus, a cube, and a cylinder. Insert probes into different parts of the solids. In the case of a cube, for example, the center of a face, the center of an edge, and a vertex are obvious points. One heats the solid to uniform temperature κ and then immerses it suddenly in ice water. One is interested in knowing the power law controlling the temperature decay for short time. And if the power is not $t^{1/2}$ or t controlling the cooling, then one knows there is a fractal phenomena occurring.

20th century – Interferometric Measurements 10^{-7} **sec.**

• Interferometer arm 1: A length of fiber-optic cable at least partly embedded within an object's region of interest (e.g., the vertex of a cube).

• Interferometer arm 2: An equal length fiber-optic cable.

• Measure the resulting time-dependent interference patterns caused by, for example, the hot object's contraction when brought into contact with a cold bath.



Evenly split coherent light from the laser source travels down arms 1 and 2 reflecting back off the fiber ends. It is then recombined and the resultant interference is subsequently detected.

A 21st century measurement

• With CAMCOR's focused Ion Beam machine make micron-sized objects (cube, torus, etc.) from a material that is transparent at wavelength λ-trap and absorbing at wavelength λ-heat. Employ the APL's optical tweezers apparatus to spatially trap and heat the object. Use the resultant fast-time decay of the object's Brownian motion as a measure of the transient heat flow from the object to its surrounding liquid bath.

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Analytic Family of Solution Operators for Degenerate Fractional Equations

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Abstract We study a differential equation in a Banach space with degenerate operator at the fractional derivative. Degenerate analytic family of solution operators are found and in a case of reflexive Banach spaces unique solvability of the Cuachy problem for the equation is proved.

Introduction Consider the Cauchy problem

$$u^{(k)}(0) = u_k, \quad k = 0, 1, \dots, m-1, \tag{1}$$

for a fractional differential equation

$$D_t^\alpha Lu(t) = Mu(t), \quad t > 0, \tag{2}$$

with linear closed operators L and M that are densely defined in a Banch space $\mathfrak{U}$ on D_L and D_M correspondingly, acting to $\mathfrak{V}$. Here D_t^α is the Caputo fractional derivative with $\alpha > 0$, m is a smallest integer greater than or equal to α. Denote the fractional integral by J_t^α.

The feature of the equation is a nontrivial kernel $\ker L$ of the operator L: $\ker L \neq \{0\}$. Such equations will be called as degenerate. The conditions is studied for a unique solution existence of problem (1)–(2). Analytic family of solution operators is constructed and it is shown that solutions belong to a subspace of $\mathfrak{U}$. This work is a continuation of [1–3].

Main results For Banach spaces $\mathfrak{U}$, $\mathfrak{V}$ the Banach space of linear continuous operators from $\mathfrak{U}$ to $\mathfrak{V}$ will be denoted by $\mathscr{L}(\mathfrak{U};\mathfrak{V})$. If $\mathfrak{V} = \mathfrak{U}$, then this denotation will have a form $\mathscr{L}(\mathfrak{U})$. The set of complex numbers $\mu \in \mathbb{C}$ such that $(\mu L - M)^{-1} L \in \mathscr{L}(\mathfrak{U})$ and $L(\mu L - M)^{-1} \in \mathscr{L}(\mathfrak{V})$ is denoted by $\rho^L(M)$.

We consider the following conditions:

(I) there exist such constants $a_0 > 0$ and $\theta_0 \in (\pi/2, \pi)$ that for all

$$\lambda \in S^L_{a_0,\theta_0}(M) = \{\mu \in \mathbb{C} : |\arg(\mu - a_0)| < \theta_0, \mu \neq a_0\}$$

inclusion $\lambda^\alpha \in \rho^L(M)$ is valid.

(II) for every $a > a_0$, $\theta \in (\pi/2, \theta_0)$ there exists such constant $K = K(a,\theta) > 0$ that for all $\mu \in S^L_{a,\theta}(M)$ we have

$$\max\left\{\|(\mu^\alpha L - M)^{-1} L\|_{\mathscr{L}(\mathfrak{U})}, \|L(\mu^\alpha L - M)^{-1}\|_{\mathscr{L}(\mathfrak{V})}\right\} \leq \frac{K(a,\theta)}{|\mu^{\alpha-1}(\mu - a)|}.$$

Keywords : *fractional derivative; degenerate evolution equation; semigroup theory; analytic family of solution operators.*
2010 Mathematics Subject Classification : **47D06; 47D09; 34G10; 26A33.**

Denote $\mathbb{R}_+ = \{t \in \mathbb{R} : t > 0\}$, $\overline{\mathbb{R}}_+ = \{t \in \mathbb{R} : t \geq 0\}$, $g_\beta(t) = t^{\beta-1}/\Gamma(\beta)$ for $\beta > 0$, $t > 0$. A function $u \in C(\mathbb{R}_+; D_L) \cap C(\mathbb{R}_+; D_M)$, such that $Lu \in C^{m-1}(\overline{\mathbb{R}}_+; \mathfrak{U})$, and

$$g_{m-\alpha} * \left(Lu - \sum_{k=0}^{m-1} (Lu)^{(k)}(0) g_{k+1} \right) \in C^m(\mathbb{R}_+; \mathfrak{V}),$$

is called as a solution of equation (1), if for all $t > 0$ equality (1) is valid.

Denote by $\mathfrak{U}^1$ (or $\mathfrak{V}^1$) a closure in $\mathfrak{U}$ (or $\mathfrak{V}$) of the image $\operatorname{im}(\mu L - M)^{-1}L$ (or $\operatorname{im}L(\mu L - M)^{-1}$) for $\mu \in \rho^L(M)$ and by $\mathfrak{U}^0$ (or $\mathfrak{V}^0$) a kernel $\ker L$ (or $\ker L(\mu L - M)^{-1}$). The restriction of the operator L (or M) on $D_L \cap \mathfrak{U}^k$ (or $D_M \cap \mathfrak{V}^k$) will be denoted by L_k (or M_k), $k = 0, 1$,

Theorem 1. *Let $\alpha > 0$, (I), (II) be satisfied, $\gamma = \partial S^L_{a_0,\theta_0}(M) + 1$, $\Sigma_{\theta_0} = \{\tau \in \mathbb{C} : |\arg\tau| < \theta_0 - \pi/2, \tau \neq 0\}$, then*

$$\left\{ U_\alpha(\tau) = \frac{1}{2\pi i} \int_\gamma \mu^{\alpha-1} (\mu^\alpha L - M)^{-1} L e^{\mu t} d\mu \in \mathscr{L}(\mathfrak{U}) : \tau \in \Sigma_{\theta_0} \right\}$$

is an analytic family of operators and for every $a > a_0$, $\theta \in (\pi/2, \theta_0)$ there exists such $C = C(a, \theta)$, that for all $\tau \in \Sigma_\theta$, $n \in \mathbb{N} \cup \{0\}$

$$\| U_\alpha^{(n)}(\tau) \|_{\mathscr{L}(\mathfrak{U})} \leq \frac{C(a,\theta) e^{a \operatorname{Re} \tau}}{\tau^n},$$

$\ker L \subset \ker U_\alpha(\tau)$, $\operatorname{im} U_\alpha(\tau) \subset \mathfrak{U}^1$ *for every $\tau \in \Sigma_{\theta_0}$.*

Besides, if Banach spaces $\mathfrak{U}$ and $\mathfrak{V}$ are reflexive, then $\mathfrak{U} = \mathfrak{U}^1 \oplus \mathfrak{U}^0$, $\mathfrak{V} = \mathfrak{V}^1 \oplus \mathfrak{V}^0$.

Let operator L_1^{-1} and at least one of operators L_1 or M_1 be bounded. Then for all $u_k \in D_M \cap \mathfrak{U}^1$ the function $u(t) = \sum\limits_{k=0}^{m-1} J_t^k U_\alpha(t) u_k$ is an unique solution of problem (1)–(2). If there exists $l \in \{0, 1, \ldots, m-1\}$ such that $u_l \notin \mathfrak{U}^1$ then there is no solution of problem (1)–(2).

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Interaction of sine-Gordon solitons in the model with attracting impurities and damping

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Abstract The dynamics of nonlinear waves of the sine-Gordon equation with a spatially modulated periodic potential and damping are studied using analytical and numerical methods. The structure and properties of multisolitons excited on attracting impurities are determined. For small-amplitude oscillations, an analytical spectrum of the oscillations is obtained, which is in qualitatively agreement with the numerical results.

Introduction In recent decades the solitons theory for nonlinear evolutionary equations has been widely used in many fields of physics. For example, the sine-Gordon equation solitons simulate different localized dynamic excitations of the physical systems [1]. However, for an adequate description of real physical processes one requires a modification of the sine-Gordon equation by addition of summands describing the presence of damping and external force in the system, spatial modulation of the periodical potential (or impurity) Advanced analytical methods for studying this problem for the modified sine-Gordon equation (MSGE) using perturbation theory for solitons tend not to give an exhaustive result. The research of the large disturbances influence on the MSGE solution, in general case, can be conducted only with the help of numerical methods [2, 3].

Main results We investigated the interaction of sine-Gordon solitons in the model with attracting impurities and damping. For the case of two impurities in the system we presented, with the help of numerical simulation, the possibility of the multisolitons states generation (such as tritons and quadrons) localized on the impurities. The following ways of the kink dynamics were found: the kink is pinned in the impurity area and oscillates in between for a certain time; it is reflected in the reverse direction or passes the impurity area. In the latter two cases oscillated localized high amplitude non-linear breather waves greatly influencing the kink energy are excited. Further interaction with the localized waves in question forms the basis for the resonance mechanism, namely "reflection from the attracting impurity". Pinning of the kink and exciting high amplitude localized non-linear waves on the impurity may be used for multisoliton excitement in the sine-Gordon equation. A triton consisting of the weakly bound kink and breather is observed at long distances between the impurities. Starting with a certain critical distance pulsation and transmission mode frequencies are synchronized with the breather oscillation frequency and a triton solution of a wobble type is observed. At very short impurity distances excitement of the strongly bound kink and soliton is possible. The dependence of the structure and excited multisoliton frequencies from the impurity distances is determined.

There is found a definite critical distance value between the impurities that provides for two possible variants of dynamic behavior of the kink. In the first one the kink behavior is similar to the behavior in a single impurity. In the second case variants of the final kink behavior change depending on the initial kink velocity. It is explained by the breather oscillation phases excited in the second

Keywords : *sine-Gordon equation; kink; multisolitons; structure and properties of solitons; numerical-analytical study; impurity.*
2010 Mathematics Subject Classification : **35C08; 35Q51; 65M06.**

impurity area. A definite critical value of impurity distances causing two quite different ways of the dynamic kink behaviour is demonstrated. The interaction of the kink with breather can lead to a resonant "quasitunneling" of the kink through the double impurity region (i.e. passing through a potential barrier with a sub-barrier initial velocity).

Using the collective coordinates method we showed, that the original problem can be reduced to a system of ordinary differential equations for two bound harmonic oscillators with an elastic type of bind, which qualitatively describes the localized waves fluctuations. Within the limit of small amplitude fluctuations of localized waves, the spectrum of possible modes, which with good accuracy corresponds with the numerical results, was calculated.

We show analytically and numerically that the damping and external force counteract the generation of kink resonant reflection from the attracting impurity. However, its cause – resonant energy exchange between solitons – still exists. For detecting reflection resonance effects and quasitunneling in real physical experiments, we proposed a method of measuring the amplitude of translational vibrations localized in kink impurity region.

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Some open problems in fractional dynamics

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Abstract Fractional calculus is an emerging and interesting branch of applied mathematics, which described the theory of derivatives and integrals of any arbitrary real or complex order [1, 2]. Fractional differential equations have gained much attention due to the fact that fractional order system response ultimately converges to the integer order system response. In this talk, we will discuss some open problems in the areas of discrete fractional calculus and fuzzy fractional differential equations.

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Keywords : *Fractional calculus; discrete fractional calculus; fuzzy fractional derivative.*
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Fractal analysis of biological signals in a real time mode

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Abstract This paper presents the investigation of biological signals by fractal analysis of heart rate variability. Method of studyis based on the construction of the dynamic system attractor whose behavior is closely related to the behavior of the organism. It is shown that one of the most precise characteristics of the functional state of biological systems is the dynamical trend of correlation dimension. On the basis of this it is suggested that a complex programming apparatus be created for calculating these characteristics on line. A similar programming product is being created now with the support of RFBR. The results of the working program, its adjustment, and further development, are also considered.

Introduction Dynamic of energy exchange in a living organism is characterized by the balance between inner thermodynamic entropy production and its exchange with environment (scattering). This phenomenon has an irregular character. The irregularity shows itself by the variable heat loss in the process of the organism development, its pathologies and decease. The feature of irregularity is connected with nonlinear and synergetic effects, which are characterized by the dynamic chaos. These considerations show that on the overall statistical fluctuation the chaos is a vitally necessary part of the normal organism activity.

Method of investigation Method of the information entropy attractor has been created for this approach realization. The method and the means are based on the fractal analysis of electrocardiogram as a sequence of the periodical physiology signals. Ffractal method of time series analysis consists in the transition from the signal to the reconstructed attractor of the dynamical system. The simplest Takens delay method builds vector coordinates of the point on the attractor with using a constant value (time delay) of the initial time series:

$\vec{x}(i) = (a(i), a(i+\tau), ...a(i+\tau(n-1)))$

where $a(i)$ is the initial time series, n - the dimension of the embedding, τ - time delay. The resultant vector is the coordinate of a point on the reconstructed attractor.

The distance between the vectors on the attractor is usually taken as:

$\rho(\vec{x}(i), \vec{x}(j)) = max\left\{\left|a(i+k\tau) - a(j+k\tau)\right|\right\}$, $0 \le k \le n-1$

The probability that the distance between any two points on the attractor is less than a given r in the n-dimensional space embedding:

$$P^n\left\{\rho(\vec{x}(i), \vec{x}(j)) \le r; i = \overline{1, M}; j = \overline{1, M}\right\} = C^n(r) = \sum_{i=1}^{M}\sum_{j=i+1}^{M-1}\frac{\theta(r - \rho(\vec{x}(i), \vec{x}(j)))}{M(M-1)/2}$$

$$\text{where } \theta(a) = \begin{cases} 1, & \alpha \ge 0 \\ 0, & \alpha < 0 \end{cases} \text{ - Heaviside function.}$$

Keywords : *biological signal, fractal analysis,dynamic system, attractor*

2010 Mathematics Subject Classification : **26A33; 34A60; 34G25; 93B05.**

Introducing logarithm of the probability: $\Lambda^n = ln(C^n(r))$ we get $D_2 = \frac{\Lambda^n}{ln r}$

The scaling attractor entropy as a measure of analyzed signal, using the probabilistic multi-fractal dimensions, characterizes its metric and statistical properties. The general expression is measured in discrete dimension Renyi,

$$D_{Rq} = \lim_{\varepsilon\to 0} \lim_{\tau\to 0} \lim_{m\to\infty} \left[\frac{1}{1-q} \frac{ln I_{Rq}(q,\varepsilon)}{ln(1/\varepsilon)} \right]$$

Here $I_{Rq}(q,\epsilon) = \sum_{i=1}^{M(\varepsilon)} p_i^q(\epsilon)$ is the generalized Renyi entropy of order q;

$M(\varepsilon)$ - the minimum number of cubes with an edge ε, covering the attractor in the n-dimensional phase space embedding; p_i - the probability of visiting the i-th cube phase trajectory; m - number of points used to estimate the dimension.

Main results The above provisions were the basis of the developed method and means of mobile diagnostics, monitoring and operational forecasting of the functional state of the human body by fractal analysis of entropy generation in its physiological rhythms. Designed package allows working in a real time and perform the calculation of important fractal HRV photographed the ECG signal. Among them is the two-and three-dimensional picture of the attractors and various fractal dimension, correlation entropy, Lyapunov exponent, Hurst exponent and others. The choice of dynamic conditions is associated not only with the operational tracking data sets in real time but also with the prognostic component of their changes to prevent threats to critical states.

In addition, the complex allows analyzing the static linear components of HRV using Holter ECG monitoring.

Also it is used for the medical studies in other human activity for the control the stability of organism dynamics by the adaptation to outer and inner impacts, including critical situations.

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Optimal control of tuberculosis and HIV/AIDS

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Abstract The human immunodeficiency virus (HIV) and mycobacterium tuberculosis are the first and second cause of death from a single infectious agent, respectively, according with the World Health Organization. Acquired immunodeficiency syndrome (AIDS) is a disease of the human immune system caused by infection with HIV. There is no cure or vaccine to AIDS. However, antiretroviral (ART) treatment improves health, prolongs life, and substantially reduces the risk of HIV transmission. Nevertheless, ART treatment still presents substantial limitations: does not fully restore health; treatment is associated with side effects; the medications are expensive; and is not curative. Individuals infected with HIV are more likely to develop tuberculosis (TB) disease because of their immunodeficiency, and HIV infection is the most powerful risk factor for progression from TB infection to disease. Collaborative TB/HIV activities (including HIV testing, ART therapy and TB preventive measures) are crucial for the reduction of TB-HIV coinfected individuals. The study of the joint dynamics of TB and HIV present formidable mathematical challenges. We propose a new population model for TB-HIV/AIDS coinfection transmission dynamics, where TB, HIV and TB-HIV infected individuals have access to respective disease treatment, and single HIV-infected and TB-HIV co-infected individuals under HIV and TB/HIV treatment, respectively, stay in a chronic stage of the HIV infection. We apply optimal control theory to our TB-HIV/AIDS model and study optimal strategies for the minimization of the number of individuals with TB and AIDS active diseases, taking into account the costs associated to the proposed control measures.

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Keywords : *Tuberculosis, human immunodeficiency virus, coinfection, treatment, equilibrium, stability, optimal control.*
2010 Mathematics Subject Classification : **Primary: 92D30, 93A30; Secondary: 34D30, 49J15.**

Regularizing algorithms for image restoration

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Abstract Image processing and restoration are very good and important for practice examples of 2D and 3D inverse problems. Mostly, such problems are ill-posed. In this paper we will discuss how to construct and apply regularizing algorithms for solving following problems:1) Image processing in astronomy. As an example we will consider image processing of the gravitational lens «Einstein cross» [1]. 2) Image processing in digital photography. We will consider restoration of defocused and smeared images [2-3]. 3) Constructing magnetic image of a ship using measurements of a magnetic field by magnetic sensors. In most general case it is necessary to solve 3D integral equations of the 1st kind by parallel computers [4]. We will consider also geophysical applications. 4) Ring artefact suppression in X-ray tomogaphy [5]. 5) Restoring the signals from an electronic microscope in the backscattered electron mode [6]. Methods for solving these inverse problems are based on regularization technique proposed in [7] and all available a priori information.

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Keywords : *inverse problems, ill-posed problems, regularization, image processing.*
2010 Mathematics Subject Classification : **65R30; 65R32; 65R20.**

TALKS

Nonlocal in Time Problem for a Class of Partial Differential Equations

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Abstract We introduce a nonlocal problem for a class of partial differential equations. Conditions on solution existence and uniqueness are obtained by methods of degenerated operator semigroups theory. A nonlocal in time problem for equations with polynomials in elliptic operators is reduced to the abstract problem in Banach space.

Introduction Consider a nonlocal problem

$$\int_0^\infty u(t)\eta(t)\,dt = u_0 \tag{3}$$

for a degenerated evolution equation

$$L\dot{u}(t) = Mu(t), \quad t \geq 0. \tag{4}$$

Here an operator $L \in \mathscr{L}(\mathscr{U};\mathscr{V})$ (linear, continuously acting from Banach space $\mathscr{U}$ to Banach space $\mathscr{V}$), $\ker L \neq \{0\}$, an operator $M \in \mathscr{C}l(\mathscr{U};\mathscr{V})$ (linear, closed and densely defined in $\mathscr{U}$ with a domain D_M, acting to $\mathscr{V}$), $\eta : (0,\infty) \to \mathbb{R}$ is a non-negative non-increasing function. The operator M is supposed to be strongly (L,p)-radial operator [1] that guarantees a degenerate strongly continuous resolving semigroup $\{U(t) \in (\mathscr{U}) : t \geq 0\}$ of equation (4) and projectors P, Q to exist. Let $\mathscr{U}^0 = \ker P$, $\mathscr{V}^0 = \ker Q$; $\mathscr{U}^1 = \mathrm{im}P$, $\mathscr{V}^1 = \mathrm{im}Q$. Operators L_k (M_k) are the restrictions of L (M) onto $\mathscr{X}^k$ $(D_{M_k} = D_M \cap \mathscr{X}^k)$, $k = 0,1$, $H = M_0^{-1}L_0$.

A generalized solution of the problem (3)-(4) is a function $u(t) = U(t)v$, $v \in \mathscr{U}$. A classical solution is a function $u \in C^1([0,\infty);\mathscr{U})$.

Theorem 2. *Let M be a strongly (L,p)-radial operator with constants $K > 0$, $a < 0$, $\eta : (0,\infty) \to \mathbb{R}$ is a non-negative non-increasing function, identically non-equal to zero. Then*

(i) $u_0 \in D_{M_1}$ *there exists a unique generalized solution $u \in C([0,+\infty);\mathscr{U})$ of the problem (3), (4), and for all $t \geq 0$*

$$\|u(t)\|_{\mathscr{U}} \leq Ce^{-|a|t}\|Mu_0\|_{\mathscr{V}},$$

where a constant C is independent from u_0 and t;

(ii) *if $u_0 \in \mathscr{U} \setminus D_{M_1}$, then a generalized solution of the problem (3), (4) does not exists*;

(iii) *a generalized solution of (3), (4) is a classical solution if and only if $u_0 \in D_{(L_1^{-1}M_1)^2}$.*

Keywords : *nonlocal problem; degenerate operator; evolution equation; operator semigrop.*
2010 Mathematics Subject Classification : **34B10; 47D06; 35M13; 34610.**

Main results Let polynomials in elleptic operators $P_n(\lambda) = \sum_{i=0}^{n} c_i\lambda^i$, $Q_m(\lambda) = \sum_{j=0}^{m} d_j\lambda^j$ be such that $c_i, d_j \in \mathbb{C}$, $i = 0,1,\dots,n$, $j = 0,1,\dots,m$, $c_n, d_m \neq 0$, $m \geq n$. And $\Omega \subset \mathbb{R}^s$ is a domain with a boundary $\partial\Omega$ of class C^∞, $\eta : [0,\infty) \to \mathbb{R}$, Δ is the Laplace operator, $\theta \in \mathbb{R}$. Consider a boundary problem

$$\int_0^\infty z(x,t)\eta(t)\,dt = z_0(x), \quad x \in \Omega, \tag{5}$$

$$P_n(\Delta)\frac{\partial z}{\partial t}(x,t) = Q_m(\Delta)z(x,t), \quad (x,t) \in \Omega \times [0,\infty), \tag{6}$$

$$\theta\frac{\partial}{\partial n}\Delta^k z(x,t) + (1-\theta)\Delta^k z(x,t) = 0,\ k = 0,\dots,m-1,\ \ (x,t) \in \partial\Omega \times [0,\infty). \tag{7}$$

Introduce a space

$$H_\theta^{2n}(\Omega) = \{v \in H^{2n}(\Omega) : \theta\frac{\partial}{\partial n}\Delta^k v(x) + (1-\theta)\Delta^k v(x) = 0, k = 0,\dots,n-1, x \in \partial\Omega\}.$$

Theorem 3. *Let $m > n$, $(-1)^{m-n}\mathrm{Re}(d_m/c_n) \leq 0$, spectrum of $\sigma(\Delta)$ does not contain common roots of the polynomials P_n and Q_m, $a = \sup_{P_n(\lambda_k)\neq 0} \mathrm{Re}\frac{Q_m(\lambda_k)}{P_n(\lambda_k)} < 0$, $\eta : (0,\infty) \to \mathbb{R}$ is a non-negative non-increasing function, identically non-equal to zero. Then there exisits a unique generalized solution of the problem (5)–(7) for any $z_0 \in H_\theta^{2m}(\Omega) \cap \mathrm{span}\{\varphi_k : P_n(\lambda_k) \neq 0\}$, and*

$$\exists C > 0 \quad \forall t \geq 0 \quad \|z(\cdot,t)\|_{H^{2n}(\Omega)} \leq Ce^{-|a|t}\|z_0\|_{H^{2m}(\Omega)}.$$

A generalized solution does not exists, if $z_0 \notin H_\theta^{2m}(\Omega) \cap \mathrm{span}\{\varphi_k : P_n(\lambda_k) \neq 0\}$. A classical solution of the problem (5)-(7) exists, if $z_0 \in H_\theta^{4m-2n} \cap \mathrm{span}\{\varphi_k : P_n(\lambda_k) \neq 0\}$.

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Cauchy Problem for a High Order Quasilinear Degenerate Equation

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Abstract At first in the work a quasilinear evolution equation in Banach space is researched that is solved on the time derivative. Mittag–Lefler function is a research tool. Then the results are used in the study of degenerate quasilinear evolution equations. Conditions for the solvability of the Cauchy problem were obtained under certain conditions on the nonlinear operator. With help the theoretical results we have possible to study of the initial-boundary value problem for the equations of Kelvin-Voigt fluid motion.

Introduction Let $\mathscr{X}$, $\mathscr{Y}$ be Banach spaces, an operator $L:\mathscr{X}\to\mathscr{Y}$ be linear and bounded, an operator M be linear, closed and densely defined in $\mathscr{X}$ ($M\in\mathscr{C}l(\mathscr{X};\mathscr{Y})$), an operator $N:\mathscr{X}\to\mathscr{Y}$ be nonlinear. Consider a Cauchy problem

$$x^{(k)}(0)=x_k,\ k=0,1,\dots,m-1, \tag{8}$$

$$\frac{d^m}{dt^m}Lx(t)=Mx(t)+N(t,x(t),x^{(1)}(t),\dots,x^{(m-1)}(t))+f(t). \tag{9}$$

Conditions of unique solution existence was derived using Mittag–Leffler's functions.

Main results Denote Mittag–Leffler's function by

$$E_{\alpha,\beta}(z)=\sum_{n=0}^{\infty}\frac{z^n}{\Gamma(\alpha n+\beta)},\ \alpha,\beta>0.$$

Consider the Cauchy problem

$$z^{(k)}(0)=z_k,\ k=0,1,\dots,m-1, \tag{10}$$

for a quasilinear differential equation

$$z^{(m)}(t)=Az(t)+B(t,z(t),z^{(1)}(t),\dots,z^{(m-1)}(t)),\quad t\in[t_0,t_1], \tag{11}$$

where $m\in\mathbb{N}$, Z is an open set in $\mathbb{R}\times\mathcal{Z}^m$, A is a bounded operator on a Banach space $\mathcal{Z}$, an operator B is nonlinear.

The function $z\in C^m([t_0,t_1];\mathcal{Z})$ is called a solution of the problem (10)–(11) on the interval $[t_0,t_1]$, if it satisfies (10), for $t\in[t_0,t_1]$ $(t,z(t),z^{(1)}(t),\dots,z^{(m-1)}(t))\in Z$ and (11) holds.

Keywords : *high order equation, quasilinear equation, degenerate evolution equation, Cauchy problem, initial boundary value problem.*
2010 Mathematics Subject Classification : **34G20; 47J35, 47J05.**

Theorem 4. *Let $A \in \mathscr{L}(\mathcal{Z})$, $m \in \mathbb{N}$, Z be an open set in $\mathbb{R} \times \mathcal{Z}^m$, an operator $B \in C(Z;\mathcal{Z})$ be locally Lipschitzian with respect to $z = (z_0, z_1, \dots, z_{m-1})$. Then there exists $t_1 > t_0$ for which problem* (10)–(11) *has a unique solution on* $[t_0, t_1]$ *for each* $(t_0, z_0, z_1, \dots, z_{m-1}) \in Z$.

Using Theorem 1 in particular, the following result was obtained.

(L, p)-boundedness of operator M guarantees the existence of projectors P, Q on the spaces $\mathscr{X}$, $\mathscr{Y}$ correspondingly. Then introduce $\mathscr{X}^0 = \ker P$, $\mathscr{Y}^0 = \ker Q$; $\mathscr{X}^1 = \mathrm{im} P$, $\mathscr{Y}^1 = \mathrm{im} Q$ and operators L_k (M_k), which are the restrictions of L (M) to $\mathscr{X}^k$ ($D_{M_k} = D_M \cap \mathscr{X}^k$), $k = 0, 1$, $H = M_0^{-1} L_0$ (see, for example, in [1]).

Theorem 5. *Let $p \in \mathbb{N}_0$, an operator M be (L, p)-bounded, Y be an open set in $\mathbb{R} \times \mathscr{X}^m$, $V = Y \cap (\mathbb{R} \times (\mathscr{X}^1)^m)$ be an open set in $\mathbb{R} \times (\mathscr{X}^1)^m$, for all $(t, y_0, \dots, y_{m-1}) \in Y$ such that $(t, Py_0, \dots, Py_{m-1}) \in Y$ $N(t, y_0, y_1, \dots, y_{m-1}) = N(t, Py_0, Py_1, \dots, Py_{m-1})$ be held, an operator $QN \in C^{\max\{0, m(p+1)-2\}}(V; \mathfrak{Y})$ be locally Lipschitzian with respect to x, $H^k M_0^{-1}(I - Q)N \in C^{mk+m-1}(V; \mathscr{X})$ for $k = 0, 1, \dots, p$, for $(t_0, x_0, \dots, x_{m-1}) \in V$ while $n = 0, 1, \dots, m-1$*

$$(I - P)x_n = -\sum_{k=0}^{p} \frac{d^{mk+n}}{dt^{mk+n}}\Big|_{t=t_0} H^k M_0^{-1}(I - Q)N(t, v(t), v^{(1)}(t), \dots, v^{(m-1)}(t))$$

are held where $v \in C^{\max\{m, m(p+2)-2\}}([t_0, t_1]; \mathscr{X}^1)$ is solution of problem

$$v^{(m)}(t) = S_1 v(t) + L_1^{-1} QN(t, v(t), v^{(1)}(t), \dots, v^{(m-1)}(t)),$$
$$v^{(k)}(t_0) = Px_k, \ k = 0, 1, \dots, m-1.$$

Then the problem (8)–(9) *has a unique solution on* $[t_0, t_1]$.

The Theorem is used to study of the initial boundary value problem for the system of a viscoelastic fluid motion. It will allow to consider optimal control problems for this system [2].

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Relaxation and optimality properties in nonlinear control problems of fractional nonlocal systems

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Abstract In this talk, we discuss essential properties of the optimality and relaxation in nonlinear control problems described by fractional differential equations with nonlocal control conditions in Banach spaces. Moreover, we consider the minimization problem of multi-integral functionals, with integrands that are not convex in the controls, of control systems with mixed nonconvex constraints on the controls. We prove, under appropriate conditions, that the relaxation problem admits optimal solutions. Furthermore, we show that those optimal solutions are in fact limits of minimizing sequences of systems with respect to the trajectory, multi-controls, and the functional in suitable topologies.

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Keywords : *fractional optimal control; relaxation; nonconvex constraints, nonlocal control conditions.*
2010 Mathematics Subject Classification : **26A33; 34B10; 49J15; 49J45.**

Investigation of Degenerate Evolution Equations with Memory Using the Methods of the Theory of Semigroups of Operators

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Abstract We consider the problem with a given story for integro-differential equation in a Banach space, taking into account the effect of the memory. It is reduced to the Cauchy problem for a stationary linear system of equations in a wider space. We found conditions for the unique solvability of the latter in the sense of classical solutions on the semiaxis using the methods of the theory of semigroups of operators.

Introduction Let $\mathfrak{U}$ and $\mathfrak{V}$ are Banach spaces, operator $L \in \mathscr{L}(\mathfrak{U};\mathfrak{V})$ (linear and continuous), operator $M \in \mathscr{C}l(\mathfrak{U};\mathfrak{V})$ (linear, closed and densely defined) has a domain D_M, $\mathbb{R}_+ = \{x \in \mathbb{R} : x > 0\}$), $\overline{\mathbb{R}}_+ = \{0\} \cup \mathbb{R}_+$, $\mathbb{R}_- = \{x \in \mathbb{R} : x < 0\}$, $\overline{\mathbb{R}}_- = \{0\} \cup \mathbb{R}_-$. Consider a problem

$$u(t) = u_-(t), \quad t \in \overline{\mathbb{R}}_-. \tag{12}$$

$$Lu'(t) = Mu(t) + \int_{-\infty}^{t} \mathscr{K}(t-s)u(s)ds + f(t), \quad t \in [0,T), \tag{13}$$

where $u_- \in L_1(\mathbb{R}_-;\mathfrak{U})$, $\mathscr{K} \in W_1^1(\mathbb{R}_+;\mathscr{L}(\mathfrak{U};\mathfrak{V}))$, $f : [0,T) \to \mathfrak{V}$, $T \leq +\infty$. Conditions for unique solvability of degenerated evolution equations with memory is obtained.

Main results Denote by $C_b(\overline{\mathbb{R}}_-;\mathfrak{U})$ the space of continuous and bounded on the negative real axis functions, $\mathfrak{U}^0 = \ker(R_\mu^L(M))^{p+1}$, $\mathfrak{V}^0 = \ker(L_\mu^L(M))^{p+1}$, $\mathfrak{U}^1$ the closure of the image of the operator $\mathrm{im}(R_\mu^L(M))^{p+1}$ in the space $\mathfrak{U}$, $\mathfrak{V}^1$ the closure of $\mathrm{im}(L_\mu^L(M))^{p+1}$ in the space $\mathfrak{V}$. Denote the restriction of the operator L (M) on $\mathfrak{U}^k$ ($\mathrm{dom}M_k = \mathrm{dom}M \cap \mathfrak{U}^k$), $k = 0,1$, by L_k (M_k), $P = s\text{-}\lim\limits_{\mu\to+\infty}(\mu R_\mu^L(M))^{p+1}$, $Q = s\text{-}\lim\limits_{\mu\to+\infty}(\mu L_\mu^L(M))^{p+1}$.

Theorem 6. *[1] Let the operator M be strongly (L,p)-radial. Then*
(i) $\mathfrak{U} = \mathfrak{U}^0 \oplus \mathfrak{U}^1$, $\mathfrak{V} = \mathfrak{V}^0 \oplus \mathfrak{V}^1$;
(ii) $L_k \in \mathscr{L}(\mathfrak{U}^k;\mathfrak{V}^k)$, $M_k \in \mathscr{C}l(\mathfrak{U}^k;\mathfrak{V}^k)$, $k = 0,1$;
(iii) *there exist operators* $M_0^{-1} \in \mathscr{L}(\mathfrak{V}^0;\mathfrak{U}^0)$ *and* $L_1^{-1} \in \mathscr{L}(\mathfrak{V}^1;\mathfrak{U}^1)$;
(iv) $H = M_0^{-1}L_0$ *is a nilpotent operator of a power at most* p.

Keywords : *evolution equation, semigroup of operators, equation with memory, integro-differential equation, boundary value problem.*
2010 Mathematics Subject Classification : **35R09; 34G10; 47D06.**

Theorem 7. *Let an operator M be strongly (L,p)-radial, a function $u_- \in C_b(\overline{\mathbb{R}}_-;\mathfrak{U}) \cap L_1(\mathbb{R}_-;\mathfrak{U})$, $u_-(0) \in D_M$, $\mathcal{K} \in W_1^1(\mathbb{R}_+;\mathcal{L}(\mathfrak{U};\mathfrak{V}))$, $\mathrm{im}\mathcal{K}(s) \subset \mathfrak{U}^1$, $s \geq 0$, $(I-Q)f \in C^{p+1}([0,T);\mathfrak{U})$, $T \leq +\infty$,*

$$(I-P)u_-(0) = -\sum_{k=0}^{p} H^k M_0^{-1}((I-Q)f)^{(k)}(0),$$

and one of the conditions holds:

(i) $Qf \in C^1([0,T);\mathfrak{U})$;

(ii) $L_1^{-1}Qf \in C([0,T);D_M)$, $L_1^{-1}\mathcal{K} \in W_1^1(\mathbb{R}_+;\mathcal{L}(\mathfrak{U};D_M))$.

Then there exists a unique solution of problem (12)–(13) *on* $[0,T)$.

Proof. Act by $M_0^{-1}(I-Q)$ on equation (13), then $\frac{d}{dt}Hu^0(t) = u^0(t) + M_0^{-1}(I-Q)f(t)$. It has a unique solution $u^0(t) = -\sum\limits_{k=0}^{p} H^k h^{(k)}(t)$ for $t \in [0,T)$, where $h(t) = M_0^{-1}(I-Q)f(t)$.

Then act by $L_1^{-1}Q$ on equation (13) and get

$$\frac{d}{dt}u^1(t) = L_1^{-1}M_1 u^1(t) + \int_{-\infty}^{t} L_1^{-1}\mathcal{K}(t-s)u^1(s)ds + g(t),$$

where

$$g(t) = \int_{-\infty}^{t} L_1^{-1}\mathcal{K}(t-s)u^0(s)ds + L_1^{-1}Qf(t).$$

Hense, problem (12)–(13) is reduced to the problem $u^1(t) = Pu_-(t)$, $t \in \overline{\mathbb{R}}_-$, for nondegenerate integro-differential equation. □

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Finite element method and Two-Grid method applied for solving electro-visco-elastic contact problem

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Abstract Purpose: In this work we study a mathematical model which describes the antiplane shear deformation of a cylinder in frictionless contact with a rigid foundation. The material is assumed to be electro-viscoelastic with long-term memory, and the friction is modeled with Tresca's law and the foundation is assumed to be electrically conductive. In this new work :

- We derive the classical variational formulation of the model which is given by a system coupling an evolutionary variational equality for the displacement field and a time-dependent variational equation for the potential field,
- We prove the existence of a unique weak solution to the model,
- We use the **Finite Element Method and Two-Grid Method** for the continuous Problem,
- We prove the existence of a unique weak discret solution to the model.

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Keywords : *Tresca's friction, Antiplane shears deformation, Electro-viscoelastic material, Variational inequality, Weak solution, Fixed point, Finite Element Method, Two-Grid Method, Discret Solution.*

2010 Mathematics Subject Classification : **74M10, 74F15, 74G25, 49J40.**

Regularization method for an abstract ill-posed bibarabolic problem

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Abstract In this paper, we are concerned with the problem of approximating a solution of an ill-posed biparabolic problem in the abstract setting. In order to overcome the stability of the original problem, we propose a regularizing strategy based on the Quasi-reversibility method. Finally, some other convergence results including some explicit convergence rates are also established under a priori bound assumption on the exact solution.

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Keywords : *Ill-posed problems; biparabolic problem; quasi-reversibility; regularization.*

2010 Mathematics Subject Classification : Primary 47A52; secondary 65J22.

Complete controllability of nonlocal fractional stochastic differential evolution equations with Poisson jumps

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Abstract The objective of this paper is to investigate the complete controllability property of a nonlinear nonlocal fractional stochastic control system with poisson jumps in a separable Hilbert space. We use fixed point technique, fractional calculus, stochastic analysis and methods adopted directly from deterministic control problems for the main results. In particular, we discuss the complete controllability of nonlinear nonlocal control system under the assumption that the corresponding linear system is completely controllable. Finally, an example is provided to illustrate the effectiveness of the obtained result.

Introduction In this paper, we consider a mathematical model given by the following fractional nonlocal stochastic differential equations with poisson jumps and control variable,

$$ {}^{C}D_t^q x(t) = Ax(t) + Bu(t) + f(t, x(t)) + \sigma(t, x(t)) \frac{dw(t)}{dt} + \int_Z g\left(t, x(t), \eta\right) \widetilde{N}\left(dt, d\eta\right), \ t \in J, \tag{14} $$

$$ x(0) + h(x(t)) = x_0, \tag{15} $$

where $0 < q < 1; {}^{C}D_t^q$ denotes the Caputo fractional derivative operator of order q, $x(\cdot)$ takes its values in the Hilbert space $X, J = [0, b]$; A is the infinitesimal generator of a compact semigroup of uniformly bounded linear operators $\{S(t), t \geq 0\}$; the control function $u(\cdot)$ is given in $L^2_\Gamma([0, b], U)$ of admissible control functions, U is a Hilbert space. B is a bounded linear operator from U into X; $f : J \times X \to X$, $g : J \times X \times Z \to X$, $\sigma : J \times X \to L_2^0$ and $h : C(J, X) \to X$ are appropriate functions; x_0 is Γ_0 measurable X-valued random variables independent of w.

Main results In this paragraph, we shall formulate and prove conditions for the controllability of fractional stochastic equations (14)–(15) by using a fixed point approach. To prove the required results, we impose some Lipschitz and linear growth conditions on the functions f, σ, g and h.

(i) For any fixed $t \geq 0, \mathscr{T}(t)$ and $\mathscr{S}(t)$ are bounded linear operators, i.e., for any $x \in X$,

$$ ||\mathscr{T}(t)x|| \leq M||x||, \quad ||\mathscr{S}(t)x|| \leq \frac{Mq}{\Gamma(q+1)}||x||. $$

(ii) The functions f, σ and g are Borel measurable functions and satisfy the Lipschitz continuity condition and the linear growth condition for some constant $k > 0$ and arbitrary $x, y \in X$ such that

Keywords : *Fractional stochastic differential equations; complete controllability; mild solution; jump process; fractional delay equations.*
2010 Mathematics Subject Classification : **34G25; 93B05.**

$$\begin{aligned} & \|f(t,x)-f(t,y)\|_X^2+\|\sigma(t,x)-\sigma(t,y)\|_{L_2^0}^2+\int_Z \|g(t,x,\eta)-g(t,y,\eta)\|_X^2\,\lambda d\eta \\ \leq\ & k\|x-y\|_X^2, \end{aligned}$$

$$\|f(t,x)\|_X^2+\|\sigma(t,x)\|_{L_2^0}^2+\int_Z \|g(t,x,\eta)\|_X^2\,\lambda(d\eta)\leq k(1+\|x\|_X^2).$$

(iii) There exists a number $\widetilde{N_0}>0$ and arbitrary $x,y\in X$ such that

$$\|g(x)-g(y)\|_X^2\leq \widetilde{N_0}\|x-y\|_X^2,\quad \|g(x)\|_X^2\leq \widetilde{N_0}\|x\|_X^2$$

(iv) The linear stochastic system is completely controllable on J.
(v) There exists a number $\widetilde{L_0}>0$ such that for arbitrary $x_1,x_2\in X$,

$$\int_Z \|g(t,x_1,\eta)-g(t,x_2,\eta)\|_X^4\,\lambda(d\eta)\leq \widetilde{L_0}\left(\|x_1-x_2\|_X^4\right),$$

$$\int_Z \|g(t,x,\eta)\|_X^4\,\lambda(d\eta)\leq \widetilde{L_0}\left(1+\|x\|_X^4\right).$$

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Degenerate impulsive fractional dynamic inclusions with nonlocal control conditions

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Abstract We introduce the notion of degenerate evolution inclusion for a class of impulsive fractional nonlinear system with nonlocal control conditions in Banach spaces. We realize multivalued maps, fractional calculus, fixed point technique and control theory for the main results. The new results required to formulate and prove an appropriate set of sufficient conditions. Finally, an example is also given to illustrate the obtained theory.

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Keywords : *Degenerate evolution inclusion; Multivalued maps, fractional derivative; Semigroup theory, control theory.*

2010 Mathematics Subject Classification : **35K65; 26A33; 34G25; 47N70.**

Stability in nonlinear delay fractional differential equations

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Abstract In this paper, we present some results for the stability of solutions for nonlinear delay fractional differential equations. The results are obtained by using Krasnoselskii's fixed point theorem in a weighted Banach space. Our result is illustrated by an example.

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Keywords : *Krasnoselskii's fixed point theorem; stability; delay fractional differential equations.*
2010 Mathematics Subject Classification : **26A33; 34K20; 45J05; 45D05.**

Complete controllability of stochastic systems with impulsive effects in Hilbert space

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Abstract We introduce a nonlocal control condition and the notion of approximate controllability for fractional order quasilinear control inclusions. Approximate controllability of a fractional control nonlocal delay quasilinear functional differential inclusion in a Hilbert space is studied. The results are obtained by using the fractional power of operators, multi-valued analysis, and Sadovskii's fixed point theorem. Main result gives an appropriate set of sufficient conditions for the considered system to be approximately controllable. As an example, a fractional partial nonlocal control functional differential inclusion is considered.

Introduction In this paper, we study the complete controllability of the system of the form

$$\begin{cases} dx(t) = Ax(t)dt + Bu(t)dt \\ \qquad +F_1\left(t, x(t), f_{1,1}(\eta x(t)), f_{1,2}(\delta x(t))\right)dt \\ \qquad +F_2\left(t, x(t), f_{2,1}(\eta x(t)), f_{2,2}(\delta x(t))\right)dw(t), \quad t \neq t_k, \quad t \in J, \\ \Delta x(t_k) = x(t_k^+) - x(t_k^-) = I_k(x(t_k^-)), \quad k = 1, 2, \ldots, r \\ x(0) = x_0 \in H. \end{cases} \tag{16}$$

in a real separable Hilbert space H, where, for $i = \overline{1,2}$:

$$f_{i,1}(\eta x(t)) = \int_0^t f_{i,1}(t, s, x(s))ds$$
$$f_{i,2}(\delta x(t)) = \int_0^T f_{i,2}(t, s, x(s))ds$$

Here, A is the infinitesimal generator of strongly continuous semigroup of bounded linear operators $\{S(t),\ t \geq 0\}$ in H, B is bounded linear operator from U into H.

$$\begin{array}{l} F_1 : J \times H \times H \times H \times H \to H \\ F_2 : J \times H \times H \times H \times H \to \mathscr{L}_2(Q^{\frac{1}{2}}E; H) \quad , \\ f_{i,1}, f_{i,2} : J \times J \times H \to H \end{array}$$

$I_k : H \to H$, $u(.) \in \mathscr{U}_2$. Furthermore, the fixed times t_k satisfies , $0 = t_0 < t_1 < \ldots < t_r < T$, $x(t_k^+)$ and $x(t_k^-)$ represent the right and left limits of $x(t)$ at $t = t_k$. And $\Delta x(t_k) = x(t_k^+) - x(t_k^-)$ represents the jump in the state x at time t_k, where I_k determines the size of the jump.The initial value x_0 is $\mathscr{F}_0$-measurable H-valued square-integrable random variable independent of w.

For the proof of the the main result, we impose the following conditions on data of the problem.

(H1) $S(t)$, $t \geq 0$, is a strongly continuous semigroup of bounded linear operators generated by A such that $\max_{0 \leq t \leq T} \|S(t)\|^2 \leq l_1$, for some constant $l_1 > 0$.

Keywords : *Complete controllability; Fixed points; Semigroup theory; Stochastic neutral impulsive systems.*
2010 Mathematics Subject Classification : **93E03, 37C25, 60H10**

(H2) The functions $F_i, f_{i,j}, \ i,j=1,2$ are Borel measurable functions and satisfies the Lipschitz condition:
there exist constants $L_1, N_1, K_1, C_1, q_k > 0$ for $x_h, y_h, v_h, z_h \in H,\ h=1,2$ and $0 \le s \le t \le T$

$$\begin{aligned}
&\left\|F_1(t,x_1,y_1,v_1,z_1) - F_1(t,x_2,y_2,v_2,z_2)\right\|^2 + \left\|F_2(t,x_1,y_1,v_1,z_1) - F_2(t,x_2,y_2,v_2,z_2)\right\|^2_{\mathcal{L}_2^0} \\
&\le L_1\left(\|x_1-x_2\|^2 + \left\|y_1-y_2\right\|^2 + \|v_1-v_2\|^2 + \|z_1-z_2\|^2\right) \\
&\left\|f_{i,j}(t,s,x_1(s)) - f_{i,j}(t,s,x_2(s))\right\|^2 \le K_1\|x_1-x_2\|^2. \\
&\left\|I_k(x) - I_k(y)\right\|^2 \le q_k\left\|x-y\right\|^2, \quad k \in \{1,\dots,r\}
\end{aligned}$$

(H3) The functions $F_i, f_{i,j}, \ i,j=1,2$ are continuous and there exist constants $L_2, N_2, K_2, C_2, d_k > 0$ for $x,y,v,z \in X$ and $0 \le t \le T$

$$\begin{aligned}
&\left\|F_1(t,x,y,v,z)\right\|^2 + \left\|F_2(t,x,y,v,z)\right\|^2_{\mathcal{L}_2^0} \le L_2\left(1 + \|x\|^2 + \left\|y\right\|^2 + \|v\|^2 + \|z\|^2\right) \\
&\left\|f_{i,j}(t,s,x(s))\right\|^2 \le K_2\left(1+\|x\|^2\right). \\
&\|I_k(x)\|^2 \le d_k\left(1+\|x\|^2\right), \quad k \in \{1,\dots,r\}
\end{aligned}$$

(HC) The linear system of (16) is completely controllable on J.

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Solution of First Order Linear Difference Equations with Uncertainty

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Abstract We study the different formulations of first order linear difference equations with uncertain initial value. We obtain the explicit expressions of the solutions for three in-equivalent difference equations while they are equivalent in the classical case.

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Keywords : *Difference equations; First order; Hukuhara difference.*

2010 Mathematics Subject Classification : **26A33; 34A60; 34G25; 93B05.**

Solitary Wave Solutions for Systems of Non-linear Partial Differential Equations

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Abstract In this paper, the tanh function method and Sech method are proposed to establish a traveling wave solution for nonlinear partial differential equations. The two methods are used to obtain new solitary wave solutions for three systems of nonlinear wave equations, namely, two component evolutionary system of a homogeneous KdV equations of order 3 (type I) as well as (type II), and the generalized coupled Hirota Satsuma KdV. Methods have been successfully implemented to establish new solitary wave solutions for the nonlinear PDEs.

Introduction This paper is motivated by the desire to find periodic wave solutions with the use of the Tanh and Sech functions. This means that the method will lead to a deeper and more comprehensive understanding of the complex structure of the nonlinear partial differential equations (NPDEs). On the one hand, to seek more formal solutions of NPDEs is needed from mathematical point of view.

Three systems of nonlinear wave equations were studied in this paper. These systems can be seen. In mathematical physics, they play a major role in various fields, such as plasma physics, fluid mechanics, optical fibers, solid state physics, geochemistry, and so on. Two systems are called component evolutionary systems of homogeneous KdV equations of order 3 (type I), (type II) respectively given by:

$$u_t - u_{xxx} - uu_x - vv_x = 0; \quad v_t + 2v_{xxx} + uv_x = 0 \tag{17}$$

Subject to initial condition:

$$u(x,0) = 3 - 6\,\tanh^2\left(\frac{x}{2}\right); \quad v(x,0) = -3i\,\sqrt{2}\,\tanh^2\left(\frac{x}{2}\right) \tag{18}$$

and

$$u_t - u_{xxx} - 2v\,u_x - v\,u_x = 0; \quad v_t - 3u\,ux = 0 \tag{19}$$

Subject to initial condition:

$$u(x,0) = -\tanh\left(\frac{x}{\sqrt{3}}\right); \quad u(x,0) = -\frac{1}{6} - \frac{1}{2}\,\tanh^2\left(\frac{x}{\sqrt{3}}\right) \tag{20}$$

The other generalized coupled Hirota Satsuma KdV system is given by

$$u_t - \frac{1}{2}u_{xxx} + 3v\,u_x - 3v_x\,w_x = 0; \quad v_t + v_{xxx} - 3u\,vx = 0; \quad w_x + w_{xxx} - 3u\,w_x = 0 \tag{21}$$

Keywords : *Nonlinear PDEs; Tanh function method; Sech function method; generalized coupled Hirota Satsuma KdV.*
2010 Mathematics Subject Classification : **74J35; 76B25; 49K20.**

With the initial condition:

$$u(x,0) = -\frac{1}{3} + 2\tanh^2(x); \quad v(x,0) = \tanh(x); \quad w(x,0) = \frac{8}{3}\tanh(x) \tag{22}$$

The generalized coupled Hirota Satsuma KdV system investigated by many autheurs using different methods such as the differential transform method, and trig-hyperbolic function method.

The modern methods of integrability Tan and Sech function methods will be applied to integrate these systems of equations. Methods will reveal solutions that will be useful in the literature of NLEEs.

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A feasible-interior point method for linear semi-definite programming problems

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Abstract In this work, we present a feasible primal-dual algorithm for linear semidefinite programming (SDP) based on the dirparagraph of Alizadeh, Haeberly and Overton (AHO). We present a different alternatives for calculate a step of displacement. In the first time, and by a new and simple manner we establish the existence and uniqueness of the optimal solution of the perturbed problem (SDP)$_\mu$ and its convergence to the optimal solution of (SDP). After, based on the dirparagraph of (AHO), we show that the perturbed objective function of (SDP)$_\mu$ decreases at each iteration which guaranteed the convergence of the algorithm. Finally, we present some numerical simulations which show the effectiveness of the algorithm developed in this work.

Introduction we present a feasible primal-dual algorithm for linear semidefinite programming (SDP) definite by

$$(SDP)\begin{cases} \min[\langle C, X\rangle = \quad tr(CX) \quad = \sum\limits_{i,j=1}^{n} C_{ij}X_{ij}] \\ \quad \mathscr{A}X = b \\ \quad X \in S_{+}^{n}, \end{cases} \tag{23}$$

where $b \in \mathbb{R}^m$, S_+^n designates the cone of the positive semidefinite matrix on the linear space of $(n \times n)$ symmetrical matrix S^n.

$\mathscr{A}$ is a linear operator of S^n in $\mathbb{R}^m$ defined by

$$\mathscr{A}X = (\langle A_1, X\rangle, \langle A_2, X\rangle, ..., \langle A_m, X\rangle)^t. \tag{24}$$

The matrices C and A_i, $i = 1, ..., m$, are in S^n. The scalar product of two matrices A and B in S^n is the trace of their product, i.e., $\langle A, B\rangle = tr(AB) = \sum\limits_{i,j=1}^{n} a_{ij}b_{ij}$. The Frobenius norm of a matrix $M \in S^n$ is given by $\|M\|_F = (\langle M, M\rangle)^{\frac{1}{2}}$. It is known that the inside of S_+^n noted by S_{++}^n, designate the set of the positive definite matrices of S^n.

This feasible primal-dual algorithm based on the dirparagraph of Alizadeh, Haeberly and Overton (AHO).
To study the problem (SDP), we replace it by the perturbed equivalent problem

$$(SDP)_\mu \begin{cases} \min[f_\mu(X) = \quad \langle C, X\rangle + \mu g(X) + n\mu \ln \mu] \quad , \mu > 0 \\ \mathscr{A}X = b, \end{cases} \tag{25}$$

where

$$g(X) = \begin{cases} -\ln(\det X) & \text{if } X \in S_{++}^n \\ +\infty & \text{otherwise.} \end{cases} \tag{26}$$

Keywords : *Linear Semidefinite Programming; Primal-Dual Interior Point Methods; Central Trajectory Methods.*
2010 Mathematics Subject Classification : **90C22; 90C51; 90C05.**

Main results

- **Lemma 8.** *Let* X_μ *an optimal primal solution of the problem* $(SDP)_\mu$*, then* $X = \lim_{\mu\to 0} X_\mu$ *is an optimal solution of* (SDP).
- The new iterate is written as follows

$$X^+ = X + \alpha\Delta X,\ y^+ = y + \alpha\Delta y,\ S^+ = S + \alpha\Delta S,$$

where α is the displacement step chosen such that $X^+, S^+ \in S^n_{++}$.
The computation of the displacement step by the classical line search methods is undesirable, and in general impossible. Because it require the computation of the eigenvalues in every iteration. In this sense, J. P. Crouzeix and B. Merikhi [2], have used the notions of the non expensive majorant functions for the dual problem of (SDP) that offers a displacements step in a simple manner. We based on results of definite positivity in linear algebra (corollary 1 Cf. Wolkowicz Styan [[3], Theorem 2.1]), we propose four new different alternatives that offers some variable steps of displacement to every iteration. The efficient of one to the other can be translated by numerical tests.

- **Lemma 9.** *Let* X^+ *and* X *be two solutions strictly feasible of* $(SDP)_\mu$ *with* $X^+ = X + \alpha\Delta X$*, where* α *is the displacement step, strictly positive, and* ΔX *is the Newton dirparagraph, Then we have:* $f_\mu(X^+) < f_\mu(X)$.

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Molecular modeling studies of the inclusion complex of Dihydroisoquinoline sulfonamide with β-cyclodextrin

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Abstract Cyclodextrin (CD) chemistry has caused much interest, not only due to its applications in pharmaceutical science and technology but also because the inclusion represents an ideal model mimicking enzyme- substrate interactions. Furthermore, The resultant inclusion complexes can induce modification of the physicochemical properties of the 'guest' molecules, particularly in terms of water solubility and solution stability. The sulfonamide group is considered as a pharmacophore, which is present in number of biologically active molecules, particularly antimicrobial agents. Hence, this work is designed to investigate the complexation of sulfonamide/β-CD through molecular modeling, using semi-empirical PM3 and PM6 methods, quantum hybrid ONIOM2, in vacuum and in water. Two different docking orientations were considered. The energetically more favorable structure obtained with the ONIOM2 method leads to the formation of intermolecular hydrogen bonds between sulfonamide and β-cyclodextrin. These interactions were investigated using the Natural Bond Orbital (NBO). Moreover, the statistical thermodynamic calculations at 1 atm and 298.15K showed that the inclusion reaction is an exothermic process.

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Keywords : *3,4-Dihydroisoquinoline-2(1H)-sulfonamide; β-Cyclodextrin; Inclusion complex; PM6.*
2010 Mathematics Subject Classification : **92C40; 92E10.**

Classification of travelling wave solutions of the fifth-order KdV equations and its stability

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Abstract In the present study, by implementing the direct algebraic method, we present the traveling wave solutions for some different kinds of the Korteweg-de Vries (KdV) equations. The exact solutions of the Kawahara, fifth order KdV and generalized fifth order KdV equations are obtained. Solutions for the Kawahara, fifth order KdV and generalized fifth order KdV equations are obtained precisely and efficiency of the method can be demonstrated. The stability of these solutions and the movement role of the waves by making the graphs of the exact solutions are analyzed. All solutions are exact and stable, and have applications in physics.

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Keywords : *Extended direct algebraic method; Traveling wave solutions; Generalized fifth order KdV equations.*

2010 Mathematics Subject Classification : **35Qxx; 35L05; 35Q53.**

Synthesis and evaluation of anti-cancer properties of new derivatives Quinolin type

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Abstract In this talk, we synthesize and evaluate some anti-cancer properties of a new kind of derivatives Quinolin. We discuss basic chemical reactivity of these new polycyclic aromatic analogs to generate alkylating species led to the hypothesis that the presence of an imine moiety impedes the formation of quinone methide intermediate and consequently abolishes in part their anti-proliferative activity.

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Keywords : *8-hydroxyquinoline; antiproliferative activity; quinone methide*
2010 Mathematics Subject Classification : **43A45; 93B50.**

Statistical analysis of surface roughness in hard turning with AL2O3 and TiC mixed ceramic tools

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Abstract In this study, the effects of cutting speed, feed rate and depth of cut on surface roughness in the hard turning were experimentally investigated. AISI 4140 steel was hardened to (56 HRC). The cutting tool used was an uncoated AL2O3/TiC mixed ceramics which is approximately composed of 70% of AL2O3 and 30% of TiC. Three factor (cutting speed, feed rate and depth of cut) and three-level factorial experiment designs completed with a statistical analysis of variance (ANOVA) were performed. Mathematical model for surface roughness was developed using the response surface methodology (RSM) associated with response optimization technique and composite desirability was used to find optimum values of machining parameters with respect to objectives surface roughness. The results have revealed that the effect of feed is more pronounced than the effects of cutting speed and depth of cut, on the surface roughness. However, a higher cutting speed improves the surface finish. In addition, a good agreement between the predicted and measured surface roughness was observed. Therefore, the developed model can be effectively used to predict the surface roughness on the machining of AISI 4140 steel within 95% confidence intervals ranges of conditions studied.

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Keywords : *Statistical analysis; mixed ceramic tools; cutting speed; feed rate; depth of cut.*
2010 Mathematics Subject Classification : **37A60; 13C15.**

A molecular modelling and spectroscopic study on the interaction between cyclomaltoheptaose and a kind of Benzoxazolinone

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Abstract Cyclic oligosaccharides are attractive design elements. Their most desirable properties are their low toxicity, their lack immunogenicity and their ability to form (Host-Guest) complexes. In pharmacy, one of the most interesting results of this property is the increase in apparent solubility of the included molecules resulting in an increase of their bioavailability. On the other hand, 2-Benzoxazolinone and their derivatives are compounds with an important role in the design of pharmacological probes, having a wide range of therapeutic applications due to the analgesic , anti-inflammatory, antipsychotic, neuroprotective and anticonvulsant activities. In present work, the interactions between 3-methyl-2-benzoxazolinone and cyclomaltoheptaose have been investigated by means of molecular modeling, FTIR and UV-vis spectroscopy methods. The computed results using semi-empirical calculations (PM6, AM1, PM3, and ONIOM2 methods), in vacuum, water and acetonitril, imposing a 1:1 stoichiometry were performed. The negative values of all the thermodynamic parameters indicated that the formation of the inclusion complexe is an enthalpy-driven process in which a crucial role is played by weak van der Waals forces and hydrogen bonds. Moreover, molecular modelling results are in better agreements with experimental observations.

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Keywords : *Cyclic oligosaccharides; Benzoxazolinone; inclusion complexes; molecular modeling.*
2010 Mathematics Subject Classification : **92C40; 92E10.**

On quasi-periodicity properties of fractional integrals and fractional derivatives of periodic functions

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Abstract We study quasi-periodic properties of fractional order integrals and derivatives of periodic functions. Considering Riemann-Liouville and Caputo definitions, we discuss when the fractional derivative and when the fractional integral of a certain class of periodic functions satisfies particular properties. We study concepts close to the well known idea of periodic function, such as S-asymptotically periodic, asymptotically periodic or almost periodic function. Boundedness of fractional derivative and fractional integral of a periodic function is also studied.

Introduction Periodic functions play a central role in mathematics since the seminal works of J. B. Fourier. Indeed, the study of existence of periodic solutions is one of the most interesting and important topics in qualitative theory of differential equations. This is due to its implications in pure or abstract areas of mathematics, but also due to its applications, ranging from physics to natural and social sciences and of course, in control theory.

However, the definition of periodic function is extremely demanding and then, the conditions to guarantee the existence of periodic solutions are very harsh. For this reason, in the past decades, many autheurs (see [1, 2] and references therein) have proposed and studied extensions of the concept of periodicity which have shown interesting and useful.

It is an obvious fact that the classical derivative, if it exists, of a periodic function is also a periodic function of the same period. Also the primitive of a periodic function may be periodic (e.g., $-\cos t$ as primitive of $\sin t$). Nevertheless, when we consider derivatives or integrals of non integer order, this fact is not true [3].

Our aim is to study quasi-periodic properties of fractional order integrals and derivatives of periodic functions. Using Riemann-Liouville and Caputo definitions, we take into consideration concepts as asymptotically periodic function, S-asymptotically periodic function or almost periodic function.

Main results In this work, we consider a T-periodic function f for some $T > 0$ and $0 < \alpha < 1$.

We begin recalling that if $I^\alpha f$, where I^α denotes the Riemann–Liouville fractional integral of order α, is a bounded function then $I^\alpha f$ is an S-asymptotically T-periodic function. Similar results can be deduced for Riemann-Liouville or Caputo fractional derivative of f.

Next, we study the concept of asymptotically periodic function. We prove the following fact.

Theorem 10. *If $I^\alpha f$ is a bounded function, then $I^\alpha f$ is an asymptotically periodic function.*

We also obtain similar results for Caputo and Riemann-Liouville fractional derivatives of f.

By virtue of the foregoing, it seems fundamentala to know when the fractional integral of order $0 < \alpha < 1$ of a periodic function is bounded. So, we prove the following theorem.

Keywords : *fractional calculus; periodic function; S-asymptotically periodic function; asymptotically periodic function; almost periodic function.*
2010 Mathematics Subject Classification : **26A33; 34A08.**

Theorem 11. *The fractional integral of order* $0 < \alpha < 1$ *of a* T*-periodic function is bounded if and only if*

$$\int_0^T f(t)\,dt = 0.$$

We continue studying the idea of almost periodic function and its role in fractional calculus. We prove that the fractional integral of a periodic function it is not an almost periodic function.

Finally, we conclude making some extra remaks about our previous results.

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Combination of a few methods of supervised classification by remote sensing for soil occupation mapping from Landsat images

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Abstract Supervised and unsupervised classification of satellite images have shown much progress in recent years. They still have discrimination difficulties between some classes by major confusions that directly affects the extraction and mapping of these surface states.
For this purpose an approach based on the fusion of four methods of supervised classifications (Support Vector Machine, Maximum Likelihood, Neural Net and Spectral Angle Mapper). The results obtained are therefore considered better for the discrimination between agglomeration, wet soils etc... Our approach aims to combine the results of the four types of classification by exploiting their complementarity. It takes into account the appearance of uncertainty and imprecision. After identifying the different stages of our classification scheme we show the fusion of classifiers. The results have shown the effectiveness of the proposed approach.

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Keywords : *confusion matrix; fusion; extraction; vectorization; maps.*
2010 Mathematics Subject Classification : **37E05; 47L25.**

Statistical methods of multilayer perceptron with applications in Medicine

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Abstract The purpose of this study allows providing the immense horizons which can be combined with statistics by networks ways of neurons.
Moreover, new approach motivated by old methods is developed within the traditional framework. Under the mode of supervised training, we managed to see how to reproduce a certain number of traditional techniques. Specifically, with the medical case study, we show that a simple perceptron and a linear classifier are equivalent to the linear discriminating analysis in the statistics.
In addition, appearance of new techniques allowing an extension of the known techniques in order to answer problems found difficult. For example, the case of the multilayer perceptron (MLP) which deal with nonlinear problems whatever its complexity. Further, the problems of nonlinear discrimination have solutions by applying network kinds.

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Keywords : *Artificial neural networks; supervised learning; perceptron; multilayer perceptron (MLP).*
2010 Mathematics Subject Classification : **92B20; 60J20.**

Novel selective mutation operator for tree-based Evolutionary Algorithm

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Abstract Evolutionary computation became known when several computer scientists tried to use the evolution for optimization problems. The idea was to evolve a population of candidate solutions to a given problem, using operators inspired by natural genetic variation and natural selparagraph. In this paper we propose a new mutation operator which does not act like the standard operator in which a random sub-tree is replaced with a new random tree as described by Koza [1]. In our approach, mutation is performed by substituting the worst sub-tree with a new random tree . The two operators will be compared within genetic programs for symbolic regression problem with different target functions to observe, when used with the standard crossover [2], how the algorithm converges toward the solution. Satisfactory results have been found and experimental evidence shows that selective sub-tree mutation requires less computational CPU time than standard sub-tree mutation.

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Keywords : *Selective mutation operator; Evolutionary Algorithm, optimization problem.*
2010 Mathematics Subject Classification : **03D32; 37N40.**

Modelling of the transport of pollutant in a groundwater aquifer

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Abstract The transport of pollutants and their evolution in the ground are determined by the hydrodynamic and hydrochemical behavior of the aquifer. Studies on the mathematical models and on the ground highlighted a dispersal upright of the source of contamination, then a side spreading in the sense of the flow of the groundwater. The digital model based on the method of the finished elements or, that of the finished differences, are very powerful in theory as regards the possibilities of representation of the real systems and the conditions in the limits which can be applied there. Our choice concerned to "Visual Modflow", who is a three-dimensional model of hydrodynamic simulation in continuous aquifer environment by the method of the finished differences. This software presents a modular structure (called also Pack, and consists in dividing the calculations into several parts, and relating between them, who allows him to adapt himself as good as possible to the problems met in the representation of the natural hydrogeological systems. It is the model the most used in the world. It is used to the construction of hydrodynamic simulation to estimate speeds and trajectories of groundwater and transport of pollutants. A case of pollution by the Chromium was observed in the low plain of Seybouse (Annaba, Algeria). Runner of an initial state (in February, 1999), when the concentrations are located around of a Steel factory (source of the pollution). The latter decrease as we go away from that this. After one year, we observe an extension of the affected zone on a surface of 8,4 km2. The chromium propagates laterally in the dirparagraph of flow of groundwater of the superficial water table and converges from borders to the center. A remarkable increase of the concentrations of the chromium after six years, Chromium can cover a surface superior to 12 km2. A long-term simulation (20 years) of the distribution of the Chromium show a spreading of the concentrations on a surface superior to 15,5 km2.

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Keywords : *Transport; pollutants; modeling; chromium.*
2010 Mathematics Subject Classification : **76F25; 00A71.**

Singular linear systems of fractional nabla difference equations

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Abstract In this talk we will study the initial value problem of a class of non-homogeneous singular linear systems of fractional nabla difference equations whose coefficients are constant matrices. By taking into consideration the cases that the matrices are square with the leading coefficient singular, non-square and square with a matrix pencil which has an identically zero determinant, we will provide necessary and sufficient conditions for the existence and uniqueness of solutions. More analytically we will study the conditions under which the system has unique, infinite and no solutions. For the case of uniqueness we will derive a formula that provides the unique solution and for the other cases we will provide optimal solutions. Finally, we study the Kalman filter for singular non-homogeneous linear control systems of fractional nabla difference equations. Numerical examples will be given to justify our theory.

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Keywords : *singular systems, fractional calculus, nabla operator, difference equations, linear, discrete time system.*

2010 Mathematics Subject Classification : 37N35; 37N40; 65F05.

Numerical modeling of natural convparagraph in rectangular inclined enclosures: influence of the aspect ratio and angles of inclination

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Abstract A numerical study on Natural convparagraph in two-dimensional rectangular inclined air-filled enclosures at various moderate aspect ratios has been performed by ADI finite difference scheme. The study covered a range of Rayleigh numbers between 10^3 and 5×10^5, the angle of inclination, measured from the horizontal, was varied between 0° and 180°, while the aspect ratio (height/width) was varied between 1 and 10. The Prandtl numbers were kept constant (air: Pr = 0.71). Nusselt number relations are derived in terms of the Rayleigh number the angle of inclination and the cavity aspect ratio. The results are in very good agreement with previous works. The effects of the Rayleigh number, the enclosure orientation and aspect ratio on the flow development and heat transfer across the cavity are studied.

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Keywords : *Natural convparagraph; heat transfer; cavity; aspect ratios; inclination; ADI finite difference scheme.*
2010 Mathematics Subject Classification : **76E06; 35Q79.**

Existence Results for Stochastic Impulsive Semilinear Neutral Functional integro-differential inclusions driven by a fractional Brownian motion

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Abstract In this paper, we prove the existence of mild solutions for impulsive semilinear stochastic neutral functional integro-differential inclusions driven by fractional Brownian motion with the Hurst index $H > \frac{1}{2}$ in a Hilbert space is studied . The results are obtained by using a fixed point theorem for a condensing map due to Martelli. The result is illustrated with an example.

Introduction In this paper, we study the existence of solutions for initial value problems for first order impulsive neutral stochastic functional integro-differential inclusions. More precisely of the form

$$\begin{cases} d[y(t)-g(t,y_t)] \in & A[y(t)-g(t,y_t)]dt + \int_0^t B(t-s)[y(s)-g(s,y_s)]dsdt & \\ & +F(t,y_t)dt + \sigma(t)dB_Q^H(t), & t \in J=[0,T], \quad t \neq t_k, \\ y(t_i^+)-y(t_i^-) = & I_i(y(t_i^-)), \ i=1,\dots,m, & \\ y(t) = \phi(t) & . & -r \leq t \leq 0 \end{cases} \tag{27}$$

in a real separable Hilbert space $\mathscr{H}$ with inner product $(\cdot,\cdot)$ and norm $\|\cdot\|$, where A is the infinitesimal generator of a strongly continuous semigroup $(S(t))_{t\geq 0}$ on a Hilbert space $\mathscr{H}$ with domain $D(A)$, $B(t)$ is a closed linear operator on $\mathscr{H}$ with domain $D(B) \supset D(A)$ which is independent of t, B_Q^H is a fractional Brownian motion on a real and separable Hilbert space $\mathscr{K}$, with Hurst parameter $H \in (1/2,1)$, and with respect to a complete probability space $(\Omega,\mathscr{F},\mathscr{F}_t,P)$ furnished with a family of right continuous and increasing σ-algebras $\{\mathscr{F}_t, t\in J\}$ satisfying $\mathscr{F}_t \subset \mathscr{F}$. Also $r>0$ is the maximum delay, and the impulse times t_k satisfy $0=t_0<t_1<t_2<\dots,t_m<T$.

Before describe the properties fulfilled by the operators F, g, σ and I_k, we need to introduce some notation and describe some spaces.

Let $\mathscr{D}$ the following Banach space defined by

$$\begin{aligned}\mathscr{D} = \{\phi\colon\ [-r,0]\to\mathscr{H}\colon\ &\phi \text{ is continuous everywhere except for a finite number} \\ &\text{of points } t \text{ at which } \psi(t^-) \text{ and } \phi(t^+) \text{ exist and satisfy } \phi(t^-)=\phi(t)\},\end{aligned}$$

endowed with the L^2−norm:

$$\|\phi\|_{\mathscr{D}}^2 = \int_{-r}^0 \|\phi(t)\|^2 dt.$$

Keywords : *mild solutions, stochastic impulsive neutral semilinear functional differential inclusions , fractional Brownian motion, fixed point.*
2010 Mathematics Subject Classification : **34A60,60H10,60H20.**

Also, whenever a problem of measurability is concern, we will equip $\mathscr{D}$ with the $\sigma-$algebra of its Borel sets. We dented by $\mathscr{D}_{\mathscr{F}_0}$ the space of all piecewise continuous processes $\psi : [-r,0]\times\Omega\to H$ such that $\psi(\theta,.)$ is $\mathscr{F}_0-$ measurable for each $\theta\in[-r,0]$ and

$$\sup_{\theta\in[-r,0]} E|\phi(\theta)|^2<\infty.$$

In the space $\mathscr{D}^0_{\mathscr{H}}$, we consider the norm:

$$\|\phi\|^2_{\mathscr{D}_{\mathscr{F}_0}}=\int_{-r}^0 E\|\phi(t)\|^2dt.$$

with for every $t\in[-r,0]$ we have $\mathscr{F}_t=\mathscr{F}_0$ there corresponds a $\mathscr{D}-$valued adapted process $y_t\in\mathscr{D}_{\mathscr{F}_0}$ defined on $\theta\in[-r,0]$ by

$$y_t(\theta,\omega)=y(t+\theta,\omega),\ \text{for}\ \theta\in[-r,0],\ \omega\in\Omega.$$

Main results Let us start by defining what we mean by a solution of problem (27). In order to define the solutions of the above problems, we shall consider the spaces

$$\begin{array}{c}\mathscr{D}_{\mathscr{F}_T}=\{y:\ [-r,T]\times\Omega\to\mathscr{H},\ y_k\in C(J_k,\mathscr{H})\ \text{for}\ k=1,\ldots m,\ y_0\in\mathscr{D}_{\mathscr{F}_0},\\ \text{and there exist}\ \ y(t_k^-)\ \text{ and }\ y(t_k^+)\ y(t_k)=y(t_k^-),\ \ k=1,\cdots,m,\\ \text{with}\ \ \sup_{t\in[0,T]}E(|y(t)|^2)<\infty\ \text{ and }\ \int_{-r}^0 E\|\phi(t)\|^2dt<\infty\},\end{array}$$

endowed with the norm

$$\|y\|_{\mathscr{D}_{\mathscr{F}_T}}=\sup_{t\in[0,T]}(E(|y(t)|^2))^{\frac12}+\|y\|_{\mathscr{D}_{\mathscr{F}_0}},$$

y_k denotes the restriction of y to $J_k=(t_{k-1},t_k]$, $k=1,2,\cdots,m$, and $J_0=[-r,0]$.
Then we will consider our initial data $\phi\in\mathscr{D}_{\mathscr{F}_0}$.

We are now in a position to state and prove our existence result for the problem (27). Along this paragraph M is a positive constant such that $\|U(t)\|^2\le M$ for every $t\in J$. For the study of this problem we first list the following hypotheses.

(H2) There is a positive constant M such that $\sup_{0\le t\le T}\|R(t-s)\|^2\le M$.

(H3) $F:[0,T]\times\mathscr{H}\longrightarrow\mathscr{P}(\mathscr{H})$ is an L^2-Carathédory function, with compact convex values.

(H4) There exists an integrable function $\eta:J\longrightarrow\mathbb{R}^+$ $\eta\in L^2(J,\mathbb{R}^+)$

$$E|F(t,\Theta)|^2=\{\sup_{v\in F(t,\Theta)}E\|v\|^2\le\eta(t)\psi(E\|u\|^2),\quad t\in J,\quad u\in\mathscr{D}\},$$

where and $\psi:\mathbb{R}^+\longrightarrow(0,\infty)$ is continuous and increasing with

$$\int_{TC_0}^\infty\frac{du}{\psi(u)}>\frac{T}{(1-5c_1T)}\int_0^T p(s)ds,$$

$$C_0\quad=\quad\frac{10M\|\phi\|^2_{\mathscr{D}_{\mathscr{F}_0}}+10M(c_1\|\phi\|^2_{\mathscr{D}_{\mathscr{F}_0}}+c_2)+5(c_1\|\phi\|^2_{\mathscr{D}_{\mathscr{F}_0}}+c_2)+10HT^{2H-1}\int_0^T\|\sigma(s)\|^2_{L_Q^0}ds+6M_1m\sum_{k=1}^m d_k}{(1-5c_1T)},$$

(H5) The function $\sigma:J\longrightarrow L_Q(\mathscr{K},\mathscr{H})$ satisfies

$$\int_0^T\|\sigma(s)\|^2_{L_Q^0}ds<\infty.$$

(H6) There exist constants d_k ,such that $|I_k(y)|^2\le d_k$,for each $y\in\mathscr{H}$

(H7) The function f is completely continuous and for any bounded set $V\in\mathscr{D}_{\mathscr{F}_T}$ the set $\{t\longrightarrow g(t,y_t)\ :\ y\in V\}$ is equicontinuous in $\mathscr{D}_{\mathscr{F}_T}$.

Theorem 12. *. Assume that hypotheses (H1)-(H7) hold. Then the problem* (27) *has at least one integral solution on* $[-r,T]$.

Proof. Transform the problem (27) into a fixed point problem. Consider the multivalued operator $\Psi : \mathcal{D}_{\mathcal{F}_T} \longrightarrow \mathcal{P}(\mathcal{D}_{\mathcal{F}_T})$ defined by

$$\Psi(y) = \left\{ h \in \mathcal{D}_{\mathcal{F}_T} \ h(t) = \left\{ \begin{array}{ll} \phi(t), & \text{if}; t \in [-r,0] \\ R(t)[\phi(0) - f(0,\phi)] - f(t,y_t) + \int_0^t R(t-s)v(s)ds & \\ + \int_0^t R(t-s)\sigma(s)dB_Q^H(s) + \sum_{0<t_k<t} R(t-t_k)I_k(y(t_k)), & \text{if } t \in J \end{array} \right. \right\}$$

where $v \in N_{F,y}$.

Clearly the fixed points of Ψ are solutions to (27).

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Fractional feed-forward networks

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Abstract We simulate a fractional feed-forward oscillator network. It is observed amplification of the small signals, by exploiting the nonlinear response of each oscillator near its intrinsec Hiof bifurcation point. Interesting features arise for decreasing values of the fractional order derivative α.

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Keywords : *nonlinear oscillator network; fractional systems; feed-forward network; amplification of signals.*

2010 Mathematics Subject Classification : 26A33; 34A60; 34G25; 93B05.

Fixed points in kleene Algebra

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Abstract Kleene algebra is an algebraic system for calculating with sequential composition, choice and finite iteration. It was first introduced by Kleene in 1956 and further developed by Conway in 1971. It has reappeared in many contexts in mathematics and computer science. Its classical application has been within the theory of formal languages, it is one of many equivalent approaches to the description of regular languages. The fixed points play an important role in mathematics and computer science specially in the while-loop semantics. In this paper, we will give some relevant results about the fixed points in Kleene algebra.

Relation Algebras We will give a definition of relational algebra and we recall some basic facts about fixed points. Most of our definitions are taken from [1, 2].

Definition 1. *An abstract homogeneous relational algebra* $(\mathscr{R}, \cup, \cap, ^-, \circ, ^T)$ *consists of a nonempty set* $\mathscr{R}$, *whose elements are called relations, such that*

a) $(\mathscr{R}, \cup, \cap, ^-)$ *is a complete, atomic Boolean algebra with zero element* O, *universal element* L, *and order* $\subseteq$,

b) $(\mathscr{R}, \circ)$ *is a semigroup with precisely one unit element* I,

c) *Schröder equivalences are valid i.e. for all relations* Q, R *and* S, *we have*

$$QR \subset S \Longleftrightarrow Q^T\overline{S} \subset \overline{R} \Longleftrightarrow \overline{S}R^T \subset \overline{Q}.$$

d) *Tarski rule is valid i.e.* $LRL = L$ *for all* $R \neq O$.

We will define fixed points of some function f and give their properties.

Definition 2. *A fixed point of a function* $f : S \longrightarrow S$ *is an element* $s \in S$ *for which* $f(s) = s$.

Kleene algebra Kleene algebra (KA) is an algebraic structure that captures axiomatically the properties of a natural class of structures arising in logic and computer science. It is named for Stephen Cole Kleene $(1909-1994)$, who among his many other achievements invented finite automate and regular expressions, structures of fundamental importance in computer science. Kleene algebra is the algebraic theory of these objects, although it has many other natural and useful interpretations.

Keywords : *Relational algebra, Kleene algebra, fixed points, matrices, reflexive transitive closure.*
2010 Mathematics Subject Classification : **03C05; 03C50; 03C52; 03B30.**

Definition 3. *A Kleene algebra is a structure* $\mathcal{K} = (K, +, \cdot, *, 0, 1)$ *where* $(K, +, \cdot, 0, 1)$ *is an idempotent semiring such that the* $*$ *operation satisfies*

$$\begin{array}{lrcl} (k_1) & 1 + xx^* & \leq & x^* \\ (k_2) & 1 + x^*x & \leq & x^* \\ (k_3) & if \;\; b + ax & \leq & x \; then \;\; a^*b \leq x \\ (k_4) & if \;\; b + xa & \leq & x \; then \;\; ba^* \leq x. \end{array}$$

(*where* $a \leq b$ *iff* $a + b = b$ *is called the natural partial order of Kleene algebra*). *The set* K *is the domain of the Kleene algebra* $\mathcal{K}$.

Now, We will give a special case of matrices which are matrices with Kleene algebra entries as an example of Kleene algebra.

Under the natural definitions of the Kleene algebra operators $+, \cdot, ^*, 0$ and 1, the family $\mathcal{M}(n, \mathcal{K})$ of $n \times n$ matrices over a Kleene algebra $\mathcal{K}$ again forms a Kleene algebra. This is a standard result proved for various classes of algebras in [1, 2].

Define $(+)$ and $(\cdot)$ on $\mathcal{M}(n, \mathcal{K})$ to be the usual operations of matrix addition and multiplication, respectively, Z_n the $n \times n$ zero matrix, and I_n the $n \times n$ identity matrix. The partial order $(\leq)$ is defined on $\mathcal{M}(n, \mathcal{K})$ by

$$A \leq B = \Leftrightarrow A + B = B.$$

Under these definitions, we conclude that.

Theorem 13. *The structure* $(\mathcal{M}(n, \mathcal{K}), +, \cdot, Z_n, I_n)$ *is an idempotent semiring.*

Using E^* for $E \in \mathcal{M}(n, \mathcal{K})$. We first consider the case $n = 2$. This construction will later be applied inductively.

Let

$$E = \begin{pmatrix} a & b \\ c & d \end{pmatrix},$$

let $f = a + bd^*c$, we have,

$$\begin{pmatrix} E^* = f^* & f^*bd^* \\ d^*cf^* & d^* + d^*cf^*bd^* \end{pmatrix}$$

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A parametric uncertainty analysis method for queues with vacations

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Abstract Queueing situations in which the idle server may take vacations encounter in flexible manufacturing systems, service systems, and telecommunication systems. A common concern with queueing models with vacations is that model parameters are seldom perfectly known. However, the parameter values themselves are determined from a finite number of observations and hence have uncertainty associated with them (epistemic uncertainty). In this paper, we study the M/G/1/N with vacations where assume that vacation parameter is computed with epistemic uncertainty. So, we propose a numerical method based on the Taylor series expansion to estimate the stationary distribution of the considered queueing model under the parametric uncertainty of the vacation rate.

Introduction Queuing models with vacations are very suitable models for modeling many real situations. During the past decade, Taylor series expansion based techniques have become an important tool for the analysis of stochastic systems that can be described by Markov chains, while focusing on the perturbation analysis.
In this paper, we develop a new queueing model with vacations under the assumption that the rate vacation is computed with epistemic uncertainty. Specifically, we estimate the stationary distribution, which is considered as a transformation or a function of the rate vacation, of the M/G/1/N-(V, E) queue with exhaustive vacation. Moreover, we focus on the estimation of its probability density function.

A parametric uncertainty analysis of the M/G/1/N-(V,E) queue Consider the $M/G/1/N-(V,E)$ queueing model with server vacation and exhaustive service. The flow of arrivals is Poisson process with parameter λ, the distribution of the service time is general distribution function S, with mean $\frac{1}{\mu}$, the duration of each V vacancy is i.i.d. with rate $\theta > 0$ and the service discipline is FIFO.

The state of this model is described by the Markov chain $X = \{X_n; n \in \mathbb{Z}_+\}$ representing the number of customers in the system at the end of the n^{th} service. Its transition matrix is defined by [3]:

$$P_\theta = \begin{pmatrix} b_0 & b_1 & b_2 & \cdots & b_{N-2} & 1-\sum_{k=0}^{N-2} b_k \\ a_0 & a_1 & a_2 & \cdots & a_{N-2} & 1-\sum_{k=0}^{N-2} a_k \\ 0 & a_0 & a_1 & \cdots & a_{N-3} & 1-\sum_{k=0}^{N-3} a_k \\ & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & a_0 & 1-a_0 \end{pmatrix}, \quad (28)$$

where, for $j \in \mathbb{Z}_+$,
$a_j = \int_0^\infty \frac{(\lambda t)^j}{j!} e^{-\lambda t} dS(t)$, $v_j = \int_0^\infty \frac{(\lambda t)^j}{j!} e^{-\lambda t} dV(t)$ and $b_j = \sum_{i=1}^{j+1} \frac{v_i}{1-v_0} a_{j-i+1}$.

Keywords : *Queues with vacations; epistemic uncertainty; Taylor series expansion; Non-parametric sensitivity analysis; Optimization; Algorithm.*
2010 Mathematics Subject Classification : **68M20; 41A58; 62G07; 65C40.**

Denote by π the stationary distribution of the Markov chain X.

In this work, we will use the Taylor series expansion method [1, 2] to analysis the above introduced queueing model, where we assume that the vacation parameter θ is not computed perfectly. Specifically, we will approximately compute $\pi(\theta+\Delta)$ by a polynomial in Δ. Indeed, by considering the transition probabilities $p_\theta(i,j)$ to be differentiable functions of a parameter θ,

$$\pi_{\theta+\Delta} = \sum_{n=0}^{\infty} \frac{\Delta^n}{n!} \pi_\theta^{(n)}, \tag{29}$$

$$\pi_\theta^{(n)} = \pi_\theta K_\theta(n), \tag{30}$$

where

$$K_\theta(n) = \sum_{\substack{1 \le m \le n; \\ 1 \le l_k \le n; \\ l_1 + \cdots + l_m = n}} \frac{n!}{l_1! \cdots l_m!} \prod_{k=1}^{m} \left(P_\theta^{(l_k)} D_\theta \right), \tag{31}$$

$D_\theta = (I - P_\theta + \Pi_\theta)^{-1} - \Pi_\theta$, where Π_θ is the stationary projector of the Markov chain X.

We introduce a new model for the vacation rate θ to describe the uncertainty on computing their values. Let $\theta = \overline{\theta} + \sigma\varepsilon$ where $\overline{\theta}$ is the mean of the random variable θ, σ its standard deviation and $\varepsilon \rightsquigarrow \mathcal{N}(0,1)$. Therefore, we will approximate the probability density function, the expectation and the variance for each components of the stationary distribution $\pi_i(\overline{\theta} + \sigma\varepsilon)$.

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Nonparametric estimation of a conditional quantile density function for time series data

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Abstract The aim of this work is to estimate nonparametrically the conditional quantile density function. A non-parametric estimator of a conditional quantile function density is presented, its asymptotic properties are derived via the estimation of the conditional distribution.

The model We consider a random pair (X,Y) where Y is valued in $\mathbb{R}$ and X is valued in some infinite dimensional semi-metric vector space $(\mathscr{F}, d(.,.))$. Let (X_i, Y_i), $i = 1,\ldots,n$ be the statistical sample of pairs which are identically distributed like (X,Y), but not necessarily independent. From now on, X is called functional random variable f.r.v. Let x be fixed in $\mathscr{F}$ and let $F_{X|Y}(\cdot,x)$ be the conditional cumulative distribution function (cond-cdf) of Y given $X = x$. Let $Q_{Y|X}(\gamma)$ be the $\gamma-$order quantile of the distribution of Y given $X = x$. From the cond-cdf $F_{Y|X}(.,x)$, it is easy to give the general definition of the γ-order quantile:

$$Q(\gamma|X=x) \equiv Q_{Y|X}(\gamma) = \inf\left\{t : F_{Y|X}(t,x) \geq \gamma\right\},\ 0 \leq \gamma \leq 1 \tag{32}$$

The condition quantile density function can be written as follows (see [4])

$$q(\gamma|X=x) \equiv q_{Y|X}(\gamma) \quad = \quad \frac{1}{f_{Y|X}\left(Q_{Y|X}(\gamma)\right)} \tag{33}$$

The estimator $\widehat{Q}_{Y|X}$ of $Q_{Y|X}$ is as follows:

$$\widehat{Q}_{Y|X}(\gamma) = \inf\left\{t : \widehat{F}_{Y|X}(t,x) \geq \gamma\right\} = \widehat{F}^{-1}_{Y|X}\left(Q_{Y|X}(\gamma)\right) \tag{34}$$

The smooth estimator of the conditional quantile density functional defined as follows:

$$\widehat{q}_{Y|X}(\gamma) = \frac{1}{\widehat{f}_{Y|X}\left(\widehat{Q}_{Y|X}(\gamma)\right)}. \tag{35}$$

where $\widehat{f}_{Y|X}(x,y)$ is a conditional kernel density estimator of $f_{Y|X}(x,y)$ and $\widehat{Q}_{Y|X}(\gamma)$ is the conditional empirical estimator of the conditional quantile function $Q_{Y|X}(\gamma)$.

Keywords : *Conditional quantile; Conditional quantile density function; Functional variable; Kernel density estimators; α-mixing.*
2010 Mathematics Subject Classification : **62G05, 62G99, 62M10**

Main results

Theorem 14. *Let $q_{Y|X}(\gamma)$ be the conditional density function corresponding to a density function $f_{Y|X}\left(Q_{Y|X}(\gamma)\right)$ and $\widehat{q}_{Y|X}(\gamma)$ denote the estimator of $q_{Y|X}(\gamma)$. Then under some assumptions and as n tends to infinity, we have*

$$\sup_{\gamma}\left|\widehat{q}_{Y|X}(\gamma)-q_{Y|X}(\gamma)\right|\longrightarrow 0 \qquad a.co.$$

The estimator of $Q_{X|Y}(\gamma)$ is defined by

$$\widehat{Q}_{Y|X}(\gamma)=\frac{1}{h_H}\int \widehat{F}^{-1}_{Y|X}(x,v)H\left(h_H^{-1}(v-\gamma)\right)dv \tag{36}$$

for an appropriate kernel function H and a bandwidth h_H. (36) suggests an estimator of $q_{Y|X}(\gamma)$ is

$$\widehat{q}^{1}_{Y|X}(\gamma)=-\frac{1}{h_H^2}\int_0^1 \frac{H'\left(h_H^{-1}(v-\gamma)\right)}{\widehat{f}_{Y|X}\left(\widehat{Q}_{Y|X}(v)\right)}dv \tag{37}$$

where H' is a kernel and h_H is the bandwidth sequence.

Next theorem proves consistency of the proposed estimator of the conditional quantile density function.

Theorem 15. *Let $q_{Y|X}(\gamma)$ be the conditional density function corresponding to a density function $f_{Y|X}\left(Q_{Y|X}(\gamma)\right)$ and $\widehat{q}^{1}_{Y|X}(\gamma)$ given by* (37) *the proposed estimator of $q_{Y|X}(\gamma)$, the conditional quantile density function. Then under some specific hypotheses as n tends to infinity, we have*

$$\sup_{\gamma}\left|\widehat{q}^{1}_{Y|X}(\gamma)-q_{Y|X}(\gamma)\right|\longrightarrow 0 \qquad a.co.$$

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Existence, Uniqueness and Stability of Solutions for Lane-Emden Fractional Differential Equations

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Abstract In this talk, we study the Lane-Emden differential equations with some arbitrary orders. We obtain new results on the existence and uniqueness of solutions using some classical fixed point theorems. We also define and study some types of U-stability for the proposed problem.

Introduction We are concerned with a generalization of the following fractional Lane-Emden differential equation:

$$\begin{cases} D^{\beta}\left(D^{\alpha}+\frac{a}{t}\right)u(t)+f(t,u(t))=g(t), \\ u(0)=\mu,\ u(1)=\nu, \\ 0<\alpha,\beta\leq 1,\ 0\leq t\leq 1,\ a\geq 0, \end{cases}$$

where, D^{γ} denotes the Caputo derivative for $\gamma>0$, f is continuous real valued function and $g\in C([0,1])$.

Main results In this paragraph, we formulate and prove sufficient conditions for the existence and uniqueness of solutions to the generalized Lane-Emden problem inspired from the above equation. Then, based on the existence and uniqueness result, we continue our study by imposing some types of Ulam stability for the proposed problem.
The following conditions are necessary to prove the main results:
(H_1) : There exist nonnegative constants $\left(\mu_k\right)_j$; $j,k=1,2,...,n$, such that for all $t\in[0,1]$ and all $(x_1,x_2,...,x_n),\left(y_1,y_2,...,y_n\right)\in S$, we have $\left|f_k(t,x_1,x_2,...,x_n)-f_k\left(t,y_1,y_2,...,y_n\right)\right|\leq\sum_{j=1}^{n}\left(\mu_k\right)_j\left|x_j-y_j\right|$.
(H_2) : The functions $f_k:[0,1]\times\mathbb{R}^n\to\mathbb{R}$ and $g_k:[0,1]\to\mathbb{R}$ are continuous for each $k=1,2,...,n$, $n\in\mathbb{N}^*$.

(H_3) : There exist nonnegative constants $(L_k)_{k=1,2...,n,}$ such that: for each $t\in J$ and all $(x_1,x_2,...,x_n)\in\mathbb{R}^n$, $\left|f_k(t,x_1,x_2,...,x_n)\right|\leq L_k$, $k=1,2,...,n$.

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Keywords : *Differential system, fixed point, existence of solutions.*
2010 Mathematics Subject Classification : **30C45, 39B72.**

Some properties on fuzzy and demonic operators

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Abstract The calculus of relations has been an important component of the development of logic and algebra since the middle of the nineteenth century. George Boole, in his *"Mathematical Analysis of Logic"* initiated the treatment of logic as part of mathematics, specifically as part of algebra. Quite the opposite conviction was put forward early this century by Bertrand Russell and Alfred North Whitehead in their *Principia Mathematica* : that mathematics was essentially grounded in logic. The logic is developed in two streams. On the one hand algebraic logic, in which the calculus of relations played a particularly prominent part, was taken up from Boole by Charles Sanders Peirce, who wished to do for the *"calculus of relatives"* what Boole had done for the calculus of sets, Peirce's work was in turn taken up by Schröder in *"Algebra und Logik der Relative"*. Schröder's work, however, lay dormant for more than 40 years, until revived by Alfred Tarski in his seminal paper *"On the calculus of binary relations"*. Tarski's paper is still often referred to as the best introduction to the calculus of relations.We first recall the concept of relational algebra and fuzzy calculus. Then we define a demonic refinement ordering and demonic operators (many of these definitions come from our previous work). Then, taking the properties of these demonic operators as a guideline, we investigate the properties of demonic operators and illustrate them by using mathematica (fuzzy logic).

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Keywords : *Fuzzy sets, demonic operators, demonic fuzzy operators, demonic fuzzy ordering.*
2010 Mathematics Subject Classification : **03C05; 03C50; 03C52; 03B30.**

On a quasi linear problem modelling the flow underlying the surfaces water before the arrival of the destructive waves like that of tsunami

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Abstract In this paper, we will study a problem that models the flow of water in the oceans underlying the water surface before the arrival of large destructive waves, like that of tsunami for example. For this phenomenon, several mathematical models have been introduced. We will use the latest mathematical model, it was introduced by [1] and [2]. In [4], the autheur has established a uniqueness result in a classic case. In our paper, we will generalize the study of [4] to more general functional framework . We will establish a result of existence and uniqueness in more richer functional spaces, this study is more general and covers the classical case. Finally, we will make a numerical validation of our work. .

Problem Formulation Let $\Omega =]r_0, R[$, $r_0 > 1$. Our problem is: Find u such that:

$$u \in H_0^1(\Omega) \cap L^{3/2}(\Omega)\ (\textbf{??}) \tag{38}$$

$$\frac{du}{dr} \in L^2(\Omega)\ (\textbf{??}) \tag{39}$$

And u is solution of the following differential equation:

$$\frac{d^2u}{dr^2} + \frac{1}{r}\frac{du}{dr} + F(u) = f \text{ in } \Omega(\textbf{??}) \tag{40}$$

With the following boundary conditions:

$$u(r_0) = 0(\textbf{??}) \tag{41}$$

$$\frac{du}{dr}(r_0) = u_1(\textbf{??}) \tag{42}$$

Where F models the vorticity, it is defined as follows:

$$F(u) = \begin{cases} u - \frac{u}{\sqrt{|u|}} & \text{if } u \neq 0 \\ 0 & \text{if } u = 0 \end{cases} \tag{43}$$

Keywords : *successive approximate method; compactness method; galerkin method; Diffusion equation;fixed points; quasilinear equation.*
2010 Mathematics Subject Classification : **26A33; 34A60; 34G25; 93B05.**

Main results To prove the existence, we have the following theorem:

Theorem 16. *Let be given*

$$f \in L^2(\Omega) \tag{44}$$

Then the problem (**??**)–(**??**) admits a least a solution checking (**??**)–(**??**).

Proof. The proof of this theorem will be made in three steps.

Step 1: Approximations We define an approximate solution $u_m = u_m(r)$of the problem (**??**)–(**??**) under the form:

$$u_m(r) = \sum_{j=1}^{m} g_{jm} w_j \tag{45}$$

Such that, for

$$\left(\frac{d^2 u_m}{dr^2}, w_j\right) + \left(\frac{1}{r}\frac{du_m}{dr}, w_j\right) + \left(F(u_m), w_j\right) = \left(f, w_j\right) \tag{46}$$

The formula (46) is an ordinary differential system. If we take into account the following conditions:

$$u_m(r_0) = u_{m0} \tag{47}$$

$$u'_m(r_0) = u_{m1} \tag{48}$$

The Problem (46)–(48) admits a least a solution in $[r_0, r_m]$, where $r_m \leq R$. The following priori estimates show that $r_m = R$.

Step 2: A priori estimates In this paragraph, we will make a priori estimates, on the sequence introduced in the previous paragraph.

Step 3: Passage to the limit In this paragraph, we will show that the sequence introduced in step 1, converges to the solution of the problem. □

Uniqueness Assume that the problem admits two solutions, after majoration and by using the Gronwall lemma, we prove that the two solutions are identical.

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Multi-quasielliptic and Gevrey regularity of hypoelliptic differential operators

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Abstract A large class of hypoelliptic linear differential operators with constant complex coefficients is considered, the so called multi-quasielliptic differential operators, see [2] for an exhaustive study. We first show the theorem of L. Zanghirati [4] and C. Bouzar and R. Chaïli [1] only in the case of the linear differential operators with constant complex coefficients. Due to a result of Hörmander [3] every hypoelliptic linear partial differential operators with constant coefficients is Gevrey hypoelliptic in some anisotropic Gevrey spaces. We first explicit this anisotropic Gevrey regularity result for multi-quasielliptic differential operators. Zanghirati [4] has also proved that there exists a Gevrey regularity result for multi-quasielliptic differential operators in the context of the so called multi-anisotropic Gevrey spaces. We prove that the multi-anisotropic Gevrey regularity result is more precise than the anisotropic Gevrey regularity for multi-quasielliptic differential operators. An illustrative example is given. The aim of this paper is to prove the multi-anisotropic Gevrey regularity of hypoelliptic linear differential operators with complex constant coefficients and consequently we precise the result of L. Hörmander and extend the result of L. Zanghirati.

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Keywords : *Differential operators; Gevrey regularity; Gevrey vectors; Multi-quasielliptic operators; Multi-anisotropic Gevrey spaces; Newton's polyhedron.*

2010 Mathematics Subject Classification : **34L30; 49N60.**

Numerical method for solving an inverse problem of identifying the source function

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Abstract We consider the inverse problem of identifying the unknown time-dependent source term in a parabolic equation. To solve this problem, method based on the direct and inverse Laplace transform is proposed. Application of the Laplace transform makes it possible to obtain an operator equation describing the explicit dependence of the unknown source function on the boundary function, and then the regularization method is used to solve this equation. This eliminates the unstable procedure of numerical inversion in the computational scheme. The proposed approach provides the basis for the numerical method of solving the inverse source problem.

Introduction The inverse source problem appears in a variety of physical and engineering applications, such as, material science, heat and mass transfer, combustion systems, heat conduction for operational engines, investigation of nanostructures. The development of numerical methods to determine unknown source function is of great interest.For example, Ismailov, Kansa and Lesnic [1], used the generalized Fourier method and Crank-Nicolson finite difference scheme to obtain unique numerical solution of the inverse source problem. Farcas and Lesnic [2] and Bushuyev [3] investigated the uniqueness of the solution to inverse source problems. Hasanov and Pektas in [4] proposed a conjugate gradient algorithm to identify the unknown time-dependent heat source of the variable coefficient heat equation.

We consider the following problem:

$$u_t = au_{xx} + f(t), \quad x \in (0,\ell), \quad t > 0, \tag{49}$$

$$u(0,t) = u(\ell,t) = u(x,0) = 0, \quad x \in [0,\ell], \quad t \geqslant 0, \tag{50}$$

$$u_x(0,t) = g(t), \quad t \geqslant 0, \tag{51}$$

where $f(t)$ is unknown. We assume that $g \in C^{2+\eta}[0,T]$ for all $T > 0$ and there exists constants M, m such that $\|g(t)\| \le Me^{mt}$ for all $t \in [0,T]$ for any $T > 0$, where $C^{2+\eta}[0,T]$ is the Hölder space and $\eta \in (0,1)$. Denote $Q_T = (0,\ell) \times (0,T)$ for $T > 0$. In this problem, it is required to determine the value of function $f(t) \in C^{2+\eta}[0,T]$ for all $T > 0$ and then to find function $u(x,t) \in H^{2,1}(\overline{Q}_T)$ satisfying (49)–(51).

It is known, that for $g(t) = g_0(t)$ there exist an exact $f_0(t)$ and an exact $u_0(x,t)$ satisfying (49)–(51) with $f(t) = f_0(t)$, but instead of g_0 we are given some approximations g_δ and an error level $\delta > 0$ such that $\|g_\delta - g_0\| \leqslant \delta$. Nevertheless, using these initial data, it is required to find source term $f(t)$, and then to obtain $u_\delta(x,t)$ satisfying (49)–(51). Uniqueness of the solution to problem (49)–(51) was proved in [3] and [4].

Keywords : *inverse source problem; system with distributed parameters; heat conduction; the Laplace transform; regularization method.*
2010 Mathematics Subject Classification : **58J35, 35R30, 65N20.**

Main results We propose to subdivide the solving of inverse source problem into two parts. The first part involves constructing the integral equation, which describes the explicit dependence of the unknown source term $f(t)$ on the known boundary condition (51). To obtain this equation, we apply the direct and inverse Laplace transforms. Then, the resulting equation is solved via the regularization method and numerical solution f_δ^α for inverse source problem is calculating. The similar approach has been used for solving a boundary value inverse heat conduction problem in paper [5].

The proposed approach provides the basis for the numerical method of solving the inverse source problem. Results of computational experiment and experimental error estimates of the obtained solutions show sufficient stability and efficiency of the proposed method.

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Impulsive hyperbolic system of partial differential equations of fractional order with delay

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Abstract This paper deals with the existence of solutions to impulsive partial hyperbolic differential equations with finite delay, involving the Caputo fractional derivative. Our results will be obtained using suitable fixed point theorems.

Introduction we study the existence of solutions for the following impulsive partial hyperbolic differential equations:

$$({}^{c}D^{r}_{z_k}u)(t,x) = f(t,x,u_{(t,x)});\ \text{ if } (t,x)\in J_k,\ \ k=0,\dots,m, \tag{52}$$

$$u(t_k^+,x) = u(t_k^-,x) + I_k(u(t_k^-,x)),\ \text{ if } x\in[0,b],\ k=1,\dots,m, \tag{53}$$

$$u(t,x) = \phi(t,x);\ \text{if } (t,x)\in\tilde{J}, \tag{54}$$

$$u(t,0)=\varphi(t),\ t\in[0,a],\ u(0,x)=\psi(x);\ x\in[0,b], \tag{55}$$

where $J_0=[0,t_1]\times[0,b]$, $J_k := (t_k,t_{k+1}]\times[0,b]$, $k=1,\dots,m, z_k=(t_k,0)$, $k=0,\dots,m$, $a,b,\alpha,\beta>0, J=[0,a]\times[0,b]$, $\tilde{J}=[-\alpha,a]\times[-\beta,b]\backslash(0,a]\times(0,b]$, ${}^{c}D^{r}_{0}$ is the Caputo fractional derivative of order $r=(r_1,r_2)\in(0,1]\times(0,1]$, $\varphi:[0,a]\to\mathbb{R}^n$, $\psi:[0,b]\to\mathbb{R}^n$ are given continuous functions with $\varphi(t)=\phi(t,0)$, $\psi(x)=\phi(0,x)$ for each $(t,x)\in J$, $0=t_0<t_1<\cdots<t_m<t_{m+1}=a$, $f:J\times C\to\mathbb{R}^n$, $I_k:\mathbb{R}^n\to\mathbb{R}^n$, $k=1,\dots,m$, $\phi:\tilde{J}\to\mathbb{R}^n$, are given functions and $C:=C([-\alpha,0]\times[-\beta,0],\mathbb{R}^n)$ is the space of continuous functions on $[-\alpha,0]\times[-\beta,0]$.
If $u:[-\alpha,0]\times[-\beta,0]\longrightarrow\mathbb{R}^n$, then for any $(t,x)\in J$ define $u_{(t,x)}$ by

$$u_{(t,x)}(s,\tau)=u(t+s,x+\tau)$$

Main results Our result is based upon the fixed point theorem due to Burton and Kirk. Let us introduce the following hypotheses which are assumed to hold in the sequel.

$(H1)$ The functions $I_k:\mathbb{R}^n\to\mathbb{R}^n$ are continuous.

$(H2)$ There exist $p,q\in C(J,\mathbb{R}_+)$ such that

$$\|f(t,x,u)\|\le p(t,x)+q(t,x)\|u\|_C,\ \text{for } (t,x)\in J \text{ and each } u\in C.$$

Keywords : *Impulsive partial differential equations, fractional order, solution, left-sided mixed Riemann-Liouville integral, Caputo fractional-order derivative, finite delay, fixed point.*

2010 Mathematics Subject Classification : **26A33, 34A37, 34K37, 35R11.**

$(H3)$ There exists $l > 0$ such that

$$\|I_k(u) - I_k(v)\| \leq l\|u - v\| \text{ for each } u, v \in \mathbb{R}^n,\ k = 1, \ldots, m.$$

Theorem 17. *Assume that hypotheses* $(H1)$-$(H3)$ *hold. If*

$$2ml < 1, \tag{56}$$

then the IVP (52)–(55) *has at least one solution on* J.

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Stability and Chaos in a Yang-Mills System

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Abstract The chaoticity of the fields of Yang-Mills systems is not a surprise as they are governed by nonlinear equations. On the other hand, there are many examples of stable solutions of nonlinear field equations. So the question of chaos in the equations of Yang-Mills systems is not a trivial and straightforward one [1]. In the contest of particle physics, Matinyan and al, were the first to show that classical Yang-Mills system is a K-one [2]. It has been conjectured by Nikolaevsky and Shchur that if chaos is present in the dynamics of homogeneous field, then it is present in the full field theory [3]. This was confirmed in the Y-M field [4]. In this work we consider a little bit complication: we add quantum to the previous study. We found that the quantum version of a Yang-Mills gauge theory returns to the classical case if we neglect the interference terms and is writing as a system of two non-linear differential equations. Then we modelize the interference with a white noise term and transform the system to a couple of stochastic differential equations and study them using the Khasminski procedure [5]. We find that our stochastic differential system has a unique solution that does not explode in a finite time. We clearly demonstrate that, if we neglect the interference part, the Quantification of a Yang-Mills theory does not remove the high sensitivity to initial conditions seen in classical case and the Non-Integrability remains.

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Keywords : *Quantum chaos; Yang-Mills system; Nonlinear differential equations.*
2010 Mathematics Subject Classification : **81Q50; 81T13; 34A34.**

A Logarithmic barrier approach for linear programming

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Abstract This paper presents a logarithmic barrier method for solving a linear programming problem. We are interested in computation of the dirparagraph by the Newton's method and in computation of the displacement step using majorant functions instead line search methods in order to reduce the computation cost. This purpose is confirmed by numerical experiments, showing the efficiency of our approach, which are presented in the last paragraph of this paper.

Introduction Let us consider the following problem:

$$(D)\begin{cases} \min b^t y \\ A^t y \geq c \\ y \in \mathbb{R}^m, \end{cases}$$

where:
$A \in \mathbb{R}^{m\times n}$ such that $rang A = m < n$, $c \in \mathbb{R}^n$, $b \in \mathbb{R}^m$.

The resolution of problem (D), is equivalent to the resolution of a perturbed problems without constraints defined by:

$$(D_r)\begin{cases} \min f_r\left(y\right) \\ y \in \mathbb{R}^m, \end{cases}$$

with $r > 0$ is a parameter barrier and f_r is a barrier function defined by:

$$f_r\left(y\right) = \begin{cases} b^t y + nr\ln r - r_{i=1}^{n}\ln\left\langle e_i, A^t y - c\right\rangle & \text{if } A^t y - c > 0, \\ +\infty & \text{Otherwise.} \end{cases}$$

Where $(e_1, e_2, ..., e_n)$ is the canonical base in $\mathbb{R}^n$.

Main results In this work, we studied the linear programming problem, denoted (D). We associate to this problem a perturbed problem, denoted (Dr). The problem (Dr) is strictly convex, the KKT conditions are necessary and sufficient. For this, we use Newton's method that allows us to calculate a good descent dirparagraph and to determine a new iterated, better than the current iterated.To calculate the displacement step, several methods have been proposed by researchers, among which, the line search methods, which are very expensive.To overcome this problem, we proposed in this work a new approach, based on the notion of majorant functions. This allows us to determine the displacement step by a simple and easy manner.

The numerical simulations confirm the effectiveness of our approaches. Our algorithm converges to the same optimal solution, using any strategy among the three proposed strategies. The first strategy is the best approach versus computing time and number of iterations.

Keywords : *Linear Programming; Interior Point Methods; Logarithmic Barrier methods.*
2010 Mathematics Subject Classification : **90C05; 90C22.**

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Solutions of Fuzzy partial differential equation

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Abstract In this paper, we study a general form of the fuzzy partial differential equations using successive of the Adomian decomposition method to concept the strongly generalized partial derivative for fuzzy-number-valued functions from $\mathbb{R}^2$ into E. Existence theorems of fuzzy solution obtain by presented method. At last, we exhibit two illustrative examples.

Introduction In [2], Bede and Gal studied several characterizations under the strongly generalized derivative for a fuzzy-number-valued function from $\mathbb{R}$ into E, and as an application obtained the existence and uniqueness theorems of the solutions for a fuzzy differential equation.

The dirparagraphal derivative has been presented and studied in [4] for a fuzzy-number-valued function from $\mathbb{R}^n$ into E. By the strongly generalized differentiability concept in [4], can introduce the strongly generalized partial derivative for a fuzzy-number-valued function from $\mathbb{R}^n$ into E. In this work, we obtain existence and uniqueness theorems of solution for a fuzzy partial differential equation by successive iterations of the Adomian decomposition method [1, 3] using concept derivative in [2, 4].

Main results In this paper we discuss about existence and uniqueness of solution of a fuzzy partial differential equation by the successive iterations Adomian decomposition method. Consider the fuzzy partial differential equation

$$u_t(t,x) = \rho(t,x,L_x)u(t,x), \tag{57}$$

subject to

$$u(t_0,x) = f(x), \tag{58}$$

where $L_x = \frac{\partial}{\partial x}$ and $(t,x) \in M = [t_0,+\infty) \times \mathbb{R}$ with $t_0 \geq 0$. The operator $\rho(t,x,L_x)$ will be a polynomial, with continuous variable coefficient respect to t and x on M, in L_x where $L_x(L_x) = L_{xx}$ and denotes the partial derivative with respect to x. Also $f:\mathbb{R}\to E$ and $u: M \to E$ are continuous fuzzy-number-valued functions where f is strongly generalized differentiable in the sense of Definition 5 in [2] and u is strongly generalized partial differentiable in the sense of Definition 1 in the paper.

Theorem 18. *Let us suppose the following conditions hold:*

(a) $f:\mathbb{R}\to E$ be a continuous and bounded function.

(b) There exist $\gamma > 0$, $\beta > 1$ and $e^{-\beta t_0}\gamma \leq 1$ such that

$$D(\rho(t,x,L_x)u(t,x), \rho(t,x,L_x)v(t,x)) \leq \gamma e^{-\beta t} D(u,v), \tag{59}$$

and $\rho(t,x,L_x)u(t,x)$ and $\rho(t,x,L_x)v(t,x)$ are continuous.

Keywords : *Fuzzy partial differential equations; Fuzzy-number-valued functions; Generalized partial derivative; Adomian decomposition method.*
2010 Mathematics Subject Classification : **35K20; 35K30; 35K45; 35K46.**

Then the fuzzy partial differential equation (57) *with the fuzzy initial condition* (58) *has two solutions (one differentiable as in Definition 1 (I) and the other differentiable as in Definition 1 (II))* $u, \overline{u} : M \to E$ *with respect to* t *and the successive iterations*

$$\begin{aligned} &\varphi_0(t,x) = f(x), \\ &\varphi_{n+1}(t,x) = f(x) \oplus \textstyle\sum_{i=1}^{n+1} \int_{t_0}^{t} \rho(s,x,L_x) u_{i-1}(s,x)\,ds, \\ &(n = 0,1,2,\ldots), \end{aligned} \tag{60}$$

and

$$\begin{aligned} &\overline{\varphi}_0(t,x) = f(x), \\ &\overline{\varphi}_{n+1}(t,x) = f(x) - (-1) \odot \\ &\qquad \textstyle\sum_{i=1}^{n+1} \int_{t_0}^{t} \rho(s,x,L_x) \overline{u}_{i-1}(s,x)\,ds, \;\; (n = 0,1,\ldots), \end{aligned} \tag{61}$$

uniformly convergent to these two solutions, respectively.

Theorem 19. *Assume the following conditions hold:*

(a) $f : \mathbb{R} \to E$ *be a continuous and bounded function.*

(b) *There exist* $\gamma > 0$, $0 < \beta \le 1$ *and* $e^{-\beta t_0} \gamma \le \frac{\beta^2}{2}$ *such that*

$$D(\rho(t,x,L_x)u(t,x), \rho(t,x,L_x)v(t,x)) \le \gamma e^{-\beta t} D(u,v), \tag{62}$$

and $\rho(t,x,L_x)u(t,x)$ *and* $\rho(t,x,L_x)v(t,x)$ *are continuous.*

Then the fuzzy partial differential equation (57) *with the fuzzy initial condition* (58) *has two solutions (one differentiable as in Definition 1 (I) and the other differentiable as in Definition 1 (II))* $u, \overline{u} : M \to E$ *with respect to* t *and the successive iterations* (60) *and* (61) *uniformly convergent to these two solutions, respectively.*

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Fuzzy transform to approximate solution of two-point boundary value problems

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Abstract In this paper, based on the fuzzy transform, we propose a new method to obtain the approximate solution of a two-point boundary value problem. We illustrate by an example the effectiveness of the method.

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Keywords : *Fuzzy transform; Two-point boundary value problem; Approximate solution.*
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Relative controllability of fractional stochastic dynamical systems with multiple delays in control

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Abstract The approximate controllability of systems represented by nonlinear evolution equations has been investigated by several autheurs, in which the autheurs effectively used the fixed point approach. Also, the approximate controllability of semilinear neutral functional differential systems with finite delay was trated. The conditions are established with the help of semigroup theory and fixed point technique under the assumption that the linear part of the associated nonlinear system is approximately controllable. In this work, we consider the relative controllability.

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Keywords : *Relative controllability; stochastic dynamical systems; delays.*
2010 Mathematics Subject Classification : **26A33; 93B05; 34K50.**

The Autocentral Automorphisms of finite groups

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Abstract Let G be group and let $\mathrm{Var}(G)$ be the group of autocentral automorphism of G. Let $\mathscr{C}_{\mathrm{Var}(G)}(Z(G))$ be the set of all autocentral automorphism of G fixing $Z(G)$ elementwise. In this paper, we characterize finite p-groups G whit this $\mathrm{Var}(G) = \mathrm{Inn}(G)$. Also, we proved that if G is a finite p-group, then $\mathscr{C}_{\mathrm{Var}(G)}(Z(G)) = \mathrm{Inn}(G)$ if and only if G is nilpotent of class 2, $G' \leqslant L(G)$ and $L(G)$ is cyclic.

Introduction Let G be a group. We denote by G', $Z(G)$, $\exp(G)$, $\mathrm{Aut}(G)$, $\mathrm{Inn}(G)$ and $\Phi(G)$, respectively, the commutator subgroup, the centre, the exponent, the automorphism group, the inner automorphism group and the frattini subgroup of G. If G and H are two groups, we denote by $\mathrm{Hom}(G,H)$ the set of all group homomorphism from G to H. Note that if H is abelian, then $\mathrm{Hom}(G,H)$ is an abelian group whit the binary operation defined by $(fg)(x) = f(x)g(x)$ for all $f,g \in \mathrm{Hom}(G,H)$ and for all $x \in G$.

Let G be a group and g_1, g_2 be element of G, then $g_1^{g_2} = g_2^{-1}g_1g_2$ and $[g_1,g_2] = g_1^{-1}g_1^{g_2} = g_1^{-1}g_1^{\varphi_{g_2}}$ denote the conjugate of g_1 by g_2 and the commutator of g_1 and g_2, respectively, where φ_{g_2} is the inner automorphism of G. If $\alpha \in \mathrm{Aut}(G)$ and $g \in G$ then the autocommutator of g and α is defined to be $[g,\alpha] = g^{-1}g^{\alpha} = g^{-1}\alpha(g)$.

We now define the subgroup $L(G)$ and G^* of G as follows

$$L(G) = \{g \in G \mid [g,\alpha] = 1 \;\; for\ all\ \alpha \in \mathrm{Aut}(G)\},$$

$$G^* = \langle [g,\alpha] \mid g \in G,\ \alpha \in \mathrm{Aut}(G)\rangle.$$

We call $L(G)$ the absolute centre of G and G^* the auto-commutator subgroup of G. One can easily check that the absolute centre is a characteristic subgroup contained in the centre of G and G^* is a characteristic subgroup of G containing the derived subgroup. The automorphism $\alpha \in \mathrm{Aut}(G)$ is said to be a central automorphism, if $[g,\alpha] = g^{-1}g^{\alpha} = g^{-1}\alpha(g) \in Z(G)$, for all $g \in G$. The set of all such automorphisms is denoted by $\mathrm{Aut}_c(G)$.

Now, we call $\alpha \in \mathrm{Aut}(G)$ to be autocentral automorphism, when $[g,\alpha] = g^{-1}g^{\alpha} = g^{-1}\alpha(g) \in L(G)$, for all $g \in G$. The set of all such automorphisms is denoted by

$$\mathrm{Var}(G) = \{\alpha \in \mathrm{Aut}(G) \mid [g,\alpha] \in L(G),\ for\ all\ g \in G\}$$

Clearly, $\mathrm{Var}(G)$ is a normal subgroup of $\mathrm{Aut}(G)$ contained in $\mathrm{Aut}_c(G)$.

The properties of $\mathrm{Aut}_c(G)$ are studied by many autheurs, see for instance [1, 2, 4, 6]. In the present article, we study some properties of autocentral automorphism of a given group G. A non-abelian group G that has no-non trivial abelian direct product factor is said to be purely non-abelian. In [5] M. R. R. Moghaddam and H. Safa show that if G is a purely non-abelian finite group, then $\mathrm{Var}(G) \cong$

Keywords : *Autocentral Automorphisms, absolute centre, Central Automorphism*
2010 Mathematics Subject Classification : **20D15, 20D45**

$\mathrm{Hom}(G, L(G))$. In [2] M. J. Curran and D. J. McCaughan characterized finite p-group G for which $\mathrm{Aut}_c(G) = \mathrm{Inn}(G)$. Also M. S. Attar [6] characterized finite p-group G for which $\mathscr{C}_{\mathrm{Aut}_c(G)}(Z(G)) = \mathrm{Inn}(G)$, where $\mathscr{C}_{\mathrm{Aut}_c(G)}(Z(G)))$ be the set of all central automorphisms of G fixing $Z(G)$ elementwise.

In this paper, we characterize finite p-groups G whit this $\mathrm{Var}(G) = \mathrm{Inn}(G)$. Also, we proved that if G is a finite p-group, then $\mathscr{C}_{\mathrm{Var}(G)}(Z(G)) = \mathrm{Inn}(G)$ if and only if G is nilpotent of class 2, $G' \leqslant L(G)$ and $L(G)$ is cyclic.

Main results Let σ be a autocentral automorphism of G. Clearly the map $f_\sigma : x \longmapsto x^{-1}\sigma(x)$ defines a homomorphism from G into $L(G)$. On the other hand, the map $\sigma_f : x \longmapsto xf(x)$ define an endomorphism of G for all f in $\mathrm{Hom}(G, L(G))$. This endomorphism is a autocentral automorphism if and only if $f(l) \neq l^{-1}$ for every l in $L(G)$. As any homomorphism $f : G \longrightarrow L(G)$ induces a homomorphism $f : G/G' \longrightarrow L(G)$, and vice versa, we see that $|\mathrm{Hom}(G, L(G))| = |\mathrm{Hom}(G/G', L(G))|$.

Theorem 0.1. *Let G is a finite p-group, with $G' \leqslant L(G)$ and $L(G)$ is cyclic. Then* $|\mathrm{Var}(G) : \mathrm{Inn}(G)| \leq |Z(G) : L(G)|$. *Moreover, if* $\exp(G/L(G)) \leq \exp(L(G))$ *then* $|\mathrm{Var}(G) : \mathrm{Inn}(G)| = |Z(G) : L(G)|$.

Theorem 0.2. *Let G be a purely non-abelian p-group, with $L(G)$ is cyclic. If* $\mathrm{Inn}(G)$ *contained in* $\mathrm{Var}(G)$, *then* $|\mathrm{Var}(G) : \mathrm{Inn}(G)| \leq |Z(G) : G'|$. *Furthermore, if* $\exp(G/G') \leq \exp(L(G))$, *then* $|\mathrm{Var}(G) : \mathrm{Inn}(G)| = |Z(G) : G'|$ *and* $L(G) = G'$.

Theorem 0.3. *If G is a non-abelian finite p-group, then* $\mathscr{C}_{\mathrm{Var}(G)}(Z(G)) = \mathrm{Inn}(G)$ *if and only if G is nilpotent of class 2, $G' \leqslant L(G)$ and $L(G)$ is cyclic.*

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Nonlinear second order fuzzy differential equations by fixed point in partially ordered sets

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Abstract This paper is devoted to prove the existence and uniqueness of solution for second order fuzzy differential equation with initial conditions admitting only the existence of a lower solution or an upper solutions.

Introduction The study of fuzzy differential equations is an area of mathematics that has recently received a lot of attention. Recently, there are some papers dealing with the existence of solution for nonlinear fuzzy differential equations by the method of upper and lower solutions and fixed point theorems[5, 2, 1]. Among of them, we can find results on existence of solution for fuzzy differential equations in presence of both lower and upper solutions relative to the problem considered. In this study, we Consider the following initial value problem for second Order fuzzy differential equation

$$u''(t) = f(t, u(t), u'(t)), \quad u(a) = u_{01}, u'(a) = u_{02}, \tag{63}$$

where $f : [a, b] \times \mathbb{R}^2_{\mathscr{F}} \to \mathbb{R}_{\mathscr{F}}$ is continuous in all of their arguments. All initial conditions are supposed to be fuzzy numbers. Here, We consider just only a lower solution or an upper solution for the above problem and use fixed point in partially ordered sets to prove the existence results.

Preliminaries In this paragraph we gather together some definitions and results from the literature, which we will use throughout this paper.

$\mathbb{R}_{\mathscr{F}}$ denotes the space of fuzzy numbers on $\mathbb{R}$. Throughout this paper, we consider $J = [a, b]$ and

$$C^2(J, \mathbb{R}_{\mathscr{F}}) = \{y : J \to \mathbb{R}_{\mathscr{F}} \mid y^{(j)} \text{ is (i)-differentiable and continuous ; } j = 0, 1\},$$

where differentiability at the endpoints a and b, is interpreted right and left differentiability at these points respectively. In vector form, for $\Phi = (\phi_1, \phi_2), \Psi = (\psi_1, \psi_2) \in C(J, \mathbb{R}_{\mathscr{F}}) \times C(J, \mathbb{R}_{\mathscr{F}})$, we define

$$\mathscr{H}(\Phi, \Psi) = max\{H(\phi_1, \psi_1), H(\phi_2, \psi_2)\},$$

where $H(u, v) = \sup_{t \in J} D(u(t), v(t))$. The metric space $(C(J, \mathbb{R}_{\mathscr{F}}) \times C(J, \mathbb{R}_{\mathscr{F}}), \mathscr{H})$ is a complete space.

Definition 4. *Suppose $x, y \in \mathbb{R}_{\mathscr{F}}$. We say that $x \le y$ if and only if $x_{\alpha l} \le y_{\alpha l}$, and $x_{\alpha r} \le y_{\alpha r}$, $\forall \alpha \in [0, 1]$.*

Let $h_1, h_2 \in C(J, \mathbb{R}_{\mathscr{F}})$ be two fuzzy functions, we say that $h_1 \le h_2$ if $h_1(t) \le h_2(t)$ for $t \in J$.

Lemma 20. *If a nondecreasing (or nonincreasing) sequence $f_n \to f$ in $C(J, \mathbb{R}_{\mathscr{F}})$, then $f_n \le f$ (or $f_n \ge f$), $\forall n$ respectively.*

Keywords : *Fuzzy differential equations; Fixed points; Monotone Method.*
2010 Mathematics Subject Classification : **34A07; 45J05; 34A12.**

Proof. Since f_n is nondecreasing sequence in $C(J,\mathbb{R}_{\mathscr{F}})$, $f_n(t)$ is nondecreasing sequence in $\mathbb{R}_{\mathscr{F}}$ for $t \in J$. Also we have $\forall \alpha \in [0,1]$, $(f_1(t))_{\alpha l} \le (f_2(t))_{\alpha l} \le ... \le (f_n(t))_{\alpha l} \le ....$ Hence $(f_n(t))_{\alpha l}$ is a nondecreasing sequence that converges to $(f(t))_{\alpha l}$ in $\mathbb{R}$. Therefore $(f_n(t))_{\alpha l} \le (f(t))_{\alpha l}$ for every n. Similarly we conclude $(f_n(t))_{\alpha r} \le (f(t))_{\alpha r}$ for every n. Thus $f_n \le f$ for every n. The similar result can be conclude for nonincreasing function. □

Definition 5. *Suppose $X = (x_1, x_2), Y = (y_1, y_2) \in \mathbb{R}_{\mathscr{F}} \times \mathbb{R}_{\mathscr{F}}$. We say that $X \preceq Y$ if and only if*

$$x_1 \le y_1, \quad \text{and} \quad x_2 \le y_2.$$

Theorem 21. *(See [3, 4].) Let $(X, \preceq)$ be a partially ordered set and suppose that d be a metric on X such that (X,d) is a complete metric space. Furthermore, let $T : X \to X$ be a monotone nondecreasing mapping such that*

$$\exists\, 0 \le k < 1 \ni d(T(x), T(y)) \le kd(x, y), \quad \forall x \ge y.$$

Suppose that either f is continuous or X is such that if $\{x_n\} \to x$ is a nondecreasing (or respectively nonincreasing) sequence in X, then $x_n \le x$ (or respectively $x_n \ge x$) for every $n \in \mathbb{N}$. If there exists $x_0 \in X$ comparable to $T(x_0)$, then T has a fixed point $\bar{x}$ and

$$\lim_{n\to\infty} T^n(x_0) = \bar{x}.$$

Main Results In this paragraph, we consider the following initial value problem for the second order fuzzy differential equation of Volterra type

$$u''(t) = f(t, u(t), u'(t)), \;\; u(a) = u_1^0, u'(a) = u_2^0, \; t \in J = [a, b], \tag{64}$$

where $f \in C(J \times \mathbb{R}^{(2)}_{\mathscr{F}}, \mathbb{R}_{\mathscr{F}})$ and u_0, u_1 are the fuzzy numbers.

Definition 6. *Let $u \in C^2(J,\mathbb{R}_{\mathscr{F}})$, we say u is a solution of* (64), *if u and u' be (i)-differentiable on the entire J and, moreover, u and u' satisfy (**??**).*

We can reduce (64) to the following system of two first order fuzzy integro-differential equations

$$v_1'(t) = v_2(t), \;\; v_2'(t) = f(t, v_1(t), v_2(t)), t \in J \;\; \text{and} \;\; v_1(a) = u_{01}, v_2(a) = u_{02}. \tag{65}$$

For convenience, we apply vector notations $V(t) = \begin{bmatrix} v_1(t) \\ v_2(t) \end{bmatrix}$, $V'(t) = \begin{bmatrix} v_1'(t) \\ v_2'(t) \end{bmatrix}$ and rewrite the problem (**??**) as

$$V'(t) = \begin{bmatrix} v_2(t) \\ f(t, v_1(t), v_2(t)) \end{bmatrix}, \quad V(a) = \begin{bmatrix} u_{01} \\ u_{02} \end{bmatrix}.$$

Lemma 22. *(See [2].) The problem* (66) *is equivalent to the following integral equations system*

$$V(t) = \begin{bmatrix} u_{01} + \int_a^t v_2(s)ds \\ u_{02} + \int_a^t f(s, V(s))ds \end{bmatrix}, \tag{66}$$

if v_1, v_2 be both (i)-differentiable on J.

Now we are in a situation to define the nonlinear mappings $\mathscr{A} : C(J,\mathbb{R}_{\mathscr{F}}) \times C(J,\mathbb{R}_{\mathscr{F}}) \to C(J,\mathbb{R}_{\mathscr{F}}) \times C(J,\mathbb{R}_{\mathscr{F}})$, which plays a main role in our discussion, as following

$$[\mathscr{A}\Phi](t) = \begin{bmatrix} [\mathscr{A}_1\Phi](t) \\ \mathscr{A}_2\Phi](t) \end{bmatrix} = \begin{bmatrix} u_{01} + \int_a^t \phi_2(s)ds \\ u_{02} + \int_a^t f(s, \Phi(s))ds \end{bmatrix},$$

where $t \in J$ and $\Phi(t) = \begin{bmatrix} \phi_1(t) \\ \phi_2(t) \end{bmatrix}$.

Definition 7. *Let* $\underline{U} = \begin{bmatrix} \underline{u}_1 \\ \underline{u}_2 \end{bmatrix}, \bar{U} = \begin{bmatrix} \overline{u}_1 \\ \overline{u}_2 \end{bmatrix} \in C(J,\mathbb{R}_{\mathscr{F}}) \times C(J,\mathbb{R}_{\mathscr{F}})$*, we say that* $\underline{U}$ *is a lower solution and* $\bar{U}$ *is an upper solution for the problem* (66) *if respectively*

$$\underline{U}(t) \preceq [\mathscr{A}\underline{U}](t), \quad and \quad \bar{U}(t) \succeq [\mathscr{A}\bar{U}](t), \qquad t \in J.$$

Theorem 23. *Consider Problem* (64) *with* f *continuous and suppose* f *is nondecreasing in all its arguments except for the first. Let exist* $l_1, l_2 > 0$ *such that*

$$D(f(t,x_1,x_2), f(t,y_1,y_2)) \leq l_1 max\{D(x_1,y_1), D(x_2,y_2)\}, \quad \forall t \in J, \tag{67}$$

for $x_1 \succeq y_1$ *and* $x_2 \succeq y_2$*. And assume* $max\{b-a, l_1(b-a)\} < 1$*. Then the existence of a lower solution* $\underline{U}$ *(or an upper solution* $\bar{U}$*) for Problem* (64) *provides the existence of a solution to* (64) *and* $\lim_{n\to\infty} \mathscr{A}^n(\underline{U}) = U$ *(or* $\lim_{n\to\infty} \mathscr{A}^n(\bar{U}) = U$*). Moreover, if* $W \in C(J,\mathbb{R}_{\mathscr{F}}) \times C(J,\mathbb{R}_{\mathscr{F}})$ *ia another fixed point of* $\mathscr{A}$ *such that* $\underline{U} \preceq W$ *(or* $W \preceq \bar{U}$*), then* $U = W$.

Proof. Since by Lemma 22, Problem (64) is equivalent to (66), we prove that the mapping (67) has a unique fixed point. To this end, We check that hypotheses in Theorem 21 are satisfied.
We consider $X = C(J,\mathbb{R}_{\mathscr{F}}) \times C(J,\mathbb{R}_{\mathscr{F}})$ that is partially ordered set by the following order relation For $G, F \in C(J,\mathbb{R}_{\mathscr{F}}) \times C(J,\mathbb{R}_{\mathscr{F}})$,

$$G \preceq F \Leftrightarrow G(t) \preceq F(t), \ \forall t \in J$$

The mapping $\mathscr{A}$, defined by (67), is nondecreasing, Since f, k are nondecreasing in all their arguments except for the first. Besides, for $\Phi \succeq \Psi$,

$$D([\mathscr{A}_1\Phi](t), [\mathscr{A}_1\Psi](t)) \leq \int_a^t D(\phi_2(s), \psi_2(s))ds \leq (b-a)H(\phi_2, \psi_2), \tag{68}$$

and also,

$$D([\mathscr{A}_2\Phi](t), [\mathscr{A}_2\Psi](t)) \leq \int_a^t D(f(s,\Phi(s)), f(s,\Psi(s)))ds \leq l_1(b-a)max\{H(\phi_1,\psi_1), H(\phi_2,\psi_2)\}. \tag{69}$$

Then from (68) and (69), we have

$$\mathscr{H}(\mathscr{A}\Phi, \mathscr{A}\Psi) \leq L\mathscr{H}(\Phi,\Psi), \tag{70}$$

where $L = max\{b-a, l_1(b-a)\}$ which by the assumption, $L < 1$. Applying Theorem 21, $\mathscr{A}$ has a fixed point $U \in C(J,\mathbb{R}_{\mathscr{F}}) \times C(J,\mathbb{R}_{\mathscr{F}})$. Since $u_1 \in C(J,\mathbb{R}_{\mathscr{F}})$ and $u_1' = u_2 \in C(J,\mathbb{R}_{\mathscr{F}})$, $u_1 \in C^1(J,\mathbb{R}_{\mathscr{F}})$. From (66), $u_1 \in C^2(J,\mathbb{R}_{\mathscr{F}})$ is a solution of Problem (64).
Now suppose $W \in C(J,\mathbb{R}_{\mathscr{F}}) \times C(J,\mathbb{R}_{\mathscr{F}})$ is another fixed point of $\mathscr{A}$ such that $\underline{U} \preceq W$. We prove that $\mathscr{H}(U,W) = 0$, where $\lim_{n\to\infty} \mathscr{A}^n(\underline{U}) = U$. Employing the nondecreasing property of the mapping $\mathscr{A}$, along with Lemma 20, $\mathscr{A}^n\underline{U}$ is comparable to $\mathscr{A}^n U = U$ and $\mathscr{A}^n W = W$ for $n = 0,1,2,\ldots$ and we have

$$\mathscr{H}(U,W) \leq \mathscr{H}(\mathscr{A}^n U, \mathscr{A}^n \underline{U}) + \mathscr{H}(\mathscr{A}^n W, \mathscr{A}^n \underline{U}) \leq L^n \mathscr{H}(U,\underline{U}) + L^n \mathscr{H}(\underline{U}, W).$$

As $n \to \infty$, the right-hand side of above equation converges to zero. Then $\mathscr{H}(U,W) = 0$. In the case $W \preceq \bar{U}$, the proof is similar. □

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On the Hochstadt–Lieberman theorem for Singular canonical Dirac operator

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Abstract The basic and comprehensive results about regular Dirac operators were given in [1]. Furthermore, direct or inverse spectral problems for Dirac operators were extensively studied in [2-8]. In this paper, by using the Hochstadt and Lieberman's method in [9], we discuss a half inverse problem for canonical Dirac operator with singular potential and show that if potential functions p(x) and q(x) are prescribed on semi interval, then the functions p(x) and q(x) on the whole interval are uniquely determined by one spectrum.

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Keywords : *Dirac operator; eigenvalue; spectrum; potential function.*
2010 Mathematics Subject Classification : **34B24; 47E03.**

The existence result of the equation of the motion of a particle in a viscous fluid

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Abstract In this paper, we will prove the existence solution of an implicit second order integro-differential equation which is the equation of moving a particle in a viscous fluid. The proof has been done without using Lipschitz condition.

Introduction In an unsteady stokes flow, the totalhydrodynamic force exerted on a moving particle in a viscous medium can be described as:

$$F = \frac{9\mu_f \nu}{2R^2} V + \frac{1}{2}\rho_f \nu \frac{dV}{dt} + \frac{9\mu_f \nu}{2R}\sqrt{\frac{\nu_f}{\pi}} \int_0^t \frac{dV}{d\tau} \frac{d\tau}{\sqrt{t-\tau}}$$

The last term which represents the effect of particle acceleration in the past tense on its current motion is known as the Basset force. In a more general condition, this term would be expressed by:

$$F_h = \int_0^t h''(\tau) K(t, t-\tau) d\tau$$

Using Newton's second low, the problem is reduced to integro-differential equation with a weakly singular kernel. So, the general form for equation of particle motion can be written as:

$$h''(t) = f(t, h(t), h'(t)) + \int_0^t k(t, s, h(s), h'(s), h''(s)) ds$$

This equation in both mathematical and mechanical points of view have been studied widely (See [1], [2] and [3]). In this paper we will prove the existence solution without using Lipschitz condition.

Main results Consider the nonlinear second order implicit integro-differential equation

$$h''(t) = f(t, h(t), h'(t)) + \int_0^t k(t, s, h(s), h'(s), h''(s)) ds, \qquad t \in [0, b] \tag{71}$$

with the initial conditions

$$h(0) = h_0, \ h'(0) = v_0 \tag{72}$$

which the kernel has a weak singularity as $s \to t$. Consider the following assumption:
(i) For $t \in [0, b]$, $s \in [0, t]$ and $h, h', h'' \in \mathbb{R}$, there exist $\nu \in (0, \frac{1}{2})$ and a continuous function k_1 such that:

$$|k(t, s, h, h', h'')| = |t-s|^{-\nu} k_1(h, h', h'').$$

Keywords : *particle motion; schauder fixed point theorem; implicit integro-differential equation.*
2010 Mathematics Subject Classification : **34K32, 34A12.**

(ii) $f \in C([0,b] \times \mathbb{R} \times \mathbb{R})$.

Let

$$u(t) := h''(t). \tag{73}$$

Then

$$h'(t) = v_0 + \int_0^t u(x)dx \tag{74}$$

and

$$h(t) = h_0 + v_0 t + \int_0^t (t-x)u(x)dx. \tag{75}$$

By substituting (73), (74) and (75) in (71), we have

$$u(t) = f\left(t, h_0 + v_0 t + \int_0^t (t-x)u(x)dx, v_0 + \int_0^t u(x)dx\right) + \int_0^t k\left(t, s, h_0 + v_0 s + \int_0^s (s-x)u(x)dx, v_0 + \int_0^s u(x)dx, u(s)\right)ds.$$

To simplify the notation, we define

$$\phi(t,u) = f\left(t, h_0 + v_0 t + \int_0^t (t-x)u(x)dx, v_0 + \int_0^t u(x)dx\right) \tag{76}$$

$$labeleq: 1.76\, \kappa(t,s,u(s)) = k\left(t, s, h_0 + v_0 s + \int_0^s (s-x)u(x)dx, v_0 + \int_0^s u(x)dx, u(s)\right). \tag{77}$$

Theorem 24. *Under the assumption (i)-(ii) the initial value problem* (71)–(72) *has a solution.*

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A new INSAR flattening algorithm based on modulation of the fringes frequency

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Abstract The interferogram flattening is an important and fundamental task in the interferometric SAR (INSAR) processing; it is the procedure for removing the flat-earth phase to extract the topographic fringe that can be used in the phase unwrapping method to generate digital elevation model (DEM). This paper describes and compares two conventional flattening methods, the geometric method and the fringe frequency method calculated from the interferogram spectrum. In the first, the orbit ephemeris can be inaccurate or even unknown; the fringe frequency algorithm is effective but doesn't respect the real length fringe which must increase between the near range to the far range. We present a novel flattening algorithm by combining the two cited algorithms with some experimental results.

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Keywords : *interferometry SAR; orbit data; flattening interferogram; flat-Earth phase.*
2010 Mathematics Subject Classification : **11Y16; 94B12**

Application of the method of integral equations in the problem of electrical sounding for a three-dimensional media with topography

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Abstract Theory of interpretation of electromagnetic fields studied in the electrical sounding with direct current, mainly developed for the case of a horizontal surface observations. However in practice we often have to work in difficult terrain surface. Conducting interpretation without the influence of topography can cause non-existent anomalies on paragraphs. This raises the problem of studying the impact of different shapes of ground surface relief on the results of electrical prospecting's research. Ground surface topography greatly complicates the task of solving the inverse problem of finding the distribution of resistivity according to physical measurement of the components of the electric field on the ground surface. Excluding the distorting factors in the solution of the inverse problem, or some version of a two-dimensional or three-dimensional inversion, it is impossible to get closer to the construction of final paragraphs that are reasonably accurate. On the other hand, purposes of geophysics have high requirements on accuracy and speed of field computations. Both of these requirements are satisfied by the method of integral equations. This research examines the numerical solutions of the direct problem of electrical sounding for three-dimensional media taking into account the terrain on the basis of the integral equations method. The Maxwell's equations for stationary field are used as a mathematical model to describe the electromagnetic processes in the medium. The idea of integral equations method is to provide the electric field as a sum of the primary field (generated by current electrodes) and the field of secondary charges (occurs when the electric current flows in points of homogeneity violation of the subsurface medium and on the surface of the medium). Contact boundaries and heterogeneous inclusions of the geoelectric paragraph act as secondary creators of the electric field. The field computation problem is reduced to the system of integral equations on the current density of secondary sources, inducted on contact surfaces of conductive mediums and on the ground surface of the medium. The mathematical description of this event leads to the Fredholm equations of type II. In study the numerical mathematical algorithm and computer program solving the direct problem of electrical sounding with direct current for three-dimensional media accounting for the terrain effects were developed. The results will improve the quality of work and the development of computer technology used in geophysics. In general, the improvement of the quality of interpretation due to better accordance with local conditions increases the efficiency with geophysical research.

Keywords : *method of integral equations; electrical impedance tomography; ground surface relief.*

2010 Mathematics Subject Classification : 31B10; 45B05.

Sumudu applications to stochastics, statistics, and correlation theory

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Abstract In this paper, we present the theoretical properties pertinent to the Sumudu transform application to stochastics related to prediction theory. Moreover , we apply the Sumudu to statistical densities and distributions thereby revealing new expression connected with moments. Furthermore, we reveal how such developed tools can help further understand statistical correlation theory. In particular. we show how this connects with student faculty evaluations vs students grades by evaluated faculty

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Keywords : *Sumudu Transform; Sumudu Laplace Transform; Fourier Transform Correlation*
2010 Mathematics Subject Classification : **4A10; 44A15; 44A35;**

On some new complex analytical solutions for the nonlinear long short-wave interaction system with complex structure

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Abstract In this paper, we develop a new analytical method called as the modified exp $(-\Omega(\xi))$-expansion function method giving more analytical solutions for partial differential equations with have powerful nonlinearity especially, and based on the $\exp(-\Omega(\xi))$-expansion method. We have applied this new approach to the nonlinear long short-wave interaction system being stand for relationships among waves such as water, gravity, high cosmic and so on. Afterwards, we have found some new hyperbolic function solution, trigonometric function solution and complex function solution for the nonlinear long short-wave interaction system by using this new the modified $\exp(-\Omega(\xi))$-expansion function method.

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Keywords : *The modified exp$(-\Omega(\xi))$-expansion function method, Long short-wave interaction system, Hyperbolic function solution, Trigonometric function solution, Complex function solution.*

2010 Mathematics Subject Classification : **41AXX; 34MXX; 35DXX; 30E05; 30E10;**

On some properties of the fractional derivative of the Riemann zeta function

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Abstract Fractional derivative of Riemann zeta function is examined along with its convergence properties. Indeed, it is shown how this fractional derivative has remarkable properties in Analytic Number Theory and its main applications in Physics, Dynamical Systems and Chaos Theory. The results prove how it could be useful to apply the Fractional Calculus to other fields of Mathematics.

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Keywords : *Fractional calculus; Ortigueira–Caputo derivative; Riemann zeta function; Convergence*
2010 Mathematics Subject Classification : **26A33.**

The application of support vector machines for the classification of multiple sclerosis subgroups

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Abstract The study has classified the subgroups of Multiple Sclerosis using Support Vector Machine. For this purpose, 120 MS patients and 19 healthy individuals have been used in our study. Through Magnetic Resonance Imaging (MRI), lesion numbers, lesion sizes and Expanded Disability Status Scale data are applied through Support Vector Machine. Having applied Support Vector Machine learning on MS subgroups, classification achievement of MS subgroups, namely that of RRMS, SPMS and PPMS has been measured.

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Keywords : *Support vector machine; multiple sclerosis; magnetic resonance imaging; expanded disability status scale; RRMS; SPMS; PPMS.*
2010 Mathematics Subject Classification : **60J45; 20K27.**

Classification Results for L_r-Biharmonic Hypersurfaces in $\mathbb{E}^{n+1}$

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Abstract A hypersurface $x : M^n \to \mathbb{E}^{n+1}$ is said to be biharmonic if $\Delta^2 x = 0$, where Δ is the Laplace operator of M^n. Chen conjecture states that every Euclidean biharmonic hypersurface is minimal. A hypersurface $x : M^n \to \mathbb{E}^{n+1}$ is called L_r-biharmonic if $L_r^2 x = 0$, where L_r is the linearized opereator of $(r+1)$th mean curvature of M^n. Since $L_0 = \Delta$, the subject of L_r-biharmonic hypersurface is an extension of biharmonic ones. In this paper, we consider the chen conjecture for L_r-biharmonic hypersurfaces in $\mathbb{E}^{n+1}$, when $n = 2, 3, 4$.

Introduction The study of biharmonic maps has several physical and geometric motivations. For instance, one can find the role of biharmonic maps in the theory of elastics and fluid mechanics in [7]. The theory of biharmonic maps plays a central role in various fields in differential geometry, computational geometry and the theory of Partial differential equations. In eighteen decade, Bang Yen Chen initiated to investigate the differential geometric properties of biharmonic submanifolds in the Euclidean spaces. He introduced some open problems and conjectures (in [4]), among them, a long-standing conjecture says that a biharmonic submanifold in a Euclidean space is a minimal one. Chen himself has proved the conjecture for surfaces in $\mathbb{E}^3$. Later on, T. Hasanis and T. Vlachos ([6]) have verified the conjecture for hypersurfaces in $\mathbb{E}^4$. Recently, in [5], it is proved that the only biharmonic hypersurfaces with three distinct principal curvatures in $\mathbb{E}^5$ are minimal ones.
The biharmonicity condition on any hypersurface $x : M^n \to \mathbb{E}^{n+1}$ is defined by $\Delta^2 x = 0$, where Δ is the The Laplace operator which can be seen as the first one of a sequence of n operators $L_0 = \Delta, L_1, \ldots, L_{n-1}$, where L_r stands for the linearized operator of the first variation of the $(r+1)$th mean curvature arising from normal variations of the hypersurface (see, for instance, [8]). These operators are given by $L_r(f) = tr(P_r \circ \nabla^2 f)$ for any $f \in C^\infty(M)$, where P_r denotes the rth Newton transformation associated to the second fundamental from of the hypersurface and $\nabla^2 f$ is the hessian of f. From this point of view, it seems interesting to generalize the definition of biharmonic hypersurface by replacing Δ by L_r.
Let $x : M^n \to \mathbb{E}^{n+1}$ be a connected orientable hypersurface immersed into the Euclidean space, with Gauss map N. By definition, M^n is called a L_r-biharmonic hypersurface if its position vector field satiesfies the condition $L_r^2 x = 0$. By the equality $L_r x = c_r H_{r+1} N$ from [3], the condition $L_r^2 x = 0$ has another equivalent expression as $L_r(H_{r+1}N) = 0$. It is clear that, r-minimal hypersurface is L_r-biharmonic. One can ask naturally :
"Is there any L_r-biharmonic hypersurface other than r-minimal ones ?"
Here, we solve the problem for hypersurfaces in $\mathbb{E}^{n+1}$ when $n = 2, 3, 4$. In special case, we prove that every L_1-biharmonic surface in $\mathbb{E}^3$ is flat, and we show that each L_r-biharmonic hypersurface in $\mathbb{E}^4$ with constant r-th mean curvature is r-minimal. We also prove that any L_1-biharmonic hypersurface in $\mathbb{E}^5$ with constant mean curvature is 1-minimal.

Keywords : *Linearized operator L_r, L_r-biharmonic hypersurfaces, r-minimal.*
2010 Mathematics Subject Classification : **53-02; 53C40; 53C42; Secondary 58G25.**

By formula in [3] page 122, we have

$$L_r^2 x = -2c_r(S\circ P_r)(\nabla H_{r+1}) - c_r\binom{n}{r+1}H_{r+1}\nabla H_{r+1} - c_r(tr(S^2\circ P_r)H_{r+1} - L_r H_{r+1})N, \tag{78}$$

where $c_r = (r+1)\binom{n}{r+1}$.

By using this formula for $L_r^2 x$ and the identifying normal and tangent parts of the L_r-biharmonic condition $L_r^2 x = 0$, one obtains necessary and sufficient conditions for M^n to be L_r-biharmonic in $\mathbb{E}^{n+1}$, namely

$$L_r H_{r+1} = tr(S^2\circ P_r)H_{r+1} \tag{79}$$

and

$$(S\circ P_r)(\nabla H_{r+1}) = -\frac{1}{2}\binom{n}{r+1}H_{r+1}\nabla H_{r+1}. \tag{80}$$

Main results

Theorem 25. *Every L_1-biharmonic surface in $\mathbb{E}^3$ is flat.*
Every L_r-biharmonic hypersurfaces in $\mathbb{E}^4$ with contant r-th mean curvature is r-minimal.
Every L_1-biharmonic hypersurfaces in $\mathbb{E}^5$ with contant mean curvature is 1-minimal.

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Computational experimentations for physical properties of multicomponent fluorohafnate glasses

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Abstract Fluoride glasses based on HfF4 have been synthesized in the HfF4-SrF2-BaF2 system. In order to decrease the crystallization rate, various fluorides (PbF2, ZnF2, AlF3 and YF3) have been introduced in the 66HfF4-22SrF2-12BaF2 composition as substituent of alkali earth cations. Glass transition temperature is close to 320 ?C, and coefficient of thermal expansion is 155 10-7 K-1. Microhardness and elastic moduli have been measured. Values of refractive index are given for wavelengths ranging from 633 nm to 1551 nm. By comparison to fluorozirconates, fluorohafnate glasses exhibit larger density, lower refractive index, lower phonon energy and extended transmission in the mid-infrared spectrum. Potential applications relate to active optical fibres and supercontinuum generation.

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Keywords : *Computational methods; Fluoride glass; infrared; refractive index.*
2010 Mathematics Subject Classification : **20B40; 74F05.**

A new improved version of optimal control solving nonlinear model of fuzzy discrete and continuous multi-objectives Gabassov (Kirillova) exact methods

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Abstract We treat in this paper the Methods (support ; Adaptee) Invented By Gabassov and Kirillova for a revision and improvement for the continuous case, essentially the discrete cases (decisions of the variables in the set N) Noting that the problems which we We are interested , for resolution , are nonlinear problems while methods were initially planned for linear cases . First, we solve a nonlinear mono objectives program , then another nonlinear Objectives multi program in real variables and Finalemment a nonlinear multi Program Objectives Discreet .We therefore get **new criteria of optimality and suboptimality for this method**. This will in fact write the new algorithms that will meet our new exigeances. We solve the mathematical programs, and then we will make applications to cases of Optimal Control , continuous type and discreet . We treat 02 cases , **determinist paramaters and fuzzy parameters** .

Introduction One of our objectives in this work is the resolution in optimal control problem following ,**with determinist paramaters and fuzzy parameters** :-

$$\begin{cases} J_k(u) = \frac{c_k' x(T_1)+a_k}{d_k' x(T_1)+e_k} \to \max \\ \frac{dx(t)}{dt} = Ax(t) + bu(t), x(0) = x_0 \\ H(x(T_1)) = g; u \in N \\ d_* \leq u(t) \leq d^*; 1 \leqslant k \leqslant p; t \in [0, T_1] \end{cases}$$

The start will be done by non-linear program resolutions mono , Objectives Multi by Method Adaptee (Gabassov) in the continuous case, modeling the existing algorithm oriented linear another faster to find the Effective solutions . The Goal next is to develop an algorithm Multi Criteria (Multi Objectives) to discrete variables (continuous) for Method Adaptee (Gabassov) this will allow us to reach quadratic problems and thus to generalize to the nonlinear case Multi Criteria along with :- .

- Addapte Method ; support Method and Anothers - Designing a Program in Deployment able to support our methods with a professional Interface .

Because our goal is the effective Obtaining Global Solutions

Consider the following problem Mono Objective :- **Problem Mono Objective**

$f(x) = \frac{c'x+a}{d'x+e} \to \max$.
$Ax = b$, $d_1 \leqslant x \leqslant d_2$

x , c, d are elements of space R^n ; e et a are scalars .

DEFINITIONS.

Keywords : *Multi -Criteria - Pareto Optimal Solutions, efficient controls -Continuous - Discreet - efficient arrises - efficient faces -control Optimal continuous Discreet, method Suppot, Adaptee, Gomory cuts, cutting Danzig, efficient cuts , site urlhttp://damoum.voila.net.*

2010 Mathematics Subject Classification : **05D05 ;49J15;68N30 ;93B03 ;93B05;93B07;90C70;90C90;90C26;90C27;90C29.**

✓ Any vector x which verifies $Ax = b$, $d_1 \leq x \leq d_2$ called plan of Problem .
✓ Any plan x^0 is optimal if x^0 realize the maximum of fonctionnel $f(x)$
✓ Any plan x^ε is called ε - optimal if $f(x^0)$ - $f(x^\varepsilon) \leq \varepsilon$

Study of the increase in fonctional and optimality criteria
Increasing Functional
We conside the support- plan { x, J_B } of Problem and $\bar{x} = x + \Delta x$ another plan and we calculate the value representing the increase of f : ie the amount :-

$$\Delta f(x) = f(\bar{x}) - f(x)$$

I = 1...m : set of line Indices ,
J = 1...n : set of column Indices ,
✓ The set m indices $J_B \subset J$ of base and $J_H \subset J$ off Base .
the maximum of increased functional with condition that LISTED above gives:-

$$\begin{cases} \Delta x_j = d_2 j - x_j & \text{if } \Omega_j < 0 \\ \Delta x_j = d_1 j - x_j & \text{if } \Omega_j > 0, j \in J_H \end{cases}$$

is equal to :- $\beta = \beta(x, J_B) = \sum_{j \in J_H{}^+} \Omega_j (x_j - d_1 j) + \sum_{j \in J_H{}^-} \Omega_j (x_j - d_2 j)$
is called value of Suboptimality Support - plan $\{x, J_B\}$. With .
$J_H{}^+ = \{j \in J_H / \Omega_j \geq 0\}$; $J_H{}^- = \{j \in J_H / \Omega_j \leq 0\}$
from :-
$\Delta f(x) = f(\bar{x}) - f(x) \leq \beta(x, J_B)$ and for $\bar{x} = x^0$ being Optimal solution .
we get $0 \leq f(x^0) - f(x) \leq \beta(x, J_B)$
This last inequality , we deduce the optimality criterion.
THEOREM / OPTIMALITY CRITERION. The following relations :-

$$\begin{cases} x_j = d_2 j & \text{if } \Omega_j < 0 \\ x_j = d_1 j & \text{if } \Omega_j > 0 \\ d_1 j \leq x_j \leq d_2 j & \text{if } \Omega_j = 0, j \in J_H \end{cases}$$

are sufficient and in the case of non-degeneracy, they are necessary for the optimality of the support-plan $\{x, J_B\}$ ..

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Global existence and blowing up of solution for non linear wave equations of Kirchhoff type with a weak fractional damping

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Abstract This paper gives the sufficient conditions of blow-up of the solution of a non degenerate non linear wave equations of Kirchhoff type with a weak fractional damping in finite time, we use an argument due to Tatar [1] and proves the global solution of this problem.

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Keywords : *Global existence; Fractional derivative; Kirrchhoff equation; Blow up; Singular kernel.*
2010 Mathematics Subject Classification : **35L20; 35L75; 45K05.**

Characterizations of classical d-orthogonal polynomials

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Abstract In this paper we stand an algebraic theory of classical d-orthogonal polynomials and we want fill in some gaps. We broaden and close some inclusions that are exist and known perhaps as consequences. Several characterizations of d-classical OPS are given in terms of (d+2)-order recurrence relation as well as in terms of functional equations. A set of tools in determining the integral representation for such class is presented.

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Keywords : *Orthogonal polynomials; d-orthogonality; classical polynomials; recurrence relation; dual sequences.*
2010 Mathematics Subject Classification : **42C05; 33C45.**

Mathematical Modeling and Optimization of Chemical Batch Reactor

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Abstract Batch reactors are used extensively for the manufacture of small volume high value added products increasingly in production facilities intended for multipurpose use. To achieve stable and reproducible operational conditions is increasingly of importance to achieve the required product purity, optimum yields and cycle times to satisfy the relevant regulatory autheurities and commercial requirements. One of the challenges in batch reactors is to ensure desired performance of individual batch reactor operations. Depending on the requirement and the objective of the process, optimization in batch reactors leads to different types of optimization problems such as maximum conversion, minimum time and maximum profit problem. The paper is focused on analysis, mathematical modeling and optimization of batch reactor. Material and energy balances are the key issues of mathematical models of chemical reactors and processes. The combination with chemical kinetics and transport effects an intellectual basis for chemical reactor design can be obtained. A mathematical dynamic model is derived and the optimal parameters were computed.

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Keywords : *Batch reactor; modeling; reaction conduct; thermal stability; optimization.*

2010 Mathematics Subject Classification : **93A30; 47N10.**

Existence of positive weak solutions for a class of nonlocal elliptic systems

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Abstract In this paper, we study the existence of positive solutions to the following nonlocal elliptic systems

$$\begin{cases} -M_i\left(\int_\Omega |\nabla u_i|^{p_i}\,dx\right)\Delta_{p_i}u_i = \lambda_i f_i(u_1,\dots,u_m) \text{ in } \Omega, i=1,\dots,m \\ \\ u_i = 0 \text{ on } \partial\Omega, \forall i = 1,\dots,m \end{cases}$$

where $\Delta_{p_i}z = div\left(|\nabla z|^{p_i-2}\nabla z\right)$, $p_i \geq 1, \lambda_i, 1 \leq i \leq m$ are a positive parameter, and Ω is a bounded domain in $\mathbb{R}^N$ with smooth boundary $\partial\Omega$. The proof of the main results is based to the method of sub-supersolutions.

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Keywords : *Weak solutions; Positive solutions; Sub-supersolutions; Nonlocal Elliptic systems.*
2010 Mathematics Subject Classification : **35J60; 35B30; 35B40.**

The heat flux identification problem for a nonlinear parabolic equation in 2D

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Abstract We consider the heat identification problem based on the boundary measurements for nonlinear parabolic equation in 2-dimensional space. The linearization algorithm is applied to nonlinear direct problem. The Conjugate Gradient Algorithm, based on the gradient formula for the cost functional, is then proposed for numerical solution of the inverse heat flux problem. Due to noisy measured data, the Tikhonov regularization is considered. Numerical analysis of the algorithm applied to the inverse problem in typical classes of flux functions is presented. Computational results, obtained for random noisy output data, show how the iteration number of the Conjugate Gradient Algorithm can be estimated. Based on these results it is shown that this iteration number plays a role of regularization parameter. Numerical results illustrate bounds of applicability of proposed algorithm, also it efficiency and accuracy.

Introduction Suppose that $\Omega_T := \Omega\times(0,T)$, $\Omega := (0,\ell_x)\times(0,\ell_y)$ is a bounded domain. We consider the heat flux identification $f :=< f_1(x,t), f_2(y,t) >$ from the additional conditions (measurements)

$$\begin{cases} h_1(x,t) = u(x,0,t), \quad (x,t)\in\Gamma_1\times(0,T] \\ h_2(y,t) = u(0,y,t), (y,t)\in\Gamma_2\times(0,T], \end{cases} \tag{81}$$

where $\Gamma_1 := (0,\ell_x)$, $\Gamma_2 := (0,\ell_y)$ and the unknown function $u(x,y,t)$, $(x,y,t)\in\Omega_T$ is solution of the following nonlinear parabolic problem:

$$\begin{cases} u_t = (k(u)u_x)_x + (k(u)u_y)_y + F(x,y,t), \quad (x,y,t)\in\Omega_T \\ u(x,y,0) = u_0(x,y), \quad (x,y)\in\Omega, \\ -k(u(x,0,t))u_x(x,0,t) = f_1(x,t), \quad (x,t)\in\Gamma_1\times(0,T], \\ -k(u(0,y,t))u_y(0,y,t) = f_2(y,t), \quad (y,t)\in\Gamma_2\times(0,T], \\ u(x,y,t) = 0, \quad (x,y,t)\in\Gamma_3\times(0,T], \end{cases} \tag{82}$$

where $\Gamma_3 := \partial\Omega - (\Gamma_1\cup\Gamma_2)$. The problem (1)-(2) is defined as *nonlinear inverse heat flux problem* (NIHFP). In this case the problem (2) is defined as *Direct* or *Forward Problem* (DP) corresponding to the NIHFP.

Approximation We use the quasisolution approach for the considered inverse problem, introducing the cost functional

$$\begin{cases} J(f) = J_1(f) + J_2(f), \\ J_1(f) := \int_{\Gamma_1\times(0,T]}[u(x,0,t;f) - h_1(x,t)]^2 dxdt \\ J_2(f) := \int_{\Gamma_2\times(0,T]}[u(0,y,t;f) - h_2(y,t)]^2 dydt, \end{cases} \tag{83}$$

Keywords : *Heat flux identification; Tikhonov regularization; conjugate gradient method; nonlinear parabolic equation.*
2010 Mathematics Subject Classification : **35R30, 35K05.**

and weak solution theory for parabolic PDEs. Based on this approach we derive an explicit gradient formula for the functional $J(f)$ via the solution of appropriate adjoint problem, and then implement the Conjugate Gradient Algorithm (CGA) for numerical solution of the NIHFP. For the linearization of the Direct Problem (2), we use standard linearization algorithm defined as follows:

$$\left\{\begin{array}{l} u_t^{(n)} = (k(u^{(n-1)})u_x^{(n)})_x + (k(u^{(n-1)})u_y^{(n)})_y + F(x,y,t), \quad (x,y,t) \in \Omega_T \\ u^{(n)}(x,y,0) = u_0(x,y), \quad (x,y) \in \Omega, \\ -k(u^{(n)}(x,0,t))u_x^{(n)}(x,0,t) = f_1(x,t), \quad (x,t) \in \Gamma_1 \times (0,T], \\ -k(u^{(n)}(0,y,t))u_y^{(n)}(0,y,t) = f_2(y,t), \quad (y,t) \in \Gamma_2 \times (0,T], \\ u^{(n)}(x,y,t) = 0, \quad (x,y,t) \in \Gamma_3 \times (0,T], \\ n = 1,2,3,\ldots. \end{array}\right. \tag{84}$$

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On stability of fractional order model of HIV infparagraph

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Abstract In this paper we are concerned with the fractional order model of HIV Infparagraph of three types of cells, non-infected activated or cycling $CD4^+$ T cells, which we consider to be target cells, T, productively infected T cells, I, and HIV virus particles V. We study the stability of the of HIV model of fractional order by varying the infparagraph rate for different values of fractional order $0 < q \le 1$. We establish the stability results for non-infected and infected equilibrium points. Finally we illustrate our results on the stability of equilibrium points by using numerical simulations.

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Keywords : *Fractional differential equations; numerical simulations; HIV model.*
2010 Mathematics Subject Classification : **34D20; 34A08; 49K15; 46N40.**

Recursive approach for generating generalised Zernike polynomials

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Abstract An algorithmic approach for generating generalised Zernike polynomials by using differential operators and connparagraph matrices is proposed. This is done by introducing a new order of Zernike polynomials such that it collects all the polynomials of the same total degree in a column vector. The connparagraph matrices between these column vectors composed by the Zernike polynomials and a family of polynomials generated by a Rodrigues formula are given explicitly. This yields a Rodrigues type formula for the Zernike polynomials themselves with properly defined differential operators. Another consequence of our approach is the fact that the generalised Zernike polynomials obey a rather simple partial differential equation. We recall also how to define Hermite-Zernike polynomials.

Introduction We establish a recursive method for computing the generalised Zernike polynomials which are known to be orthogonal on the unit disc of $\mathbb{R}^2$ with respect to the weight function $\rho(x, y; \lambda) = (1-x^2-y^2)^\lambda$. The use of Zernike polynomials [4] for describing the classical aberrations of an optical system is well known [1]. There have been many other applications, such as to describe the statistical strength of aberrations produced by atmospheric turbulence, atmospheric thermal blooming effects, optical testing, opthalmic optics, corneal topography, just to mention a few.

The real generalised Zernike polynomials are defined as a product of angular functions and radial polynomials for $0 \le j \le 2m$, $0 \le \varrho < 1$, and $0 \le \theta < 2\pi$

$$Z_{m,j}(\varrho,\theta;\lambda) = \begin{cases} R_m^0(\varrho;\lambda), & m = [j/2], \\ R_m^{m-[j/2]}(\varrho;\lambda)\cos\left(\theta\left(m-[j/2]\right)\right), & m-[j/2] > 0,\ j+m^2 \text{ odd}, \\ R_m^{m-[j/2]}(\varrho;\lambda)\sin\left(\theta\left(m-[j/2]\right)\right), & m-[j/2] > 0,\ j+m^2 \text{ even}, \end{cases} \tag{85}$$

where $[x]$ denotes the integer part of x, and ordered in accordance with their total degree, with radial part

$$R_n^m(\varrho;\lambda) = \sum_{s=0}^{n-m} \frac{(-1)^s(\lambda-m+n+1)_n \varrho^{-m+2n-2s}(\lambda+n+1)_{-m+n-s}}{s!(n-s)!\binom{2n-m}{n-m}(-m+n-s)!}, \quad 0 \le m \le n, \quad \lambda > -1, \tag{86}$$

where $(A)_s = A(A+1)(A+s-1)$, $(A)_0 = 1$, denotes the Pochhammer symbol. For $\lambda = 0$ they coincide with Zernike polynomials [4]. They are the real and imaginary parts of those in complex coordinates introduced in [3]. Let $\mathbb{Z}_n^\lambda$ be the column vector of all Zernike polynomials of a fixed total degree s.

Keywords : *Generalised Zernike polynomials; Rodrigues-type formula; Ordering of Zernike polynomials; Bivariate orthogonal polynomials; Hermite-Zernike polynomials*
2010 Mathematics Subject Classification : **41A21; 41A27; 37K10; 47N20; 42C05; 33D45; 39A13.**

Main results We obtain a the Rodrigues-type formula, where the operators are defined recursively and can be constructed explicitly. It is clear from their definition that they are homogeneous partial differential operators with constant coefficients that do not depend on λ. This yields the equivalent but alternative way of building the generalised Zernike polynomials $\mathbb{Z}_n^\lambda$.

Let us also consider the bivariate polynomials defined by the Rodrigues-type formula [2]

$$P_{n,m}(x,y;\lambda) = \frac{1}{(1-x^2-y^2)^\lambda} \frac{\partial^{n+m}}{\partial x^n \partial y^m} \left[(1-x^2-y^2)^{n+m+\lambda} \right], \tag{87}$$

and $\mathbb{P}_n^\lambda = \left(P_{n,0}(x,y;\lambda), P_{n-1,1}(x,y;\lambda), \ldots P_{0,n}(x,y;\lambda) \right)^T$, as well as the connparagraph problem between (87) and (85).

It is worth emphasising that the connparagraph matrices $\mathbb{A}_n$ link the family of polynomials $\mathbb{P}_n^\lambda$ orthogonal only in subspaces to a full orthogonal system $\mathbb{Z}_n^\lambda$. A remarkable feature of the algorithm proposed is that the matrices $\mathbb{A}_n$ connecting the vector column of generalised Zernike polynomials $\mathbb{Z}_n^\lambda$ and the bivariate orthogonal polynomials $\mathbb{P}_n^\lambda$ do not depend on the parameter λ. This immediately implies another interesting result: the generalised Zernike polynomials $\mathbb{Z}_n^\lambda$ are solution of the same second-order linear partial differential equation of hypergeometric-type as $\mathbb{P}_n^\lambda$.

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Existence and multiplicity results for elliptic equation with singular growth

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Abstract In this work, we study an elliptic problem with singular coefficients and critical growth. We prove the existence and multiplicity results using variational methods. Resonant and non resonant cases are considered.

Introduction In this work we study the existence of nontrivial solutions to the semilinear elliptic problem

$$\begin{cases} -\Delta u - \mu \frac{u}{|x|^2} = \lambda f(x) u + |u|^{2^*-2} u & \text{in } \Omega \setminus \{0\}, \\ u = 0 & \text{on } \partial\Omega, \end{cases} \tag{88}$$

where Ω is a bounded domain in $\mathbb{R}^N$ $(N \geq 3)$ with $0 \in \Omega$, λ and μ are positive parameters such that $0 \leq \mu < \bar{\mu} = \left(\frac{N-2}{2}\right)^2$, $\bar{\mu}$ is the best constant in the Hardy inequality, $2^* = \frac{2N}{N-2}$ is the critical Sobolev exponent and f is a positive measurable function which will be specified later.

The study of this type of problems is motivated by its various applications, for example, it has been introduced as a model for nonlinear schrödinger equation with a singular potential. The mathematical interest lies in the fact that these problems are doubly critical due to the presence of the critical exponent and the Hardy potential.

Moreover if $\lambda \leq 0$ and Ω is starshaped, using Pohozaev identity; we prove that (88) has no solution. When $f \equiv 1$ the problem (88) has been deeply investigated in literature, we cite for example [2, 1, 4, 6]. The starting point to study these types of problems is the paper of Jannelli [6]. He proved the following results for $f \equiv 1$:

1) If $0 \leq \mu \leq \bar{\mu} - 1$, then (88) has at least one solution $u \in H_0^1(\Omega)$ for all $0 < \lambda < \lambda_1^\mu$ where λ_1^μ is the first eigenvalue of the operator $\left(-\Delta - \frac{\mu}{|x|^2}\right)$ in $H_0^1(\Omega)$.

2) If $\bar{\mu} - 1 < \mu < \bar{\mu}$, then (88) has at least one solution $u \in H_0^1(\Omega)$ for all $\mu^* < \lambda < \lambda_1^\mu$.

Ferrero and Gazzola [4] have showed the existence of solutions for $\lambda \geq \lambda_1^\mu$. Cao and Han [2] have completed the results obtained in [4].

When $f \not\equiv 1$, is a positive measurable function, Nasri [8] has extended the results of Jannelli [6] when f can be singular.

Borrowing ideas from [2] and [4], we give existence and multiplicity results when f is a singular function. Resonant and non resonant cases are considered.

Keywords : *Elliptic equations; Critical exponents; Singular terms.*

2010 Mathematics Subject Classification : **35J65; 35J20.**

Main results Our main reults, are the following:

Theorem 26. *Suppose that* $\mu \in \left[0, \bar{\mu} - \left(\frac{2-\beta}{2}\right)^2\right]$ *and* $\lambda \notin \sigma_\mu(f)$. *Then the problem* (**??**) *has at least one solution.*

Theorem 27. *Suppose that* $\mu \in \left(\bar{\mu} - \left(\frac{2-\beta}{2}\right)^2, \bar{\mu}\right)$ *and there exists* $\lambda_k^\mu(f) \in \sigma_\mu(f)$ *such that* $\lambda \in \left(\lambda^*, \lambda_k^\mu(f)\right)$ *with* $\lambda^* = \lambda_k^\mu(f) - S_\mu \left(\int_\Omega |x|^{-\beta N/2} dx\right)^{-2/N}$. *Then the problem* (**??**) *admits* v_k *pairs of nontrivial solutions, where* v_k *denotes the multiplicity of* $\lambda_k^\mu(f)$.

Theorem 28. *Suppose that* $\mu \in \left[0, \bar{\mu} - \left(\frac{N+2}{N}\right)^2 \left(\frac{2-\beta}{2}\right)^2\right[$. *Then for all* $\lambda > 0$, *the problem* (**??**) *admits at least one solution.*

We prove our results using critical point theory. However the energy functional associated to (88) does not satisfy (P.S) due to the lack of compactness. We follow Brezis-Nirenberg's arguments [1] to verify that the energy functional to (88) satisfies $(\mathrm{P.S})_c$ condition on suitable compactness range.

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Estimation of discrete probability density associated with the kernel method

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Abstract This paper is a contribution to the problem of the choice of the type of kernel and parameter of smoothing h to estimate the density of probability by the method of the associated kernel. The method of estimate with kernel of the density of probability is a very important technique in the statistical analysis of the data. This estimate by the method of the core starting from a sample requires the choice of the kernel K and the parameter of smoothing h. This tool became today, very popular and much more used. This is due to its simple interpretation and its good asymptotic properties. The performance of the estimator is examined and compared, while combining between the various kernels and the methods for the choice of the parameter of smoothing h (Validation-Cross, Approach bayesian) by studies of simulations for samples of small size, average size and big size, and on real data. The results obtained show that there does not exist a preference between the methods of estimate of the parameter of smoothing h.

Introduction One of the problems usually met in statistics is that of the functional estimate such as the estimate of the function of density $f(x)$, let $X_1 \ldots X_n$ an independent random N-sample and identically distributed of density of unknown probability $f(x)$. A discret symmetric or asymmetric kernel estimator can be defined as follows:

$$\hat{f}_h(x) = \frac{1}{n}\sum_{i=1}^{n} K_{x,h}(X_i) \tag{89}$$

where $K_{x,h}$ is a discret associated kernel function, and $h > 0$ is a parameter of smoothing $K_{x,h}$ is known as kernel associated of target x and smoothing parameter h. Although the two issues cannot be treated separately, it is widely accepted that the performance of kernel density estimator is mainly determined by the bandwidth, while the impacts of kernel choices on the performance of the resulting density estimator was examined by lot of autheurs where they deducted that the standard symmetric kernel estimator to a bias problem. Several autheurs have been proposed asymmetric kernels to solve this bias problem.
In this work, It should be noted that a fixed global bandwidth selected by different classical method, the minimization of criterion mean integrated squared error (MISE) and cross-validation (CV) method does not generally provide an optimal solution to the problem of smoothing, because it is likely simultaneously under and over smooth in different parts of the function. and the second method for the choise of bandwidth is global approach bayesian.

Keywords : *Kernel density estimation; Bayesian bandwidth selector; MCMC method; Cross validation; Prior distribution.*
2010 Mathematics Subject Classification : **47N30; 62G07.**

Some properties The expressions of Mean Integrated Squared Error (MISE) and (ISE) are defined as:

$$MISE(\hat{f}_h(x)) = \mathbf{E}\sum_{x\in\aleph}\{\hat{f}_h(x)-f(x)\}^2 \quad (90)$$

$$= \sum_{x\in\aleph}\mathbf{Var}[\hat{f}_h(x)]+\sum_{x\in\aleph}\mathbf{Biais}^2[\hat{f}_h(x)]. \quad (91)$$

$$ISE(\hat{f}_h(x)) = \sum_{x\in\aleph}\{\hat{f}_h(x)-f(x)\}^2 \quad (92)$$

$$(93)$$

For a Binomial kernel $\mathscr{B}(N,p)$ we consider the associated discrete kernel $B_{x,h}$, where $\mathscr{B}(x+1,(x+h)/(x+1))$,$\aleph_x=\{0,1,...,x+1\}$, $x\in\mathbb{N}$ and $h\in]0,1]$, $\cup_x\aleph_x=\mathbb{N}$ such that:

$$B_{x,h}(y)=\frac{(x+1)!}{y!(x+1-y)!}(\frac{x+h}{x+1})^y(\frac{1-h}{x+1})^{x+1-y}, \quad y\in\mathbb{N}. \quad (94)$$

Cross validation

$$h_{cv}=arg\min_{h>0}CV(h),$$

Where,

$$CV(h) = \sum_{x\in N}\hat{f}_h^2(x)-\frac{2}{n}\sum_{i=1}^{n}\hat{f}_{h,-i}(X_i)$$

$$= \sum_{x\in N}[\frac{1}{n}\sum_{i=1}^{n}K_{x,h}(X_i)]^2-\frac{2}{n(n-1)}\sum_{i=1}^{n}\sum_{j\neq i}K_{X_i,h}(X_j).$$

Global approach Bayesian $\star$ **conditional likelihood estimator:**

$$L(X_1,...,X_n;h)=\pi(X_1,...,X_n|h)=\prod_{i=1}^{n}f_h(X_i),$$

$$\pi(h)\propto\frac{1}{1+h^2}$$

$\star$ **posterior distribution of h:**

$$\pi(h|X_1,...,X_n)=\frac{\pi(X_1,...,X_n|h)\pi(h)}{\pi(X_1,...,X_n)},$$

Where $\pi(X_1,...,X_n)=\int\pi(X_1,...,X_n|h)\pi(h)dh$.

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Behaviour at the non positive integers of a class of multiple Dirichlet series

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Abstract We relate the special value at a non positive integer $\underline{\mathbf{s}} = (s_1, \ldots, s_r) = -\underline{\mathbf{N}} = (-N_1, \ldots, -N_r)$ obtained by meromorphic continuation of the multiple Dirichlet series

$$Z(\underline{\mathbf{P}}, \underline{\mathbf{s}}) = \sum_{\underline{m} \in \mathbb{N}^{*n}} \frac{1}{\prod_{i=1}^{r} P_i^{s_i}(\underline{m})}$$

to special values of the function

$$Y(\underline{\mathbf{P}}, \underline{\mathbf{s}}) = \int_{[1,+\infty[^n} \prod_{i=1}^{r} P_i^{-s_i}(\underline{\mathbf{x}})\, d\underline{\mathbf{x}}$$

where $\underline{\mathbf{P}} = (P_1, \ldots, P_r)$ are polynomials of several variables which verified a certain conditions.
We prove a simple relation between $Z(\underline{\mathbf{P}}_{\underline{\mathbf{a}}}, -\underline{\mathbf{N}})$ and $Y(\underline{\mathbf{P}}_{\underline{\mathbf{a}}}, -\underline{\mathbf{N}})$, such that for all $\underline{\mathbf{a}} \in \mathbb{C}^n$, with the notation $\underline{\mathbf{P}}_{\underline{\mathbf{a}}} := (P_{1\underline{\mathbf{a}}}, \ldots, P_{r\underline{\mathbf{a}}})$, where $P_{\underline{\mathbf{a}}}(\underline{\mathbf{x}}) := P(\underline{\mathbf{x}} + \underline{\mathbf{a}})$ is the shifted polynomial.

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Keywords : *Meromorphic continuation; integral representation; special values.*
2010 Mathematics Subject Classification : **11M32; 11M41.**

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Statistical evaluation of the distribution of the population in the Algerian physical space

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Abstract One goal of statistics is to study the properties of digital sets with many individuals or statistical units. The objective of all is to formulate statistical laws valid for a set of beings or materials, in which we give the name of population. The meeting of all possible statistical units constitutes all statistics or the statistical population. In Algeria the contrasts are so strong between empty areas in several parts of the Sahara and the overcrowded northern regions and the significance of these average densities should be taken with great caution. Indeed, nine out of ten Algerians live in the north of the country (the coast at the northern edge of the Saharan Atlas) on a little over a tenth of the country's area (12.6%). The adopted methodology in this study consists of two elements and is shown in 02composantes:
-The methods of collecting data.
-Statistical analysis of the data from a survey by ONS 2008 "employment survey from households" The purpose of this study is to try to put the issue of statistical measurement of the distribution of population in the Algerian physical space in relation to the transformations that affected the economic and social system.

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Keywords : *Statistical evaluation; population trends; distribution; Algerian territory.*
2010 Mathematics Subject Classification : **91B82; 92D25.**

The Variational Method for the Solution of an Inverse Problem

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Abstract We consider an inverse problem of determining the unknown coefficient of nonlinear time-dependent Schrödinger equation. Let's consider the following first and second type boundary value problems described by nonlinear Schrödinger equation

$$i\frac{\partial\psi_1}{\partial t}+a_0\frac{\partial^2\psi_1}{\partial x^2}+ia_1(x)\frac{\partial\psi_1}{\partial x}-a_2(x)\psi_1+v(t)\psi_1+ia_3|\psi_1|^2\psi_1=f_1(x,t) \tag{95}$$

$$\psi_1(x,0)=\varphi_1(x),\ x\in(0,l), \tag{96}$$

$$\psi_1(0,t)=\psi_1(l,t)=0,\ t\in(0,T) \tag{97}$$

and

$$i\frac{\partial\psi_2}{\partial t}+a_0\frac{\partial^2\psi_2}{\partial x^2}+ia_1(x)\frac{\partial\psi_2}{\partial x}-a_2(x)\psi_2+v(t)\psi_2+ia_3|\psi_2|^2\psi_2=f_2(x,t) \tag{98}$$

$$\psi_2(x,0)=\varphi_2(x),\ x\in(0,l), \tag{99}$$

$$\frac{\partial\psi_2(0,t)}{\partial x}=\frac{\partial\psi_2(l,t)}{\partial x}=0,\quad t\in(0,T), \tag{100}$$

respectively, where $\psi_1(x,t)=\psi_1(x,t;v)$ and $\psi_2(x,t)=\psi_2(x,t;v)$ are wave functions, $l>0,\ T>0$ are given numbers, $\Omega=(0,l)\times(0,T)$, $i=\sqrt{-1}$ is a imaginary unit, $a_0,a_3>0$ are given real numbers, $a_2(x)$ is a measurable real-valued function that satisfies the condition $0<\mu_0\le a_2(x)\le\mu_1$ *for almost all* $x\in(0,l)$, $\mu_0,\mu_1=$ *const.*>0, $a_1(x)$ is a measurable real-valued function that satisfies the conditions $|a_1(x)|\le\mu_2$, $\left|\frac{da_1(x)}{dx}\right|\le\mu_3$,*for almost all* $x\in(0,l)$, $\mu_2,\mu_3=$ *const.*>0, $a_1(0)=a_1(l)=0$, the functions $\varphi_1(x)$, $\varphi_2(x)$, $f_1(x,t)$, $f_2(x,t)$ are given complex-valued functions such that $\varphi_1\in\mathring{W}_2^2(0,l),\varphi_2\in W_2^2(0,l)$, $\frac{d\varphi_2(0)}{dx}=\frac{d\varphi_2(l)}{dx}=0$, $f_k\in W_2^{0,1}(\Omega)$ *for* $k=1,2$. The inverse problem is formulated as follows: is to determine the unknown coefficient $v(t)$ and the functions $\psi_1(x,t),\psi_2(x,t)$ from the additional condition

$$\psi_1(x,t)=\psi_2(x,t),\ (x,t)\in\Omega. \tag{101}$$

The unknown coefficient $v(t)$ is investigated on the set

$$V=\left\{v=v(t):\ v\in W_2^1(0,T),\ |v(t)|\le b_0,\ \left|\frac{dv(t)}{dt}\right|\le b_1 \text{ for almost all } t\in(0,T),\ b_0,b_1=const.>0\right\}.$$

If the coefficient $v(t)$ is assumed to be unknown, then the additional condition (101) can be treated as an observation for determination of the coefficient $v(t)$. For this reason, using the method in [**?**], in this paper, we will present a variational formulation of the above inverse problem as the following: Consider the problem of minimizing the functional

$$J_\alpha(v)=||\psi_1-\psi_2||^2_{L_2(\Omega)}+\alpha||v-w||^2_{W_2^1(0,T)}$$

on set V under conditions (95)-(97) and (98)-(100), where $\alpha\ge0$ is a given number, $w\in W_2^1(0,T)$ is a given element. The aim of paper is to prove the existence and uniqueness of solution of the variational problem and to obtain a necessary condition for the solution of the variational problem.

Keywords : *inverse problem; Schrödinger equation; variational method.*
2010 Mathematics Subject Classification : **35R30; 35J10; 49J40.**

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Nonconvex optimization based on DC programming and DCA in the search of a global optimum of a nonconvex function

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Abstract In this paper we present an algorithm for solving a DC problem nonconvex on an interval [a, b] of R. We use the DCA (Difference of Convex Algorithm) and the minimum of the average of two approximations of the function from a and b. This strategy has the advantage of giving in general a minimum to be situated in the attraction zone of the global minimum searched. After applying the DCA from this minimum we certainly arrive at the global minimum searched.

Introduction

The fminbnd function from MATLAB is a standard method for resolution of a real function minimization defined on a bounded closed interval $[a, b] \subset R$.

It realizes a golden paragraph search and parabolic interpolation. It provides us with only a local minimum, not necessarily global if the function is not unimodal [1,2,3].
In this paper, we propose an alternative method based on the decomposition of the function in a difference of convex functions (DC) and the application of DCA algorithm [4,5].
The DCA also generally provides a local minimum that is not necessarily global (or even a critical point) [6,7,8].
From a good initial point DCA furnishes a global minimizer, we propose to find a good initial point [9,10,11,12].
Instead, we minimize the average of two approximations of the function from a and b.
This strategy has the advantage of giving generally a minimum to be located in the attraction zone of the global minimum searched.
We apply the DCA from the minimum found, we arrive certainly to the global minimum searched.

Problem Formulation

As follows us consider the optimization DC problem:

$$(P_{dc}) \Longleftrightarrow \min\{f(x) = g(x) - h(x), x \in [a, b]\}$$

With:
$f : R^n \longrightarrow R$ nonconvex
$g : R^n \longrightarrow R$ convex
$h : R^n \longrightarrow R$ convex
We want to solve the problem P_{dc} by applying the DCA to the minimum of the average of the two approximations of f from a and b (MDC).

Keywords : *Optimization DC and DCA; global optimization; nonconvex optimization.*
2010 Mathematics Subject Classification : **90C26; 90C34; 90C25.**

The Principle of the Method (MDC) From a good initial point the DCA furnishes a global minimizer [5],[6].
In the case of minimizing a real function defined on $[a, b]$, the minimum found when starting from a will generally be different from that found when starting from b.
We propose to find a good initial point. For that we want to minimize the average of two approximations to f from a and b (MDC) let:

$$min \frac{1}{2}(f_k(x,a) + f_k(x,b))$$

with:

$$f_k(x,a) = g(x) - h'(a)(x-a) - h(a)$$
$$f_k(x,b) = g(x) - h'(b)(x-b) - h(b)$$

$h'(a)$ and $h'(b)$ are the vector gradient. This strategy has the advantage of providing in general a minimum to be located in the attraction zone of the minimum global searched.

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Statistical analysis of solution accuracy for inverse problems in electrodynamics

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Abstract Calculation of electric/magnetic field parameters and source intensity based on measurements is discussed. It is required to assess the accuracy of obtained solution. Upper estimates of the measurement error limits and analytical approaches to error calculations give considerable overestimations and prove to be inefficient for practical applications. The approach based on statistical estimate of solution error is proposed. The obtained error estimate is in good agreement with experimental data.

Let us describe the field sources by function u considered as an element of the normed space U ($u \in U$), the measured field values (input data f) are considered as an element of the normed space F ($f \in F$). The relation between u and f is governed by the operator equation of 1st kind

$$Au = f,$$

where operator A continuously represents U in F.

An inverse problem is solved, i.e. the known values of field f are used to determine sources u. The found sources u can be used to calculate the field parameters g [1]. The determination of functional g is formulated as

$$g = Cu,$$

where C — operator acting from the space U to the normed space G.

Magnetic fields of various engineering objects are considered. For stationary electric fields the similar results are obtained. In case of the magnetic field the function u may represent magnetization or magnetic charge, f — magnetic induction in measurement points, and g — multipole moments or field values in locations other than measurement points (field extrapolation).

The main attention is paid to the accuracy of functional g. Relative error is calculated

$$\delta_g = \frac{|g - g^{Exact}|}{|g^{Exact}|},$$

where g^{Exact} — reference value.

For error estimation the functional g was calculated for numerous test problems. To compose the test set Monte Carlo method was used. It is based on generation of large number of random process implementations. Under this approach, the error δ_g is considered as a random variable.

The total error of function g calculation depends on two basic components. They are method error and measurement error.

The method error δ_g was estimated using a model of sources where point magnetic dipoles were distributed over length, surface or volume. The number of dipoles and their location were fixed. The random variable was source intensity. It was assumed that the values of each magnetic dipole components had uniform distribution on the interval $[-M_{max}; M_{max}]$, where M_{max} — specified value.

Monte Carlo method was also used for estimating δ_g due to measurement error. In this case the system of sources was fixed, and the measurement error was treated as a random variable.

Keywords : *inverse problems; magnetic field; field sources; error of solution; statistical analysis; Monte Carlo method.*
2010 Mathematics Subject Classification : **65C05; 65R32; 78M25.**

If required it is possible to combine the simulation of sources and input data error for estimation of the total error of functional determination.

From analysis of δ_g values obtained by solving a set of test problems it is seen that the variable δ_g has the normal distribution. Mathematical expectation and standard deviation of random variable δ_g are calculated. The three-sigma rule is used for final error estimation.

The accuracy δ_u of solution u itself can be evaluated using the similar algorithm.

Laboratory experiments were performed on a system of magnets with known magnetic dipole moments. Measurements of the magnetic field were taken. Based on measurements the system's dipole moment was calculated and compared with the exact value. The series of laboratory tests indicated that the actual error of the dipole moment did not exceed the error determined by the statistic method.

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Non existence of solutions of a third boundary value problem

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Abstract Existence and non-existence of solutions of a class of boundary value problems of differential, integral and difference equations have received a large amount of attention in the recent literature. In this note, We are interested to non existence of solutions to a higher order fractional boundary value problems The aim is to present a Lyapunov-type inequality for a Caputo fractional differential equation of order $n-1<\alpha<n$ for $n\geq 2$ subject to mixed boundary conditions.

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Keywords : *Lyapunov's inequality; Riemann–Liouville derivative, Caputo fractional derivative; Mixed boundary conditions.*
2010 Mathematics Subject Classification : **34A08; 34A40; 26D10; 33E12.**

Modelling of semi-infinite practical problem in connparagraph with a humanoid robot

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Abstract We study in this paper a humanoid robot trajectory, this problem is modelled as a semi-infinite optimization problem i.e. with a finite number of variables and an infinite number of constraints. A cutting plane method and exchange method are proposed to solve this problem efficiently.

Introduction The optimization has several algorithms which are limited to problems of considerable size (finite),any time in practice and following the evolution we are often confronted with situations that bring to infinite size problems (or the number of constraint is infinite). The semi-infinite programming follows from this concept.
The semi-infinite programming consists in solving problems characterized by a finite number of variables and an infinite number of constraints....

Problem Formulation The general form of the semi-infinite problem is as following:

$$(P)\begin{cases} \min f(x) \\ g(x,s) \leq 0 \quad \forall s \in S \\ x \in R^n \end{cases}$$

- S : is compact of R^p
- f: $R^n \to R$
- $g : R^n \times S \to R$

Practical example: Great interest draws researchers for the class of semi-infinite optimization problems, which are characterized by a finite number of variables and an infinite number of constraints such problems appear, for example in the reduction air pollution, the solution weakly singular integral equations, in the probability distribution and robotics[1]...

Robotics has become a very important tool in different research areas. In our example, we are interested in planning of the movement of a robot humano?de within the framework to validate the developed methods of restoration of the movement in a paraplegic patient as shown in this figure

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Keywords : *Convex semi-infinite problem, cutting plane method, new exchange method.*
2010 Mathematics Subject Classification : **90C26; 90C34; 90C25.**

Simulation and modeling of a wall loss measurement through combined FBG sensors for pressurized pipes

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Abstract Pressure vessels including pressurized pipes in oil and gas industry are all subject to sustained load cracking, corrosion, erosion and hence, material wall losses. Measurement of material wall losses at early stages is primordial to oil and gas industry. Efficient and online monitoring of pipe wall thickness is paramount for engineers as it provides valuable information about failure probability and hence, downtime prevention. In this paper, a model of wall loss measurements with combined FBG (Fiber Bragg Grating) sensors is presented. The proposed model design make use of three FBG sensors, where two are dedicated to axial and hoop strain measurements, while the other sensor serves for temperature compensation. Stress and strain simulations, obtained for 1 to 5 MPa inner pipe pressures, are used to identify the positions in elbows corresponding to maximum wall losses. Simulation of FBG$'s$ optical spectrum reflectivity shows a wavelength shift directly related to the thickness of the pipe material.

Introduction Pipes and pressure vessels have been used extensively for decades to transport and store liquids and gases at high pressure. PWT (Pressure Wall Thinning) in pressure vessels or in any piping system can be produced in various mechanisms. Highly corrosive fluid carrying acid gas or petrochemicals produces in time wall thinning in carbon steel piping systems in oil and gas refineries and other industrial plants due to Flow Accelerated Corrosion (FAC)[1]. Maximum Allowable Working Pressure (MAWP) for process components is set by appropriate codes, ASME paragraph VIII (1989). These codes must be consulted for design decisions. Many researchers have studied (MWAP) by Finite Element Analysis (FEA) of several steel pipes using 3-D analysis, where a pipeline rehabilitation concept and reported rehabilitation of corroded pipes has been reported by putting steel back into pipelines [2, 3]. Inside pipe pressure or internal pressure induces, longitudinal, radial and circumferential stresses. Very recently, after the discovery of fiber Bragg grating strain sensors, many researches forwarded their efforts to fiber optic design in several applications such as airspace, aircraft, civil structure, oil and gas industry and in many areas in engineering. In the following paragraphs, a strain analysis of pressure for pipes and elbows are presented. A model based on three sensors, where two are reserved for strains in two dirparagraphs and the other for temperature compensation. Simulation of the wavelength shift, in reflectivity spectra, for both axial and circumferential dirparagraphs are shown. An analysis of wall loss measurement is also derived for this model of group sensors.

Main results The Bragg wavelength of a grating is a function of the effective index of the guided mode (neff) and the period of index modulation Λ written as $\lambda_B = 2$ neff Λ. Normalized reflectivity has been obtained from coupled mode theory[4, 5].

Figure 1 shows Bragg wavelength shift of 3.3 nm in the simulation of the reflectivity spectrum at 2000 micro strain.

The main spectrum is obtained with the following relevant parameters: Bragg wavelength λ_B = 1550 nm, effective index neff = 1.4455, grating length L = 10 mm, grating period Λ = 532 nm, fiber optic index variation δn = 1e-4. The wavelength shift is linear versus strain. It has been obtained with a strain optic tensor components

Keywords : *FBG sensor; pipe$'$s thickness; strain; stress; wall loss measurements; reflectivity spectrum.*

2010 Mathematics Subject Classification : 00A72; 00A79; 78A25; 78M10.

P11 = 0.113 and P12 = 0.252, a Poisson ratio of ν = 0.275, Thermal coefficients of α = 10.8e-6 /degree C and μ = 8.6e-8 /degree C for fused silica.

The wall thickness and pipe$'s$ radius were respectively r = 20 mm and b = 0.5 m. The shifted Bragg wavelength produced by the imposed strain and temperature change ΔT is given by $\Delta\lambda_B = \lambda_B(1-P_e)\Delta\epsilon + \lambda_B(\alpha+\mu)\Delta T$

Where $\Delta\lambda_B, P_e$ and $\Delta\epsilon$ are respectively Bragg wavelength shift, strain optic constant, and axial strain change along fiber axis and P_e is the strain optic constant. Results show a bijparagraph relation between pipe$'s$ wall thickness and induced strain which could be derived easily from stress and strain equations. Wall thickness was found inversely proportional to strain. Results of the wall loss simulation are summarized in figure 2.

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One Ring or More Rings? Corona Ring Design for 400kV High Voltage Non - Ceramic Insulators

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Abstract Transmitting large volumes of electricity from long distances efficiently can be achieved only through high voltage. However high voltage transmission lines need to be protected with insulators and the latter must be protected by corona ring(s). Corona ring is also very important for non-ceramic insulators since it reduces the electric field stress around the two ends of the insulator and distributes the voltage uniformly along the insulator. Efficiency and life span of the insulator can be increased significantly through optimal design of the corona ring(s)/optimal dimensions, optimal positioning and optimal number of rings used. We have obtained a non-linear programming (NLP) model of corona ring for a 400 kV non-ceramic insulator based on the data collected through simulations by applying Comsol Multiphysics software. Applying the model, one can either find out the most cost effective dimensions of the ring for the given load or for the given amount of material one could determine the dimensions of the optimal ring which protects the insulator with the maximum safety. In this paper, we provide quantified answer to the question on the optimal number of rings in terms of amount of material used. This will then settle the question if a single corona ring or multiple corona rings such as double or triple corona rings would be more effective in reduction of maximum electric field along the insulator.

Introduction Currently 400 kV high voltage insulators are manufactured with double (France/EU) or triple corona rings (UAE) to ensure the safety of insulators. If these number of corona rings were really necessary and how many rings for a given insulator is optimal, could be disputable and needs detailed exploration. Thanks to simulation softwares such as Comsol Multiphysics, one could build very realistic model of electric appliances while integrating all the physics into the model and could conduct detailed study about the model. Appliance makers such as Miele are already using Comsol Multiphysics based simulation to reduce the development to production time and the number of experiments to finalize the product significantly [1]. Since it is very expensive or almost impossible to study and test the real appliances for wide range of values for all important parameters, simulation based studies before hand are the most cost effective and safe especially for high cost and not easily replaceable appliances such as high voltage insulators. By the studies [2], [3], it is revealed that applying full scale simulation studies, one could reduce the cost significantly while increasing the performance with one optimal corona ring.

In the case of multiple rings, one has to study if more rings would increase the performance of the insulators and determine the optimal positions of these rings. Once we established the model using the simulation data, we determine if it is possible to achieve further reduction in electric field values by placing double or triple optimal corona rings on the optimal positions along the insulator. Since the electric field values peak at the two ends of the insulator, our previous study [3] has focused on global optimization of single corona ring and provided the optimal dimensions of the ring with high efficiency and fractional cost compared to the existing ones used in practice. Further we have found that it is most effective to place the corona ring at the position where electric field value is maximum, so the optimal position for the ring is exactly the energized end of the insulator. It turns out that the size of the ring should not be too big, otherwise it causes a peak at the ground end of the insulator. Complexity of the multiple corona ring problem is significant in general. For the sake of simplicity and practicality, we make the optimal single corona ring as starting point, keeping the optimal single corona ring at the energized end unchanged, we place the second corona ring at the ground end and run simulations to find out the optimal dimension the second ring. This way complexity of the problem remains manageable.

Keywords : *COMSOL Multiphysics, Simulation, Modeling, Optimization, Matlab.*
2010 Mathematics Subject Classification : **34C60; 93A30; 97M10; 97M50, 97N80.**

Main results Our study revealed that the reduction in electric field norm could be insignificant even if the same size ring which is optimal for the insulator with single corona ring is used at both ends at the optimal positions. The optimal dimensions of the double corona rings should be determined through studies rather than randomly decided. Otherwise the multiple corona rings would not bring any further reduction in electric field norms compared to single corona ring. As a conclusion of our findings, we state that at first the optimal corona ring for energized end has to be determined (it should not be too small or too big), then the optimal dimension of the ring for ground end should be found, there is a find balance for the the optimal dimensions of the rings.

Our study may offer a novel mathematical methodology for optimal design of related devices such as other types of high voltage insulators and grading rings.

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Controlling the wave movement on the surface of shallow water with the Caputo–Fabrizio derivative with fractional order

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Abstract In order to control the movement of waves on the area of shallow water, the newly derivative with fractional order proposed by Caputo and Fabrizio was used. To achieve this, we first proposed a transition from ordinary to fractional differential equation. We proved the existence and uniqueness of the coupled-solutions of the modified system using the fixed-point theorem. We derive the special solution of the modified system using an iterative method. We proved the stability of the used method and also the uniqueness of the special solution. We presented the numerical simulations for different values of alpha.

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Keywords : *Shallow water model; Caputo–Fabrizio fractional derivative; fixed-point theorem; stability and uniqueness.*
2010 Mathematics Subject Classification : **26A33; 34A08.**

The solutions of Nonlinear Evaluation equations via Hermite Approximation

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Abstract It is well recognized that new types of exact travelling wave solutions to nonlinear partial differential equations can be obtained by modifications of the methods which are in hand. In this study, we extend the class of auxiliary equations using Hermite differential equation so the solution space of nonlinear partial differential equations is extended too. The proposed Hermite differential equation plays an important role in quantum mechanics, probability theory, statistical mechanics, and in solutions of Laplace's equation in parabolic coordinates. Consequently, we introduce new exact travelling wave solutions of some physical systems in terms of the solutions of the Hermite differential equation.

Introduction The inspparagraph of nonlinear wave phenomena of diffusion, convparagraph, dispersion and dissipation appearing in engineering is of great interest from both mathematical and physical points of view. In most case, the theoretical modeling based on nonlinear partial differential equations (NLPDEs) can accurately describe the wave dynamics of many physical systems. Of critical importance is to find closed form solutions for NLPDEs of physical significance. This could be a very complicated task and, in fact, is not always possible since in various realistic problems in physical systems. So, searching for some exact physically significant solutions is an important topic because of wide applications of NLPDEs in biology, chemistry, physics, fluid dynamics and other areas of engineering. There are several theoretical results about local and global solutions of differential equations that establish existence, uniqueness etc., yet makes every effort to find exact solution formulas [1, 2]. Since many of the most useful techniques in analysis are formal or heuristic the trend in recent years has also been to justify and provide the new procedures or methods rigorously. Hence, over the past decades, a number of approximate methods for finding travelling wave solutions to nonlinear evolution equations have been proposed/or developed and furthermore modified. The solutions to various evolution equations have been found by one or other of these methods. Among all these methods, one of the prominent methods is so called auxiliary equation method. The technique of this method consist of the solutions of the nonlinear evolution equations such that the target solutions of the nonlinear evolution equations can be expressed as a polynomial in an elementary function which satisfies a particular ordinary differential equation along with is named as auxiliary equation in general so the solutions of nonlinear evolution equations are depended on solution of auxiliary equations. Recently, to determine the solutions of nonlinear evolution equations, many exact solutions of various auxiliary equations have been utilized [3]. In this paper, we will examine the differences of the choice of different auxiliary equation which is Hermite equation for determining the solutions of the nonlinear evolution equation in consideration and more we search for additional forms of new exact solutions of nonlinear differential equations which satisfying Hermite equation.

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2010 Mathematics Subject Classification : **35A22; 35A25; 35A25; 58J72.**

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Statistical information on the squatter housing

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Abstract Statistical information on the squatter housing problem being dispersed in Annaba city, situated in North-East of Algeria, is considered as the major constraint to the city spatial expansion. The disproportionality between population size and squatter houses expansion makes the situation very intricate, especially on social, economic and environment aspects. After the country independence in1962, the problem of housing was resolved by succession process by hoses left by frensh conquerors. However, by the industrialisation, housing need seems to be more and more serious. Annaba is a good example representing the housing issues in algerian cities. Among them we can introduce : municipal land speculation, marginalisation, low standard housing, illicit constructions, etc. The abscence of well tought urban policy and inadequated urban legislations are the major causes for the no response to the housing needs in Annaba metropolitan area.

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Keywords : *Squatter housing; Environment; Industrialisation; Urban legislation; Urban marginality.*
2010 Mathematics Subject Classification : **62B15; 62B10.**

A Comparison of the Lattice Solitons Governed by the NLS Equation and NLSM Systems

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Abstract Fundamental solitons of the two-dimensional nonlinear Schrödinger (NLS) equation and NLS equation with a mean term (NLSM) are numerically obtained in the existence of external lattices. Although the first nonlinear band-gap structure of the models are almost overlapping, the soliton power of the NLSM systems are larger than soliton power of the NLS equation. The linear stability spectrum and nonlinear evolution of these solitons are investigated by numerical methods. It is demonstrated that the solitons of NLS equation can be nonlinearly stable for a wide range of the propagation constant in the gap but solitons of NLSM Systems can be stable for a narrow part of the propagation constant near edge of the gap. Also, it is noticed that unstable solitons of the NLSM systems have a shorter blow-up (or collapse) distance than solitons of the NLS equation for the same potential depth and propagation constant.

Introduction In recent years, there has been considerable interest in studying solitons in system with periodic potentials or lattices, in particular, those that can be generated in nonlinear optical materials [1].

Solitons in the presence of a (optically or magnetically) induced lattice have been investigated analytically and experimentally in Bose-Einstein condensates (BECs) and in optical Kerr media [2, 3]. Such structures appear as special solutions of the focusing two-dimensional cubic nonlinear Schrödinger (NLS) equation with an external potential.

In many applications the leading nonlinear polarization effect in optical materials are quadratic; they are referred to as $\chi^{(2)}$ materials. The pulse dynamics in multidimensional nonresonant $\chi^{(2)}$ materials can be described by generalized nonlinear Schrödinger (NLS) equation with coupling to a mean term (hereafter denoted as NLSM Systems) [4].

Recently, the regions of collapse and collapse dynamics in the NLSM systems have been investigated [5, 6]. Also, it was pointed out that NLSM collapse can be arrested by small nonlinear saturation [7]. Another way of arresting wave collapse is adding an external potential (lattice) to the governing equation. There have not been any study that considered the solutions of the NLSM systems with an external potential yet.

The purpose of this study is to compare the soliton properties of the NLS equation and NLSM systems in the existence of external lattices. The model is given by

$$iu_z + \sigma\Delta u + |u|^2 u - \rho\phi u - V(x,y)u = 0, \quad \phi_{xx} + \nu\phi_{yy} = (|u|^2)_{xx}. \tag{102}$$

where $u(x,y,z)$ is the normalized amplitude of the envelope of the electric field (which associated with the first-harmonic), $V(x,y)$ is external potential, $\phi(x,y,z)$ is the normalized static field, ρ is a coupling constant, and ν is the coefficient that comes from the anisotropy of the material [4, 6]. NLS equation can be obtained by setting $\rho = 0$ in the NLSM system (102).

Main results Using a modification of spectral renormalization method [8], we numerically obtained the fundamental solitons of the NLS equation and NLSM systems.

The comparison shows that, although the first nonlinear band-gap structures are almost same for the NLS equation and NLSM system, there is a marked difference between the soliton powers of these models. The soliton

Keywords : *NLS equation; mean term; soliton power; soliton stability.*
2010 Mathematics Subject Classification : **37K40; 30C30.**

power of the NLSM systems is greater than NLS equation. It is known that increased power value has a negative effect on stability properties of lattice solitons and, after a critical value of the power, none of the solitons can be stable [9].

The linear and nonlinear (in)stabilities are also examined for these solitons by direct computations of the models and their linearized form. Results of the stability analysis show that although the solitons of NLS equation can be nonlinearly stable away from the band-gap bound (which means taking lower values of the propagation constant), the stable solitons of the NLSM systems can be obtained by choosing larger values of the propagation constant. In addition, it is seen that unstable solitons of NLSM systems have a shorter blow-up (or collapse) distance than solitons of NLS equation for the same potential depth and propagation constant.

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Using estimated fuzzy linear parameters for electrical load estimation

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Abstract A formulation of the fuzzy linear estimation problem is presented. It is formulated as a linear programming problem. The objective is to minimize the spread of the data points, taking into consideration the type of the membership function of the fuzzy parameters to satisfy the constraints on each measurement point and to ensure that the original membership is included in the estimated membership. Different models are developed for a fuzzy triangular membership and the fuzzy numbers of LR-type. The proposed models are applied to different examples from the area of fuzzy linear regression and finally to different examples for estimating the electrical load on a busbar. It had been found that the proposed technique is more suited for electrical load estimation, since the nature of the load is characterized by the uncertainty and vagueness.

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Keywords : *Fuzzy regression; load estimation*

2010 Mathematics Subject Classification : **62H12; 62J05; 62J86.**

Conserved currents and exact solution of some systems of nonlinear PDEs

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Abstract This article constitutes the Lie symmetries, conservation laws and exact solutions of generalized non-linear system and $(2+1)$-dimensional generalized Nizhink-Novikov-Veselov (NNV) equaion. The multiplier approach is employed to compute the conservation laws. Then the Lie point symmetries are derived and the association between symmetries and conserved vectors are established using symmetries conservation laws relationship. We apply the double reduction theory to find the exact solutions of systems under consideration.

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Keywords : *Lie symmetries; conservation laws; double reduction theory.*

2010 Mathematics Subject Classification : **35L65; 37K05.**

A partial Lagrangian method for models of Economics

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Abstract It is shown how one can utilize the Legendre transformation in a more general setting to provide the equivalence between a current value Hamiltonian and a partial Lagrangian when it exists. As a consequence we develop a discount factor free Lagrangian framework to deduce reductions and closed-form solutions via first integrals for ordinary differential equations (ODEs) arising from economics by proving two important propositions. The approach is algorithmic and applies to many state variables of the Lagrangian. In order to show its effectiveness, we apply the method to models, one linear and two nonlinear, with one state variable. We obtain new exact solutions for the last model. The partial Lagrangian naturally arises in economic growth theory and many other economic models when the control variables can be eliminated at the outset which is not always possible in optimal control theory applications of economics. We explain our method via three widely used economic growth models: the Ramsey model with a constant relative risk aversion (CRRA) utility function and the Cobb Douglas technology, a one-sector AK model of endogenous growth and Ramsey model with logarithmic utility preferences. We point out the difference of this approach and that of the more general partial Hamiltonian method proposed earlier for the current value Hamiltonian [R. Naz, F. M. Mahomed and A. Chaudhry, A partial Hamiltonian approach for current value Hamiltonian systems, Commu. Nonlinear. Sci. Numer. Simulat. 19 (2014) 3600-3610.] which is applicable in a general setting involving time, state, costate and control variables.

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Keywords : *Partial Lagrangian; Partial Hamiltonian; Partial Noether condition; Economic growth models; first Integrals.*
2010 Mathematics Subject Classification : **70H03; 06F10.**

Economic dispatch with fuzzy load and cost function coefficients of all thermal power systems

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Abstract In this paper fuzzy load demand and fuzzy cost function parameters for economical dispatch problem is proposed. The classical economical dispatch parameter will be fuzzified in order to obtain an optimal economical solution for the cost function. To evaluate the performance and the capability of reducing cost in a varying cost function coefficient a synthetic system example of a three generation unit is tested with the fuzzy economical dispatch formulation.

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Keywords : *Economic dispatch; all thermal power system; fuzzy load and fuzzy cost function parameters.*
2010 Mathematics Subject Classification : **62H12; 62J05; 62J86.**

On Aluthge transform and non normality of operators

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Abstract In this talk, we present the Aluthge transform of operators defined on a separable complex Hilbert space. We give an application of this notion on some non normal operators with introducing the famous Fuglede-Putnam theorem applied on some classes of such operators. Other results are also presented.

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Keywords : *Aluthge transform; Fuglede-Putnam theorem;* log*-hyponormal operator; p-hyponormal operator.*
2010 Mathematics Subject Classification : **47B47; 47B20.**

On the structure of the set solution of a class of Paratingent equation with delayed argument

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Abstract In this paper we will study the main properties of the set solutions of the paratingent equation (type differential inclusion) with delayed argument of the form: $(Ptx)(t) \subset F([x]_t)$ for $t \geq 0$ with the initial condition: $x(t) = \zeta(t)$ for $t \leq 0$. We will be interested particularly in the topological properties of emission and zone of emission.

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Keywords : *Attainable set; conve; delayed argument; differential inclusion; emission; paratingent; set solutions.*
2010 Mathematics Subject Classification : **34A60; 49J24; 49K24.**

The Band Collocation Problem and Its Combinatorial Model

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Abstract In this paper, we propose a new combinatorial optimization problem named Band Collocation (BC) problem which is a modification of the Bandpass problem introduced by Babayev et al. [1]. The Band Collocation problem consists of finding the minimum cost of the system in optical communication networks by exploring the optimal order of channels in WDM (Wavelength division multiplexing) systems. We give a combinatorial model of the BC problem by taking into consideration the technical limitations of WDM systems.

Introduction The Bandpass Problem (BP) which is an optimization problem and arises in telecommunication networks was first modelled by Babayev et al. in 2009 [1]. According to this model, on a communication network, there are sending point and destination points. A sending point has m information packages to be sent to n different destination points. If an information package is sent to a destination point, it will be shown as 1, otherwise 0. This situation is described by a boolean matrix $A = (a_{ij})$ of dimension $m \times n$. If the information package i ($i = 1,...,m$) is destined for point j ($j = 1,...,n$), then $a_{ij} = 1$; otherwise, $a_{ij} = 0$ [1]. The standard BP has a fixed number B to be grouped packages (consecutive rows) which are called "bandpass" in a matrix. Note that in a every non-zero entry of a column can be included in only one bandpass and several bandpasses in the same column cannot have any common rows. Whereas the Bandpass Problem consists of finding a permutation of rows of a binary matrix that maximizes the total number of bandpasses of given bandpass number B in all columns. In our present day, however, It is not mandatory that all elements of a bandpass are one. Therefore, the model given by Babayev et al. [1] is not practical. In this paper, we propose a new combinatorial optimization problem named Band Collocation (BC) problem to find the optimal permutation of rows without the condition that all elements of a bandpass are one. Then, we give its combinatorial model.

The Band Collocation Problem and Its Combinatorial Model Let $A = (a_{ij})$ be an $m \times n$ boolean matrix ($i = 1,...,m, j = 1,...,n$), $B_k = 2^k$ be a length of bands ($k = 0,..,t$), c_k be a cost of the B_k-length band ($k = 0,..,t$) and π be a permutation of the rows, $\pi = (\pi(1), \pi(2),...,\pi(m))$. The combinatorial formulation of the BC is as follows:

$$Minimize \sum_{k=0}^{t} \sum_{i=1}^{m-2^k+1} \sum_{j=1}^{n} c_k y_{\pi(i)j}^k \tag{103}$$

subject to

$$2^k y_{\pi(l)j}^k \le \sum_{i=l}^{m} z_{\pi(i)j}^k, \; k = 0,...,t, \; j = 1,...,n, \; l = 1,...,m-2^k+1, \tag{104}$$

$$\sum_{k=0}^{t} \sum_{i=1}^{m-2^k+1} 2^k y_{\pi(i)j}^k \ge \sum_{i=1}^{m} a_{ij}, \; j = 1,...,n, \tag{105}$$

$$y_{\pi(l)j}^k + \sum_{i=l+1}^{l+2^k-1} \sum_{p=0}^{t} y_{\pi(i)j}^p \le 1, \; k = 0,...,t, \; j = 1,...,n, \; l = 1,...,m-2^k+1, \tag{106}$$

Keywords : *combinatorial optimization, bandpass problem, , dense wavelength-division multiplexing technology, telecommunication*
2010 Mathematics Subject Classification : **90C09; 90B18; 90C90; 68R05.**

$$\sum_{k=0}^{t} y_{\pi(i)j}^{k} \leq 1,\ i=1,...,m,\ j=1,...,n, \tag{107}$$

$$\sum_{k=0}^{t} z_{\pi(i)j}^{k} \geq a_{ij},\ i=1,...,m,\ j=1,...,n, \tag{108}$$

$$\sum_{k=0}^{t} z_{\pi(i)j}^{k} \leq 1,\ i=1,...,m,\ j=1,...,n, \tag{109}$$

$$y_{ij}^{k}, z_{ij}^{k} \in \{0,1\},\ i=1,...,m,\ j=1,...,n,\ k=0,...,t, \tag{110}$$

where

$$y_{ij}^{k} = \begin{cases} 1 & \text{if row } i \text{ is the first row of a k-length band in column } j \\ 0 & \text{otherwise.} \end{cases}$$

$$z_{ij}^{k} = \begin{cases} 1 & \text{if } a_{ij} \text{ is an element of a k-length band} \\ 0 & \text{otherwise.} \end{cases} \quad j=1,...,n;\ k=1,...,m$$

Under the constraints (104)-(110), the goal is to minimize the cost of k-length bands in the matrix whose rows are permuted.

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Effect of Slow Layer on the Propagation of Surface Elastic SH Waves in a Double Layered Nonlinear Elastic Half Space

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Abstract In this work the propagation of nonlinear SH waves in a double layered nonlinear elastic half space is examined . It is assumed that linear shear velocity of the bottom layer is slower than velocities of the top layer and the half space. By employing an asymptotic perturbation method, it is shown that nonlinear modulation of SH waves is governed by a nonlinear Schrödinger equation. It is remarked that propagation of bright and dark solitons is affected strongly by the nonlinear material parameter of the slow layer.

Introduction It is assumed that in the rectangular Cartesian system of axes (X,Y,Z), the top layer P_1, the bottom layer P_2 and the half space occupy the regions; $0<Y<h_1$, $-h_2<Y<0$ and $-\infty<Y<-h_2$., respectively. It is also assumed that the top surface $Y=h_1$ is free of traction, the stresses and displacements are continuous at the interfaces $Y=0$ and $Y=-h_2$; moreover the displacement in the half space goes to zero as $Y\to\infty$. Then an SH wave described by

$$x=X,\quad y=Y,\quad z=Z+u^{(r)}(X,Y,t)\quad r=1,2,3 \tag{111}$$

Where (x,y,z) and (X,Y,Z) are, respectively, the spatial and material coordinates of a point referred to the same rectangular Cartesian system of axes. In (111) the superscripts r refers to the region P_r. It is assumed that constituent materials are homogenous, nonlinear, isotropic, incompressible elastic and their strain energy functions are of the form $\Sigma^{(r)}=\Sigma^{(r)}(I^{(r)})$ where $I^{(r)}$ is the first invariant of the Green's deformation tensor $C_{KL}=x_{k,K}x_{K,L}$ (Teymur, 1996). These metarials are called generalized neo-Heoken metarials. The nonlinear self modulation of a group of surface SH-waves centered around a wave number k and a frequency ω has been investigated. Thus the harmonic-resonance phenomena is excluded in this examination. The amplitude of waves is assumed to be small but finite, and therefore we employ the method of multiple scales by introducing the following new independent variables

$$x_i=\epsilon^i X,\quad t_i=\epsilon^i t,\quad y=Y,\quad i=0,1,2 \tag{112}$$

in which $\epsilon>0$ is a small parameter which measures the weakness of the nonlinearity and (x_1,x_2,t_1,t_2) are the slow variables describing the slow variations in the problem whereas (x_0,t_0,y) are fast variables describing the fast variations. Then $u^{(r)}$, $r=1,2,3$, are taken to be functions of these new independent variables and they are expanded in the following asymptotic series in ϵ:

$$u^{(r)}=\sum_{n=1}^{\infty}\epsilon^n u_n^{(r)}(x_0,x_1,x_2,y,t_0 t_1,t_2) \tag{113}$$

Writing the governing equations and the boundary conditions in terms of the new independent variables (112) and then employing the expansions (113) yield a hierarchy of problems from which it is possible to determine $u_n^{(r)}$, successively. These problems, at each step, are linear and first order problem is simply the linear wave problem investigated by (Stoneley, 1950). It was shown that for the existence of the generalized SH wave, the phase velocity c of this wave must satisfy either the condition $c_2<c\le c_1<c_3$ or the one $c_2<c_1\le c<c_3$ where c_r's represent linear shear velocities in the regions P_r. We proceed first by assuming that the first inequality is

Keywords : *SH Waves, Schrödinger Equation, Nonlinear Waves*
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satisfied by the phase velocity of the surface SH wave. Since the harmonic resonance phenomena is also excluded in the analysis, the displacements of the first order problem are found to be

$$u^{(1)} = \mathbf{A}_1(R_1 e^{-kv_1 Y} + R_2 e^{kv_1 Y})e^{i\theta} + c.c \tag{114}$$

$$u^{(2)} = \mathbf{A}_1(R_3 e^{ikp_2 Y} + R_4 e^{-ikp_2 Y})e^{i\theta} + c.c \tag{115}$$

$$u^{(3)} = \mathbf{A}_1 R_5 e^{kv_3 Y} e^{i\theta} + c.c \tag{116}$$

where a c.c symbol denotes the complex conjugate of the proceeding terms, $\theta = kx_0 - \omega t_0$, $v_1^2 = \left(1 - c^2/c_1^2\right)$, $p_2^2 = \left(c^2/c_2^2 - 1\right)$, $v_3^2 = \left(1 - c^2/c_3^2\right)$, R_1, R_2, R_3 are some constants. $\mathbf{A}_1 = \mathbf{A}_1(x_1, t_1, x_2, t_2)$ is a complex function representing the first order slowly varying amplitude of the wave modulation, to be determined in higher order perturbation problems. A compatibility condition in the second order perturbation problem shows that $\mathbf{A}_1 = \mathbf{A}_1(x_1 - V_g t_1, x_2, t_2)$ where V_g is the group velocity of the waves. Then the compatibility condition in the third order problem yields the following NLS equation for $\mathbf{A} = k\mathbf{A}_1$;

$$i\frac{\partial \mathbf{A}}{\partial \tau} + \Gamma \frac{\partial^2 \mathbf{A}}{\partial^2 \xi^2} + \Delta |\mathbf{A}|^2 \mathbf{A} = 0, \quad \Gamma = \frac{k^2}{2\omega}\frac{d^2\omega}{dk^2} \tag{117}$$

where $\tau = \omega t_2$, $\xi = k(x_1 - V_g t_1)$. The coefficient Δ depends on the nonlinear parameters of the two layered half space. The analysis is also carried out for the case in which $c_2 < c_1 \leq c < c_3$, and for nonlinear wave modulation of the waves again an NLS equation is obtained whose coefficients Γ and Δ are different from the previous one.

Main results It is known that the properties of the solutions of NLS equation strongly depend on the sign of the product $\Gamma\Delta$. Therefore the variation of this product with the nondimensional wave number $k(h_1 + h_2)$ is evaluated for the lowest branch of dispersion relation giving appropriate values to the materials constants and the ratio h_2/h_1. As a result of the numerical evaluation of $\Gamma\Delta$ for fixed linear material properties, it is observed that the propagation is affected strongly by the the nonlinear constitution of the slow bottom layer as well as the ratio h_2/h_1.

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Relationship between Convex and B-convex Functions

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Abstract B-convex sets and B-convex functions have been studied by different autheurs ([1, 2, 3, 4, 5]). In this paper, to reveal the relation between convex and B-convex functions, some examples of B-convex and convex functions are given. These examples are shown that the class of B-convex functions and the convex functions class are not include each other.

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Keywords : *abstract convexity; B-convexity; B-convex functions; convexity.*
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Solitary-Wave Solutions of the GEW Equation Using Quintic B-spline Collocation Method

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Abstract Numerical solution of the generalized equal width wave (GEW) equation is obtained by using quintic B-spline collocation method with two different linearization techniques. Test problems including single soliton, interaction of solitons and Maxwellian initial condition are studied to validate the proposed method by calculating the error norms L_2 and L_∞ and the invariants I_1, I_2 and I_3. A linear stability analysis based on the von Neumann method of the numerical scheme is also investigated. As a result, our findings indicate that our numerical scheme is preferable to some recent numerical schemes.

Introduction This study has focused on the following generalized equal width wave (GEW) equation:

$$U_t + \varepsilon U^p U_x - \delta U_{xxt} = 0, \tag{118}$$

with physical boundary conditions $U \to 0$ as $x \to \pm\infty$, where p is a positive integer, ε and δ positive constant, t is time and x is the space coordinate. Boundary and initial conditions are chosen

$$\begin{array}{ll} U(a,t) = 0, & U(b,t) = 0, \\ U_x(a,t) = 0, & U_x(b,t) = 0, \\ U_{xxx}(a,t) = 0, & U_{xxx}(b,t) = 0, \\ U(x,0) = f(x), & a \le x \le b, \end{array} \tag{119}$$

where $f(x)$ is a localized disturbance inside the considered interval and will be determined later. In the fluid problems, U is related to the wave amplitude of the water surface or similar physical quantity. In the plasma applications, U is the negative of the electrostatic potential.

GEW equation is nonlinear wave equation with $(p+1)$th nonlinearity and has solitary solutions, which are pulse-like Raslan (2006). Eq. (118) is an alternative model to the generalized RLW equation and GKdV equation. So, the solitary wave solution of the GEW equation has an important role in understanding the many physical phenomena.

GEW equation has been solved with various methods. Evans & Raslan (2005) solved the equation numerically by using quadratic B-spline collocation method. Raslan (2006) obtained the numerical solutions of the equation with collocation method using cubic B-spline. Roshan (2011) studied the equation numerically using linear hat function by Petrov-Galerkin method.

In the present work, we have applied the quintic B- spline collocation method using two different linearization techniques to the GEW equation.

Main results We have obtained the solitary-wave solutions of the GEW equation based on the quintic B spline collocation method using two different linearization techniques. The error norms L_2, L_∞ for single soliton and the invariants I_1, I_2, I_3 for the three test problems including single soliton, interaction of solitons and Maxwellian initial condition have been calculated. The obtained results are tabulated. As seen from these tables, for each linearization technique, the changes of the invariants are reasonably small and the values of invariants are consistent with the other results. The quantity of obtained error norms are better than the ones in previous numerical methods.

Keywords : *GEW equation; collocation method; quintic B-spline; soliton; solitary waves.*
2010 Mathematics Subject Classification : **41A15; 65L60; 76B25.**

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Multiple Solutions of Second Order Differential Equations

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Abstract In this paper, we use the reproducing kernel Hilbert space method (RKHSM) for finding multiple approximate solutions of second order differential equations. The numerical approximations to the exact solutions are computed. Multiple solutions have not been found by this method till now. The comparison of the results with exact ones is made to confirm the validity and efficiency.

Introduction We consider the boundary value problems

$$\begin{cases} u''(x) = \lambda \exp(\mu u(x)), & 0 \le x \le 1, \\ u(0) = u(1) = 0, \end{cases} \tag{120}$$

and

$$\begin{cases} \left(\exp(x) v'(x)\right)' + |\ln x| = 0, & \forall x \in (0,\infty), \quad x \neq 1, \\ \Delta v' |_{x_1=1} = v^2(1), \\ v(0) = 0, \quad v(\infty) = 0. \end{cases} \tag{121}$$

The problem (120) arises in applications involving the diffusion of heat generated by positive temperature dependent sources. For instance, if $\mu = 1$ it arises in the analysis of Joule losses in electrically conducting solids, with λ representing the square of constant current and $\exp(u)$ the temperature-dependent resistance, or frictional heating with λ representing the square of the constant shear stress and $\exp(u)$ the temperature-dependent fluidity. In particular if $\lambda = 1$ and $\mu = -1$ the boundary value problem (120) has two solutions $u_1(x)$ and $u_2(x)$. Solution $u_1(x)$ drops below upto $-0.14050941\ldots$ and solution $u_2(x)$ upto $-4.0916146\ldots$. Boundary value problem (121) has at least two positive solutions v_1, v_2 satisfying $0 \le \|v_1\| \le \frac{1}{2} \le \|v_2\|$.

In this paper, the RKHSM [1] will be used to investigate the problems (120) and (121). The theory of reproducing kernels [2], was used for the first time at the beginning of the 20th century by S. Zaremba in his work on boundary value problems for harmonic and biharmonic functions. In recent years, a lot of attention has been devoted to the study of RKHSM to investigate various scientific models. The RKHSM which accurately computes the series solution is of great interest to applied sciences. The method provides the solution in a rapidly convergent series with components that can be elegantly computed. The book [1] provides excellent overviews of the existing reproducing kernel methods for solving various model problems such as integral and integro-differential equations.

Main results We obtain the solutions of (120) and (121) in the reproducing kernel space $W_2^3[0,1]$. On defining the linear operator $L : W_2^3[0,1] \to W_2^1[0,1]$ as

$$Lu(x) = u''(x) \tag{122}$$

model problem (120) takes the form:

$$\begin{cases} Lu = f(x,u), & x \in [0,1], \\ u(0) = u(1) = 0, \end{cases} \tag{123}$$

where $f(x,u) = \lambda \exp(\mu u(x))$.

Keywords : *reproducing kernel method; series solutions; second order differential equations; multiple solutions; reproducing kernel space.*
2010 Mathematics Subject Classification : **30E25; 34B15; 47B32; 46E22; 74S30.**

In Eq. (122) since $u(x)$ is sufficiently smooth we see that $L : W_2^3[0,1] \to W_2^1[0,1]$ is a bounded linear operator. For model problem (121) similar things can be done.

Put $\varphi_i(x) = T_{x_i}(x)$ and $\psi_i(x) = L^*\varphi_i(x)$, where L^* is conjugate operator of L. The orthonormal system $\{\widehat{\Psi}_i(x)\}_{i=1}^{\infty}$ of $W_2^3[0,1]$ can be derived from Gram-Schmidt orthogonalization process of $\{\psi_i(x)\}_{i=1}^{\infty}$,

$$\widehat{\psi}_i(x) = \sum_{k=1}^{i} \beta_{ik}\psi_k(x), \quad (\beta_{ii} > 0, \quad i = 1,2,\ldots). \tag{124}$$

Theorem 29. *If u_1 and u_2 are the exact solutions of* (123)*, then*

$$u_1(x) = \sum_{i=1}^{\infty}\sum_{k=1}^{i} \beta_{ik} f(x_k, u_{1_k})\widehat{\Psi}_i(x) \tag{125}$$

and

$$u_2(x) = \sum_{i=1}^{\infty}\sum_{j=1}^{i} \gamma_{ij} f(x_j, u_{2_j})\widehat{\Psi}_i(x), \tag{126}$$

where $\{(x_i)\}_{i=1}^{\infty}$ *is dense in* $[0,1]$.

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Asymptotic Behaviour of Eigenvalues and Eigenfunctions of a Boundary Value Problem with Retarded Argument

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Abstract We investigate the asymptotic formulas for eigenvalues and eigenfunctions of the boundary value problem with the differential equation

$$y''(x) + \lambda^2 \rho(x) y(x) + q(x) y(x - \Delta(x)) = 0, \quad 0 \le x \le \pi \tag{127}$$

and the boundary conditions

$$y(0)\cos\alpha + y'(0)\sin\alpha = 0, \tag{128}$$

$$y(\pi)\cos\beta + y'(\pi)\sin\beta = 0, \tag{129}$$

$$y(x - \Delta(x)) \equiv y(0)\phi(x - \Delta(x)), \tag{130}$$

where the real valued functions $q(x)$ and $\Delta(x)$ are continuous on the interval $[0,\pi]$, λ is a real parameter, α and β are arbitrary real numbers, $\phi(x)$ is a continuous initial function on the initial set $E_0 = \{x - \Delta(x) : x - \Delta(x) < 0, t > 0\}$ with $\phi(0) = 1$ and

$$\rho(x) = \begin{cases} 1, & 0 \le x < a, \\ \alpha^2, & a < x \le \pi, \end{cases} \tag{131}$$

where $0 < \alpha \ne 1$.

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Keywords : *asymptotics of eigenvalues and eigenfunctions; boundary value problems; differential equation with retarded argument; discontinuous coefficient.*

2010 Mathematics Subject Classification : **34B09; 34L05; 34L20.**

Application of the Sumudu transform to the Riemann Zeta function and consequences

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Abstract In light of the autheurs previous combined work we propose to apply the Sumudu transform and its various properties and powers to the Zera function. In the process, we observe the consequences on the roots of the zeta function, and various other ramifications. Some of these ramifications include the preservation of the chaotic but fastening decay to zero of the fractional derivative of the Riemann Zeta function under positive powers of the Sumudu transform.

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Keywords : *Sumudu transform; Riemann Zeta function.*
2010 Mathematics Subject Classification : **11E45; 34A25.**

Energetic Method of Solving Nonlinear Established Tasks of Thermomechanics for the Rod Superalloys

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Abstract Bearing elements are made of special heat-resistant materials in modern energetic installations. One of the physical characteristics of these materials is that the thermal expansion coefficient α depends on the temperature. The field of temperature distribution is a function of coordinates. In this case for the heat-resistant materials $\alpha = \alpha(T(x))$. Except that for the rods from heat-resistant materials potential energy of elastic deformation has the following form:

$$\Pi = \int_V \frac{\varepsilon_x^2(x)}{2} dV - \int_V \alpha(T(x)) E T(x) \varepsilon_x dV \tag{132}$$

Here $\varepsilon_x = \frac{\partial U}{\partial x}$ is the elastic component of deformation, $u = u(x)$ is the movement field, E is the modulus of elasticity, $T(x)$ is temperature field. The temperature field longwise of the rod is defined from the total thermal energy. $J = \int_{S_i} q_i T(x) dS + \int_V \frac{K_{xx}}{2} \left(\frac{\partial T}{\partial x}\right)^2 dV + \int_{S_j} \frac{h_j}{2} (T_{(x)} + T_{ocj})^2 dS$ where S_i is the local surfaces where heat flows are brought with intensity q_i; S_j is the local surfaces through which happens to the environment, at the same time the coefficients of heat-exchange is h_j, and environments temperature is T_{ocj}; K_{xx} is coefficient of thermal conductivity of material of a rod.

As a result the field of temperature distribution, movement, elastic, temperature and thermoelastic components by deformation and stress are defined. Also the field of coefficient distribution of thermal expansion is defined. The size of lengthening of a rod taking into account a certain field of temperatures and a koefiitsent of thermal expansion are calculated. Also the size of the arising axial force is calculated. In such a way the offered computing algorithm and a method allows to research deeply the thermomechanical condition of rods of limited length made of heat resisting materials and located under simultaneous influence diverse types of sources of heat.

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Keywords : *local temperature, heat-exchange, axial force, coefficients of thermal conductivity, thermal expansion, modulus of elasticity, the total thermal energy, deformation, stress, lengthening, coefficient of heat-exchange.*

2010 Mathematics Subject Classification : **81T80; 34A34.**

Localization and pseudospectrum of some matrix

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Abstract In this talk, let A and B be complex matrix, not necessarily of the same dimension; and let $\|.\|$ be a matrix norm. Consider the conditions

$$\|(zI-A)^{-1}\| = \|(zI-B)^{-1}\|$$

and

$$\|\sqrt{A}\| = \|\sqrt{B}\|$$

. Our main result is to study some essential properties concerning the above conditions.

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Keywords : *Pseudospectrum; perturbation; resolvent.*
2010 Mathematics Subject Classification : **47A10; 34D10.**

Group analysis of the time-fractional Buckmaster equation with Riemann-Liouville derivative

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Abstract Finding the symmetries of the nonlinear fractional differential equations plays an important role in studying of fractional differential equations. In this manuscript, our purpose is to find the Lie point symmetries of the time-fractional Buckmaster equation. After that we use the infinitesimal generators for obtaining their corresponding invariant solutions.

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Keywords : *Fractional differential equation; Lie group; time-fractional Buckmaster equation; Riemann-Liouville derivative; Group-invariant solutions.*

2010 Mathematics Subject Classification : **35R11;34A60**

Chaos Measures for Autoregressive Fractionally Integrated Moving Average Difference Equations

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Abstract Autoregressive fractionally integrated moving average (ARFIMA) arises in modeling of financial time series. ARFIMA is governed by a linear stochastic difference equations of the form:

$$\Phi(B)(1-B)^d X_t = \Theta(B)\epsilon_t \tag{133}$$

where

$$\Phi(B) = 1 - \sum_{j=1}^{p} \phi_j B^j \quad \Theta(B) = 1 - \sum_{k=1}^{q} \theta_k B^k,\ \phi_j \in \mathbb{R} \text{ and } \theta_k \in \mathbb{R}\,.$$

$\{\epsilon_t\}_{t\in\mathbb{Z}^+}$ is the white noise process with zero mean and variance $\sigma_\epsilon > 0$. $\{X_t\}_{t\in\mathbb{Z}^+}$ is the discrete time real valued stochastic process which represent ARFIMA (p,d,q). Moreover, B is the backward shift operator, i.e. $B^d X_t \equiv X_{t-d}$ (d is the factional differencing parameter $-1/2 < d < 1/2$). In this work we have computed the Lyapunov exponents, correlation dimensions and mutual information for both the stochastic difference equation given in Equation 1 and for the financial time series to detect the chaos in ARFIMA (p,d,q) processes. We have shown that largest Lyapunov exponents are positive in both cases, therefore, ARFIMA (p,d,q) exhibits chaos. We also comment memory behaviour and forecasting issues based on the chaos measures.

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Keywords : *autoregressive fractionally integrated moving average; lyapunov exponents, correlation dimension, mutual information*
2010 Mathematics Subject Classification : **91G70; 37M10; 37D45; 39A50; 37M25.**

Conservation Laws of Ermakov-Pinney Equation

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Abstract In this study, we investigate Noether and λ-symmetries of the Ermakov-Pinney equation. Firstly, the Lagrangian for the equation is constructed and then the determining equations are obtained based on the Lagrangian approach. Noether symmetry classification is implemented and the first integrals, conservation laws and group invariant solutions are obtained and classified for the Ermakov-Pinney equation. Secondly, based on the λ-symmetry method, we analyze λ-symmetries Then, we investigate λ-symmetry properties and the corresponding reduction forms, integrating factors, and first integrals by using the mathematical relationship with Lie symmetries.

Introduction The investigation of solution of differential equations, one of the most powerful methods for nonlinear differential equations is based on the study of Lie group of transformations play an important role in the literature. The applications of Lie groups to the problems in mechanics, mathematics, physics is useful method to obtain solution of equation. The Lie symmetry of Ermakov-Pinney equation are investigated. Also the application of Noether theorem in the concept of theory of Lie groups to differential equations introduces the Noether symmetry of the action of a physical system has a corresponding conservation law. A conservation law means a quantity associated with a physical system that stays unaltered as the system evolves in time. The natural form of the Lagrangian is defined in Noether theory and an important property of the Lagrangian is that conservation laws can be extract by using it. Noether theorem gives variational symmetries are correspondence with conservation laws for the associated Euler-Lagrange equations. In addition, in the literature, there is another method called λ-symmetry, which is introduced by Muriel and Romero. They introduce a new prolongation formula to investigate λ-symmetries for second order differential equations.

Main results We assume that x is the independent variable and $y = (y^1, \ldots, y^m)$ is the independent variable with coordinates y^α with respect to x are given as following form

$$y_x^\alpha = y_1^\alpha = D_x(y^\alpha), \quad y_s^\alpha = D_x^s(y^\alpha), \quad s \geq 2, \quad \alpha = 1, 2, \ldots, m, \tag{134}$$

where D_x is the total derivative operator [2-5], with respect to x, which is defined as

$$D_x = \frac{\partial}{\partial x} + y_x^\alpha \frac{\partial}{\partial y^\alpha} + y_{xx}^\alpha \frac{\partial}{\partial y_x^\alpha}. \tag{135}$$

Here, the vector space of all differential functions of all finite orders is represented by $\mathscr{A}$ that is universal space. Also, operators apart from total derivative operator (2.2) are defined on space $\mathscr{A}$.

Definition 1. The operator

$$\frac{\delta}{\delta y^\alpha} = \frac{\partial}{\partial y^\alpha} + \sum_{s\geq 1} (-D_x)^s \frac{\partial}{\partial y_x^\alpha}, \quad \alpha = 1, 2, \ldots, m, \tag{136}$$

is called the *Euler operator* or *Euler-Lagrange operator*.

Definition 2. X is a Noether point symmetry corresponding to Lagrangian of the system of differential equations

Keywords : *Noether theory; first integral; λ-symmetry; integrating factor; invariant solution.*
2010 Mathematics Subject Classification : **26A33; 34A60; 34G25; 93B05.**

(2.8) if there exists a function $B(x,y)$. In addition, X is a Noether point symmetry corresponding to a Lagrangian of the fin equation, then I is a first integral associated with X, which is given by the expression [10]

$$I = \xi L + (\eta - y'\xi)L_{y'} - B. \tag{137}$$

We now consider Noether symmetry classification of the nonlinear Ermakov-Pinney

$$y'' + w^2(x)y = \frac{1}{x^3}, \tag{138}$$

in which the overdot denotes represents differentation with respect to the independent variable t, which in manyapplication is the time.

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Structural electronic and optical properties of alloys with first principles

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Abstract First principles of total energy calculations have been performed using full potential linear augmented plane wave method (FP-LAPW) within density functional theory to study the structural, electronic and optical properties of MgSxSe1-x, MgSxTe1-x and MgSexTe1-x alloys in the rock salt phase. The local density approximation (LDA) and generalized gradient approximation (GGA) for the exchange-correlation (XC) are used. The equilibrium lattice constants are in agreement with the available experimental results. The electronic properties of ternary alloys MgSxSe1-x, MgSxTe1-x and MgSexTe1-x (0.25 < x < 0.75) are calculated. Optical bowing for these semiconductors alloys has been discussed with the approach of Zunger to explain the bowing parameter in the alloys. Results are compared with experimental and other theoretical data with reasonable agreement.

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Keywords : *Alloys; ab initio calculations; electronic structure; optical properties.*
2010 Mathematics Subject Classification : **78A60; 74G65.**

Lattice Fractional Diffusion Equation of Random Order

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Abstract The discrete fractional calculus is used to fractionalize difference equations. The fractional logistic map is illustrated that the chaotic solution can be conveniently obtained. Then a Riesz fractional difference is defined for fractional partial difference equations on discrete finite domains. A lattice fractional diffusion equation of random order is proposed to depict the complicated random dynamics and an explicit numerical formulae is derived directly from the Riesz difference.

Introduction Partial difference equations are a class of discrete models particularly for the mentioned topics. The discrete fractional calculus (DFC)[1, 2, 3, 4, 5, 6, 7] is a fractionalization tool for difference equations.

In this paper, a novel Riesz fractional difference is defined on time scales. Then a space discrete fractional diffusion equation of variable order is proposed. The paper is organized in the following paragraphs. In paragraph 2, preliminaries of the discrete fractional calculus are introduced and a novel Riesz difference is defined. In paragraph 3, direct finite difference method (DFDM) is introduced and is used to obtain chaotic solutions of the fractional map. The fractional chaotic dynamics are discussed. In paragraph 4, a discrete fractional diffusion of variable order is defined on the lattice $\{0, h, 2h, ..., (N-1)h, Nh\}$. The random solutions are given by the DFDM.

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Keywords : *discrete fractional calculus; lattice fractional diffusion equations; random order.*

2010 Mathematics Subject Classification : 76Rxx; 34A60; 78M20; 35R11.

Investigation of the Keller-Segel model with a fractional derivative

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Abstract The prototypical suggested by Keller and Segel was prolonged to the concept of fractional derivative recently proposed by Caputo and Fabrizio. We presented in detail the existence of the coupled-solutions using the fixed-point theorem. A detail analysis of the uniqueness of the coupled-solutions is presented. Using an iterative approach, we derived special coupled-solutions of the modified system and we presented some numerical simulations to see the effect of the fractional order.

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Keywords : *Keller-Segel model; Caputo-Fabrizio fractional derivative; Fixed-point theorem; special solution.*

2010 Mathematics Subject Classification : **26A33; 58J20.**

Nonuniform $(\mu, \nu)-$Trichotomy of Linear Discrete-Time Systems in Banach Spaces

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Abstract The aim of this paper is to give characterizations for the concept of $(\mu, \nu)-$trichotomy of time-varying linear systems described by difference equations with noninvertible operators in Banach spaces. This concept contains as particular cases the classical properties of (uniform and nonuniform) exponential trichotomy and polynomial trichotomy. Also we consider the robustness property, in the sense that the existence of such a trichotomy for a given linear discrete-time system persists under sufficiently small linear perturbations.

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Keywords : *nonuniform $(\mu, \nu)-$ trichotomy; robustness; linear discrete-time systems; difference equations.*

2010 Mathematics Subject Classification : **34D09, 34D10.**

Schrodinger Equation in Nano Science

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Abstract In this talk, I will revisit the Schrodinger equation and cite the impact on twenty first century physical sciences. This differential equation paved the way to nano science and nano technology which are flourishing nowadays. This success is the result from a huge boost from the recent advances in computation hardware technology and software. Micro budget laboratories can do fair quality fundamental or applied research due to the low cost of the computation machines, free open software platforms like LINUX, free compilers like GFORTRAN, computation libraries as BLAS and LAPACK and free real time support from worldwide research community through the internet. To illustrate the impact I will go over some examples of use which are important in everyday life of mankind in the twenty first century and are of interest to me. The first example is the rechargeable batteries [1] and the second is magnetic materials in information backup [2]. I demonstrate from ground state energy and wave function determined from Schrodinger equation that I can map all the physical properties of the concerned material. Moreover, I can use Schrodinger equation to search for new artificial materials to suite some application needs.

Acknowledgments: I am thankful to all my students past and present for being my companions in this wonderful journey of scientific research.

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Keywords : *Schrodinger equation; computation; Li Ion Batteries; magnetic recording.*
2010 Mathematics Subject Classification : **93B40; 82D40.**

On a New Class of Operators Satisfying the Kato Conjecture

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Abstract We consider a differential-difference equation with incommensurable shifts with Dirichlet boundary condition. The smoothness of solutions of this problem can be violated almost everywhere. Nevertheless, we prove that the corresponding operator satisfies the Kato conjecture.

Introduction

For simplicity, we consider one-dimensional case. Introduce operator

$$(Ru)(x) = au(x-1) + bu(x+\tau), \tag{139}$$

where a and b are some real numbers and $0 < \tau < 1$ is an irrational number. We consider the boundary value problem

$$-(u - Ru)''(x) = f(x), \qquad x \in (0, \pi), \tag{140}$$

$$u(x) = 0, \qquad x \in [-1, 0] \cup [\pi, \pi+1]. \tag{141}$$

The function f belongs to $\left[H_0^1(Q)\right]'$, a dual space to $H_0^1(0,\pi)$.

Introduce operator $A\colon H_0^1(Q) \to \left[H_0^1(Q)\right]'$:

$$\langle Au, \overline{v}\rangle = \left((u - R_Q u)', v'\right)_{L_2(0,\pi)}, \qquad \forall v \in H_0^1(0,\pi), \tag{142}$$

and the symbol of the operator:

$$a(\xi) = \left(1 + ae^{-i\xi} + be^{i\tau\xi}\right)\xi^2. \tag{143}$$

Here the subscript Q means that we continue the function by zero outside of $(0,\pi)$, apply operator R, and than take its projection on $(0,\pi)$.

Definition 8. *The function $u \in H_0^1(0,\pi)$ is called the weak solution of the problem* (140)–(141)*, if $\langle Au, \overline{v}\rangle = \langle f, \overline{v}\rangle$ for all $v \in H_0^1(0,\pi)$.*

In [1] (Ch. 1, Sec. 3), there was constructed an example, when $u \notin H^2(\alpha,\beta)$ for all $0 \le \alpha < \beta \le \pi$.

Main results

Lemma 30. *Operator A is strongly coercitive iff there exists a constant c such that*

$$\mathscr{R}e\ a(\xi) \ge c\xi^2, \qquad \forall 0 \ne \xi \in \mathbb{R}. \tag{144}$$

Note that the left-hand side of Lemma 1 holds, e.g., if $|a| < 1/4$ and $|b| < 1/4$. The next lemma gives us another sufficient condition.

Keywords : *differential-difference equations; Kato conjecture.*
2010 Mathematics Subject Classification : **39B02.**

Lemma 31. *Operator A is strongly coercitive if the matrix*

$$\begin{pmatrix} 1-a & b/2 & 0 & 0 \\ b/2 & 1-a & b/2 & 0 \\ 0 & b/2 & 1-a & b/2 \\ 0 & 0 & b/2 & 1-a \end{pmatrix}$$

is positive defined.

Theorem 32. *Let operator A be strongly coercitive, then for every $f \in [H_0^1(0,\pi)]'$ problem 140–141 has a unique weak solution.*

Consider the restriction of operator A, operator $\mathscr{A}: D(\mathscr{A}) \to L_2(0,\pi)$.

Theorem 33. *Let operator A be strongly coercitive, then $D(\mathscr{A}^{1/2}) = D(\mathscr{A}^{*1/2})$.*

The poof of this theorem is similar to [2] (Theorem 3).

Some generalizations

- We can consider more shifts and complex coefficients;
- We can consider multidimensional case;
- We can consider Neumann boundary conditions, but Lemmas 30 and 31 do not hold in this case.
- We can consider parabolic problems with operator A.

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A Two-Level Method for Emulating Parameterized Dynamic Partial Differential Equation Models

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Abstract We introduce a novel framework for emulating parameterised, time dependent partial differential equation models. The approach is based on a two-level approximation, using nonlinear dimensionality reduction to learn the snapshots for a proper orthogonal decomposition. This method can be used with Galerkin projection schemes or finite volume/difference methods.

Introduction Partial differential equations (PDE) models are ubiquitous in science and engineering. The computational cost of solving such models is often prohibitive, e.g., in design optimisation and inverse parameter estimation. Model order reduction (MOR) techniques (e.g., proper orthogonal decomposition (POD), balanced truncation and Krylov subspace methods [1, 4]) can be employed in such cases. POD is based on a reduced-basis (RB) representation of the output space using data from carefully selected simulations. The numerical formulation is restricted to the truncated basis and the coefficients in the basis become the targets for the numerical scheme. State-of-the-art methods are greedy RB [3] and global basis POD [2]. The focus of our research is to extend POD approaches for applications to nonlinear, time-dependent PDEs with multiple parameters using a novel statistical emulator based on nonlinear dimensionality reduction to learn the snapshots for a new parameter value; the standard methods described above are either unfeasible or unlikely to succeed. The method itself is not restricted to standard POD based methods; it is readily modified for application with other methods such as balanced truncation. Examples include 2D heat conduction-convection (via finite volume) and a 1D burgers equation (using Galerkin finite element). It is shown that our method are capable of dealing with such non-linear problems across a broad range of parameters.

Main results The algorithm we propose emulates the outputs of a computer model $\boldsymbol{y}(t)$ for inputs $\boldsymbol{x}$. The computer model is treated as a function $\boldsymbol{\eta}(\boldsymbol{x}, t)$ of the inputs and time. The outputs are time-dependent, vectorised values of a spatial field (d can be on the order of 10^6). In standard (or global basis) POD snapshots are computed in order to define the orthogonal components (using principal component analysis). In our method, Gaussian process emulation (GPE) is used to extract the POD bases for new parameter values. In order to avoid prohibitive computational cost due to the high dimensionality of the output space, we used nonlinear dimensionality reduction on the output space $\mathscr{O}$. This novel method was developed in [5]. A standard POD using the learned basis is then performed. A pseudo code is provided below:

1. Select design points $\boldsymbol{x}^{(j)} \in \mathscr{X} \subset \mathbb{R}^l$, $j = 1, \dots, m$, using design-of-experiment and select times t_n, $n = 1, \dots, N$. Collect outputs $\boldsymbol{y}^{(j)}(t_n) = \boldsymbol{\eta}(\boldsymbol{x}^{(j)}, t_n) \in \mathscr{O} \subset \mathbb{R}^d$ from computer model
2. Perform nonlinear dimensionality reduction on $\boldsymbol{y}^{(j)}(t_n)$, $j = 1, \dots, m$, to obtain coeffcients in a feature space: $z_i^{(j)}(t_n)$, $i = 1, \dots, d$, for each $n = 1, \dots, N$ and $j = 1, \dots, m$

Keywords : *parameterised partial differential equations; proper orthogonal decomposition; nonlinear dimensionality reduction; Galerkin projection; finite volume.*

2010 Mathematics Subject Classification : **65C20; 74S05; 65C60; 68P99.**

3. Select a new input $\boldsymbol{x}$ for prediction
   for $n = 1 : N$
   for $i = 1 : r$ $(r \ll d)$
   scalar GPE on $\{\boldsymbol{x}^{(j)}, z_i^{(j)}(t_n)\}_{j=1}^m$ to yield predicted coefficient $z_i(t_n)$ in a feature space basis
   end
   Reconstruct $\boldsymbol{y}(t_n)$ using $\boldsymbol{z}_r = (z_1(t_n), \ldots, z_r(t_n))^T$
   end
4. Apply POD on $\boldsymbol{y}(t_n)$, $n = 1, \ldots, N$, to extract the POD basis. Perform standard POD

The method was tested on a linear 2D heat conduction-convection problem using the finite volume method and a nonlinear 1D burgers equation using a Galerkin finite element discretization. The results (presented in the full paper) demonstrate the accuracy and computational efficiency of our approach for multiple parameters (currently not feasible using other methods).

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Design, Workshop Development Interior Design and Furniture Section

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Abstract In this research, we will focus on the best methods to develop and equip the experimental laboratory to train university students on basic skills with the aim to prepare efficient engineers and to cope up with the requirements of training programs and plans in order to achieve training qualitative transition, and to utilize spaces optimally as well as developing plans for security and safety, and developing a future vision to apply quality and accreditation system in training. Fortunately, all these issues are part of our professional specialty and expertise for over thirty years.

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On the Approach of a Mixed Type Functional Differential Equation from Physiology

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Abstract It is introduced a numerical scheme which approximates the solution of a particular non-linear mixed type functional differential equation from physiology, adapted from the work introduced in [3, 2, 1]. The mathematical equation models a superficial wave propagating through the tissues.

1 Introduction

We are concerned about the numerical solution of the mathematical model, represented by the following non-linear equation with delay and advanced arguments ((145))

$$Mx''(t) + Bx'(t) + Kx(t) = \frac{P_L}{K_t}\frac{x(t-\tau) - x(t+\tau)}{x_0 + x(t+\tau)}, \tag{145}$$

where $x_0 + x(t+\tau) > 0$ and M, B, K, P_L and τ known parameters.

After an adequate change of variable, equation ((145)) can be rewritten as a non-dimensional model (146)

$$u''(t) + \alpha u'(t) + \omega^2 u(t) = p\frac{u(t-\tau) - u(t+\tau)}{1 + u(t+\tau)}, \tag{146}$$

where $1 + u(t+\tau) > 0$, k_t a Known coeficient, $p = \frac{P_L}{k_t x_0 M}$, $\alpha = B/M$ and $\omega = \sqrt{K/M}$.

Two different approximations of (146) are considered when it is assumed a small time delay [4] or an arbitrary time delay.

To solve numerically ((146)), we developed a process adapted from the one applied in [1].

The problem is reduced to a BVP on a limited interval, asymptotic approximations are obtained and an approximation of solution of the problem ((146)) subject to some natural constrains is computed. The nonlinear problem can be reduced to a sequence of linear problems using the Newton method. An initial approximation is obtained in order to guarantee the convergence of the Newton iteration process. We choose adequate values of a set of parameters which allows to obtain asymptotic approximations, imposing regularity conditions and boundary conditions respectively. The system is updated for each iterate of the NM.

2 Main results

The numerical scheme based on an adapted method of steps, was rebuilt and reajusted from [3, 2, 1] so one could solve numerically the equation on study. Different sets of parameters were tested and, in general, the results obtained were consistent. The convergence is guaranteed, for some set of parameter values, and compatible with the expected order. When we consider larger intervals, the accuracy is reduced. The validation of the proposed method is still going on, considering other sets of parameters.

Keywords : *mixed-type functional differential equations, non linear equations, vibration of elastics tissues, numerical approximation*
2010 Mathematics Subject Classification : **95Q05**

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A New Family of Large Sets of t-Designs and Cryptography

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Abstract We construct several new large sets of t-designs that these large sets give rise to further new large sets by means of known recursive constructions. Then we investigate the combinatorial properties of threshold schemes. In formally a (t,w)-threshold schemes is a way of distributing partial information (shadows) to w participants, so that any t of them can easily calculate a key, but no subset of fewer than t participants can determine the key. We express the relationship between recent concept and large sets of t-designs.

3 Introduction

The study of large sets of t-designs constitutes an important part of combinatorial design theory that have applications in cryptography[9]. Let t,k,v and λ be positive integers such that $t \le k \le v$. Let X be a set of size v (or a v-set called point set) and $P_i(X)$, $0 < i \le t$, denotes the set of all i-subsets of X. A t-(v,k,λ) design (or in short a t-design) is a collection D of $k-$subsets (blocks) of a $v-$set X such that every $t-$subset of X is contained in exactly λ blocks [1]. A t-design is simple if no two blocks are identical. Here, we consider only simple designs. We recall some necessary preliminaries from [6]. A large set of t-(v,k,λ) designs, denoted by $LS[N](t,k,v)$, is a partition of the $P_k(X)$ into N disjoint t-(v,k,λ) designs and hence $N = \binom{v-t}{k-t}/\lambda$. Consequently a necessary condition for the existence of an $LS[N](t,k,v)$ is that

$$N \mid \binom{v-i}{k-i}, \quad 0 \le i \le t\,. \tag{147}$$

This necessary condition (147) is not always sufficient, for in 1850, Cayley showed that it is possible to have two disjoint 2-(7,3,1) designs and no more [3]. So there are no $LS[5](2,3,7)$ and $LS[5](3,4,8)$, while $5 \mid \binom{7-i}{3-i}, 0 \le i \le 2$ and $5 \mid \binom{8-i}{4-i}, 0 \le i \le 3$.

Let S_v be the group of all permutations on X, where as defined above X is a v-set. Let $\sigma \in S_v$, $x \in X$, $B \in P_k(X)$ and $\beta \subseteq P_k(X)$. We denote by x^σ, B^σ and β^σ the images under σ of x, B and β, respectively. If $\beta^\sigma = \beta$, then σ is called an automorphism of β. If G is a subgroup of S_v such that $\beta^\sigma = \beta$ for every $\sigma \in G$, we say that β is G-invariant. The set of all automorphisms of β forms a subgroup of S_v. β is called rigid if its automorphism group is trivial.

In order to define the same concept for large sets, consider the set $D = \{\beta_i\}_{i=1}^N$, such that each β_i is a t-(v,k,λ) design and $\beta_i \cap \beta_j = \emptyset$ for all $i \neq j$. A permutation $\sigma \in S_v$ is said to be an automorphism of D if $D^\sigma = D$, that is, $\beta_i^\sigma \in D$, for each $\beta_i \in D$. The set of all automorphisms of D is, of course, a subgroup of S_v denoted by $AutD$. If G is a subgroup of $AutD$, we say that D is G-invariant. Notice that a G-invariant large set may contain designs which are not G-invariant by themselves [4].

Let G be a subgroup of S_v and let $T_1, T_2, \ldots, T_s$ and $K_1, K_2, \ldots, K_r$ (for positive integers r and s) be the orbits of $P_t(X)$ and $P_k(X)$ under the action of G, respectively. The Kramer-Mesner matrix is the $s \times r$ matrix $A_{t,k}^v(G)$ whose

Keywords : *Large Set, Kramer-Mesner Matrix, Block Design, Threshold Scheme.*
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(i,j)-th entry is $|\{k \in K_j \; ; \; T \subseteq k\}|$, where T is any representative in T_i. The following theorem, due to Kramer and Mesner gives more details [8].

Theorem A There exists a G-invariant t-(v,k,λ) design β if and only if there exists a vector $u \in \{0,1\}^r$ satisfying the equation $A^v_{t,k}(G)u = \lambda J$, where J is the s-dimensional all one vector.

Remark B. One may also adapt the same method for finding large sets. We pick up one from the set of solutions of u and remove the corresponding columns from $A^v_{t,k}(G)$. The resulting matrix $A'^v_{t,k}(G)$ is used in a similar way to find designs via the equation $A'^v_{t,k}(G)u' = \lambda J$. We repeat the procedure until all orbits on k-subsets are used [7].

The purpose of this paper is to construct some large sets of t-designs whose existence was perviously not known. A few of these large sets are used to obtain further new large sets.

Suppose that a bank has a vault that must be opened every day. The bank employs three senior tellers, but they do not want to trust any individual with the combination. Hence, they would like a system whereby any two of the three senior tellers can gain access to the vault. This problem can be solved by means of threshold schemes [5].

A perfect (t,w)-threshold scheme is a method of sharing a secret key k among a finite set P of w participants, in such a way that any t participants can compute any information about the value of k from the shares they hold [5].
In the bank vault example described above, we desire a $(2,3)$-thresold scheme.
We now turn our attention to a special type of threshold scheme which is known as anonymous threshold scheme. A perfect threshold scheme is called anonymous if the following two properties are satisfied:

- The w participants receive w distinct shares.
- The key can be computed solely as a function of t shares, without the knowledge of which participant holds which share. Thus, the key computation can be performed by a black box that is given t shares and does not know the identities of the participants holding those shares.

As an illustration of applications of large sets of t-designs in cryptography, let us see how an $LS[7](2,3,9)$ can be used to construct an anonymous $(3,3)$-threshold scheme with seven possible keys. The large set consists of seven disjoint $2-(9,3,1)$ designs. Denote them by $\beta_i \;\; (0 \le i \le 6)$. Now, the seven keys correspond to the seven designs in the large set; the shares are the points in $X = \{0,1,2,...,9\}$. If the dealer's secret is K then he chooses a random block B from β_K and gives the three points in B to the three participants as their shares. Now, three shares uniquely determine the block B, which occurs in exactly one of the seven designs (since we have a large set). Hence three participants can compute K. But K, there is a (unique) block in β_K that contains x and y [5].

4 Main results

In this section first we state the necessary condition (1.1), in terms of some congruence relations. Then we prove the existence of a family of large sets. Let p be a prime number and k_p be the smallest power of p such that $k < p^{k_p}$ [10]. The following theorem helps us to identify the existence of some large sets with special parameters.

Theorem C.[10] Let p^α be a prime power. Then $v \in B[p^\alpha](t,k)$ if and only if one of the following conditions hold:

i. $v \equiv t,\dots,k-1 \pmod{p^{k_p+\alpha-1}}$,

ii. $v \equiv v_0 \pmod{p^{k_p+\alpha-1}}$, $k < v_0 < p^{k_p+\alpha-1}$ and $v_0 \in B[p^\alpha](t,k)$.

The following theorem due to Khosrovshahi et al. [7], presents a recursive method and provides an important mechanism for constructing families of large sets from small cases.

Theorem D.[7] If there exist $LS[N](t,i,v)$ for all $t+1 \le i \le k$, then there exist $LS[N](t,i,l(v-t)+j)$ for all $l \ge 1$,

$t+1 \le i \le k$ and $t \le j < i$.

A t-(v,k,λ) design D is said to be *cyclic* if it has an automorphism σ which is a cycle of order v on the point set X. If D is cyclic it is clear that we can regard the points of D as the elements of the additive cyclic group Z_v, in such a way that σ is simply the cycle $x \to x+1$. In this case, Z_v is a subgroup of $Aut(D)$ [2].

Remark E. Consider Theorem A and Remark B above and let B be a matrix of size $s \times q$, where $q \le r$ and all q columns of B are chosen arbitrarily from r columns of $A^v_{t,k}(G)$, then any solution of matrix equation $Bv = \lambda J$ easily yields to a solution u of Kramer-Mesner Matrix equation $A^v_{t,k}(G)u = \lambda J$. Just let u be the zero-one vector each of its entry correspond to each column of $A^v_{t,k}(G)$ such that for all columns in B the vector u has the same entry as v and corresponding for the other columns has zero entry.

Finally, based on Theorem D and the existence of $LS[N](t,k,v)$ for some special parametrs that we have made them with Remark B, proved the existence of some new family of large sets.

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POSTERS

Solving a Stochastic Inverse Problems using an iterative method with unknown operator

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Abstract In this paper, we consider a linear equation $Ax = u$, where A is a unknown compact operator in Hilbert space $\mathbf{H}_1$. To solve this problem arising from many experimental fields of science, we propose an iterative method with Gaussian errors which converges almost completely.

Introduction The resolution of ill-posed inverse problem often requires a regularization, that transforms it into a family of well-posed problems whose solution is an approximation to the exact solution of the original model. Then, we choose, then one of these solutions by adjusting the regularization parameters in order to gain the maximum stability while controlling the error caused by the approximation of the modified problem.

To solve problems that are not well-posed and inverse problems, the most used regularization method is Tikhonov regularization. In different methods of Tikhonov regularization, we always use the inversion of the operator, which is not always easy to do, this operation may be very costly in practice in terms of computing. We propose an iterative method for solving a stochastic inverse problems in the case where the operator is unknown. We establish exponential inequalities. These inequalities yield the almost complete convergence and the convergence rate of approximate solution.

A large variety of problems arising from different areas of applied sciences can be often regarded mathematically as an equation with an operator taking the following form

$$Ax = u \tag{148}$$

where $A : \mathbb{H}_1 \to \mathbb{H}_2$ describes a compact operator ie $A \in \mathscr{K}(\mathbb{H}_1, \mathbb{H}_2)$. x is unknown solution in Hilbert space $\mathbb{H}_1$ and $u \in \mathbb{H}_2$.

Setting of the problem Consider the following iterative process, for any first iteration solution x_0, the n-th $(n \geq 1)$ iteration solution is defined by

$$x_n = x_{n-1} - \epsilon\left(\lambda x_{n-1} - A^*\left(u_e - Ax_{n-1}\right)\right) + a_n \xi_n \tag{149}$$

where $\left(a_i\right)_{i\geq 1}$ a sequence of real positive numbers such that na_n converges to a constant when n tends to infinity and $\left(\xi_i\right)_{i\geq 1}$ is a sequence of independent Gaussian, zero mean and identically distributed random elements, satisfying $\mathbb{E}\|\xi_i\|^2 = \sigma^2 < +\infty$. We assume that the operator A is unknown. We consider an estimator of A such that $\left\|A - \widehat{A}\right\| < \vartheta$.

Theorem 34. *Let A be a linear compact operator and x_e be the unique exact solution of equation* (148) *with exact right hand side u_e. Furthermore, considering the iterative process*

$$\widehat{x}_n = \widehat{x}_{n-1} - \epsilon\left(\lambda x_{n-1} - \widehat{A}^*\left(u_e - \widehat{A}x_{n-1}\right)\right) + a_n \xi_n$$

where $\left(\xi_i\right)_{i\geq 1}$ is a sequence of independent Gaussian, zero mean and identically distributed random elements. Assume that $1 < \epsilon\lambda < 2$. $\widehat{A}$ is as, $\left\|A - \widehat{A}\right\| < \vartheta$. Then, $\forall \varepsilon > 0$

$$\mathbb{P}\left(\|x_n - x_e\| > \varepsilon\right) \leq 2\exp\left(-\frac{\left\|B^{1-n}\right\|^2 \varepsilon^2}{18\left(\sum_{i=1} a_i^2\right)\sigma^2\lambda_1}\left[1 - 3\sigma\frac{\sqrt{S_p\widehat{B}^*\widehat{B}\sum_{i=1} a_i^2}}{\varepsilon\left\|\widehat{B}^{1-n}\right\|}\right]^2\right) \tag{150}$$

Keywords : *Inverse problem; Linear operator; Tikhonov regularization; Iterative methode.*
2010 Mathematics Subject Classification : **34A55; 46A32; 47A52.**

where $S_p \widehat{B}^* \widehat{B}$ *is a spectrum of* $\widehat{B}^* \widehat{B}$, *and* $\widehat{B} = I - \epsilon\left(\lambda I + \widehat{A}^* \widehat{A}\right)$.

Corollary 35. *Under the assumptions of Theorem 1, the sequence* (x_n) *converges almost completely (a.co.) to the exact solution* x_e *of the equation* (148).

Corollary 36. *Under the assumptions of Theorem 1, we have*

$$x_n^\alpha - x_e = O\left(\sqrt{\frac{\log n}{n}}\right) \quad a.co. \tag{151}$$

Corollary 37. *Under the assumptions of Theorem 1, for a given significance threshold* γ, *it exists an integer* n_γ *for which*

$$\mathbb{P}\left\{\left\| x_{n_\gamma}^\alpha - x_e \right\| \le \varepsilon\right\} \ge 1 - \gamma, \tag{152}$$

ie, the exact solution x_e *of equation (148) belongs to the closed ball with center* $x_{n_\gamma}^\alpha$ *and radius* ε *with probability greater than or equal to* $1 - \gamma$.

We have conducted a simulation study to show that the estimator x_n given by the iterative method (149) is consistent.

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Global optimization: the Alienor mixed method with branch and bound technique

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Abstract In this paper we propose an iterative algorithm to solve the global optimization problem of polynomial functions with several variables. Presents a variant of a deterministic method. This is the method of reducing processing Alienor coupled with the branch and bound method. This transformation allows to reduce a function of n variables to a function of a single variable that would retain the global minimizers at least so approximated. The technique used is based on the reduction of the dimension using "space filling curves". These curves have the advantage of being of class C1 and preserve the properties of the objective function. The Eleanor method has proven to be highly effective by partnering with some one-dimensional methods such as algorithms recovery. The coupling of Alienor with these algorithms was applied to multivariate testing functions with a global minimum hard to find by conventional methods. We study in our work the coupling method Alienor with the branch and bound algorithm in the case where the objective function is defined on a polunome and P Rn pad. Interesting results concerning Minimum of approximation and the computing time has been made.

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Keywords : *Global Optimization, polynomial function, method branc and bound, Method of reductive transformation, curves a-dense..*
2010 Mathematics Subject Classification : **90C26; 90C30;90C34.**

Supramolecular Study on the Interaction between Doxorubicin and β-cyclodextrin

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Abstract Doxorubicin agent is one of the most frequently used in anticancer treatment. Cyclodextrins are known to form inclusion complex with many kinds of compounds. In this study the Doxorubicin - βCD interactions were simulated using the semi empirical technique PM6 and the hybrid ONIOM/2 method at three levels (B3LYP, MPW1PW91 and WB97X-D).The binding energies (Ebind) and the total stabilization energies (EONIOM) were used to confirm the most favorable configurations and the stability sequence of the inclusion complexes. Natural bond orbital (NBO) analysis reveals that the most important hydrogen bonds interactions are of type N-H...O and O-H...O with stabilization energies around 5-17 kcal/mol indicating that intermolecular hydrogen bonds is main driving forces of inclusion complexes formation.

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Keywords : *Doxorubicin; β-cyclodextrin; inclusion complexes; quantum mechanics.*

2010 Mathematics Subject Classification : 82C10; 35Q40.

A computational study of Host-guest interaction between Benzoxazolinone and β-cyclodextrin

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Abstract In present work, the inclusion complex of Benzoxazolinone with β-cyclodextrin was investi-gated experimentally and by molecular modeling study. Semi-empirical calculations using PM3, PM6, ONIOM2 (B3LYP/6-31G : PM3) and ONIOM2 (B3LYP/6-31G : PM6) methods, in vacuum and in water were performed. The negative energy values obtained demonstrate clearly that the benzoxazolinone molecule can form a stable inclusion complex with β-CD, and the *B* orientation is signi?cantly more favorable than that of *A* orientation.

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Keywords : *β-Cyclodextrin; Benzoxazolinone; inclusion complex PM6; ONIOM2; UV-vis spectrophotometry.*
2010 Mathematics Subject Classification : **74F10; 81V10.**

Riesz–Martin representation for positive polysuperharmonic functions in a harmonic space

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Abstract In the context of the axiomatic potential theory, we introduce the notions of polyharmonic Green domains and polyharmonic functions of order m on a Brelot space Ω. For these functions we prove that if u is a positive polyharmonic function in a polyharmonic Green domain ω, then u has a representation analogous to the Riesz–Martin representation for positive harmonic functions on Ω.

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Keywords : *polyharmonic functions; polyharmonic Green domains.*
2010 Mathematics Subject Classification : **31A30; 34B27.**

Influence of morphology and human activities on the evolution of water chemistry a semi arid aquifer: The Tamlouka plain case study

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Abstract To understand the spatial evolution of the groundwater quality in Tamlouka plain, piezometric surveys and geochemical analyzes have been made since 2011 in nearly 50 wells. Hydrodynamic and hydrogeological data have also been used. The interpretation, modeling and graphical representation of hydrochemical data appear are three main types of facies (with chloride, sulphates and bicarbonates) which follow a well-defined spatial distribution. Indeed all concentrations increase from the periphery to the center of the plain (from 367 to 4000 mg / l). The morphological character (bowl) is the main cause of the increase. During their journey underground, the water undergoes a double influence; that of the lithology and of the flow velocity. Reporting features and exchange index base (EIB) show that the lithology of the reservoir rock is extremely diverse (carbonate, clay and marl) which determines a certain zoning reflecting the spatial variation of parameters aroused.

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Keywords : *Tamlouka plain; geomorphology; lithology; mineralization.*
2010 Mathematics Subject Classification : **47N60; 80A50.**

Application of the method of analysis in principal component (APC) on the waters of the region of Bekkouche Lakhdar

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Abstract The Analysis in Principal Components (APC) makes after minority interests multidimensional descriptive methods called factorial methods. The APC proposes, from a rectangular picture of data containing the values of (p) quantitative variables for (n) units (called also individuals), geometrical representations of these units and these variables. These data can be stemming from a procedure of sampling either from the observation of the whole population. The application of the A.P.C on waters of the region of Bekkouche Lakhdar allowed us to say that waters were individualized and grouped according to a similar evolution of the parameters of pollution and drinkability.

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Keywords : *pollution; drinkability; APC; Bekkouche Lakhdar.*

2010 Mathematics Subject Classification : **91B76.**

An inequality of an arithmetical function related to the number of divisors of an integer

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Abstract In this work we provide an inequality of an arithmetical function f attached to the number of divisor of an integer n. The method consists to use some fundamental functions in order to obtain an estimation of an arithmetical function f.

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Keywords : *Arithmetical function; inequalities.*
2010 Mathematics Subject Classification : **15A39; 26D20.**

An M/G/1 Retrial Queue with Negative Customers and Bernoulli Feedback: Stochastic Comparison Approach

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Abstract We propose to use a mathematical method based on stochastic comparisons of Markov chains in order to derive performance indices bounds. In this paper, we consider a single server retrial queue with negative customers and two types of Bernoulli feedback and we derive several stochastic comparison properties in the sense of strong stochastic ordering and convex ordering. The stochastic inequalities provide simple insensitive bounds for the stationary queue length distribution. Numerical illustrations are provided to support the results.

Introduction In recent years, there has been significant contribution to the retrial queueing systems. The special feature of the retrial phenomenon is that an arriving customer who finds the server busy upon arrival may join the virtual group of blocked customers, called orbit and retry for service after a random amount of time. The queueing system with retrial phenomenon is called a retrial queue. Retrial queues are widely and successfully used as mathematical models of several computer network systems, telephone switching systems and wireless network systems. For instance, peripherals in computer networking systems may make retrials to receive service from a central processor. Hosts in local area networks may attempt many retrials in order to access the communication medium, which is clearly indicated in the Carrier Sense Multiple Access (CSMA) protocol that controls this access.

In the recent past, several researchers have studied a new class of queueing networks in which customers are either positive or negative. The queueing systems with negative customers is called G-queues. Positive customers enter a queue and receive service as ordinary queueing customers. A negative customer will vanish if it arrives at an empty queue, and it will reduce by one the number of positive customers if the queue is nonempty. Negative customers can not accumulate in a queue and do not receive service. Positive customers which leave a queue to enter another queue can become negative or remain positive. For example, in computer networking systems, if a virus enters a node, one or more files may be infected, and the system manager may have to go through a number of backups to recover the infected files. In some cases, they may not be recoverable. A virus may originate from outside the network, e.g., through a floppy disk, or by an electronic mail [2].

One additional feature which has been widely discussed in retrial queueing systems is the Bernoulli feedback of the customers. The retrial queueing system with Bernoulli feedback of the customers occurs in a computer system which operates in a time sharing model. In a time shared computer system, each job is allocated a small time interval for uninterrupted processing at the CPU. If the total required processing time of a job exceeds the length of this interval, it is fed back to the orbit containing waiting jobs, where the jobs waiting in the orbit are permitted another turn in processing facility according to retrial. This procedure is repeated until the job has obtained its required processing time before leaving the system [1].

In the last decade there has been a tendency towards the research of approximations and bounds. Qualitative properties of stochastic models constitute an important theoretical basis for approximation methods. One of the important qualitative properties and approximation methods is monotonicity which can be studied using the general theory of stochastic ordering.

Stochastic order is useful for studying internal changes of performance due to parameter variations, to compare distinct systems, to approximate a system by a simpler one, and to obtain upper and lower bounds for the main performance measures of systems [1].

Keywords : *retrial queue; negative customers, Bernoulli feedback; Markov chain; stochastic comparison; monotonicity.*
2010 Mathematics Subject Classification : **60E15; 60J10; 60K25.**

An examination of the literature reveals the remarkable fact that the non-homogeneity caused by the flow of repeated attempts is the key to understand most analytical difficulties arising in the study of retrial queues. Many efforts have been devoted to deriving performance measures such as queue length, waiting time, busy period distributions, and so on. However, these performance characteristics have been provided through transform methods which have made the expressions cumbersome and the obtained results cannot be put into practice.

Main results In this paper, we use a stochastic comparaison approach to establish insensitive bounds for some performance measures of an $M/G/1$ Bernoulli feedback retrial queueing system with negative customers by using the theory of stochastic orderings (see [1]). The proposed technique is quite different from that in Krishna Kumar et al. [2], in the sense that our approach provides from the fact that we can come to a compromise between the role of these qualitative bounds and the complexity of resolution of some complicated systems where some parameters are not perfectly known (e.g. the service times and repair times distributions are unknown). We prove the monotonicity of the transition operator of the embedded Markov chain relative to strong stochastic ordering and convex ordering. We obtain comparability conditions for the distribution of the number of customers in the system. The main result of this paper consists in giving insensitive bounds for the stationary distribution of the considered embedded Markov chain. Such a result is confirmed by numerical illustrations.

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Existence of periodic solutions for second order nonlinear neutral differential equations with functional delay

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Abstract In this paper, we study the existence of periodic solutions of the second order nonlinear neutral differential equation with functional delay

$$\frac{d^2}{dt^2}x(t)+p(t)\frac{d}{dt}x(t)+q(t)x^3(t)=\frac{d}{dt}g(t,x(t-\tau(t)))+f\left(t,x^3(t),x(t-\tau(t))\right),\ t\in\mathbb{R}.$$

The main tool employed here is the Burton Krasnoselskii's by the fixed point theorem dealing with asum of two mapping, one is a large and the other is compact.

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Keywords : *Krasnoselskii's theorem; large contraction; nonlinear neutral differential equations; integral equation; periodic solution.*
2010 Mathematics Subject Classification : **34C25; 39A23.**

Optimization of Energy-Efficient Control Power in wireless networks : A Multi-Criterion Game-Theoretic Approche

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Abstract We consider a wireless communications network with N uses and a single receiver (multiple access channel), where the users choose themselves their best power control strategy in order to selfishly maximize their energy-efficiency. Power control is modeled as a non-cooperative game in which each user decides how much power to transmit over each carrier to maximize its own utility. The utility function considered here is a multi-criterion function that measures the number of reliable bits transmitted on all carriers while considering the amount of energy consumed. Existing work to suggest the existence and uniqueness of the Nash equilibrium of mono-criterion power control game. In this paper, we have modeled the problem as a multi-criterion power control game. A search algorithm of optimal transmit powers (Nash equilibrium) which optimizes the utility of each user has been proposed.

Introduction Currently, the development of wireless technology continues to grow and manage their parameters becomes an increasingly difficult task. The demands in wireless transmissions, providing reliable communications of the voice and of the data "anywhere, anytime" has increased massively in recent years. The explosion of this market, its sustained growth and the emergence of new services bring existing the mobile networks at their limit. Unlike wired networks, a number of problems such as call admission control, resource allocation, location management and routing are more difficult to solve in mobile networks and must their complexity to the imperfections of the support wireless [1].

It is therefore necessary to establish strategies to spread the load over different wireless access points, for example. Currently, this is achieved by static policies in which the mobile is primarily connect via access points. It takes no account of the quality of service required by the mobile or the overall state, which changes over time, other networks to which the mobile can connect.

The recent revival of interest in the application of game theory tools to problems encountered in telecommunications networks is owing to the development of wireless communications. In this context, the multiple devices, transmitters and receivers, share the same communications environment and the same resources (such as : frequency bands, the time intervals, ...) [1, 2].

However, over the last decade, energy consumption has become an issue more and more important in wireless networks. Furthermore, in wireless networks where change batteries for devices is very inconvenient, or in some cases impossible, the power consumption becomes a critical issue. Such that each user terminal adopts strategies based on the energy level of his battery in order to maximize its transmission rate (number of packets sent by success) upon interaction with other users.

We study wireless networks in which user devices are autonomous in their choice of communication configurations. This autonomy of decision may relate in particular to the choice of network access technology, the choice of the access point, the frequency bands occupied, the power of the transmitted signal, etc. Typically, these choices are made in order to maximize the transmission rate while spending as little energy as possible for each network device. Under the assumption that the users take their decisions rationally to maximize their performance.

Keywords : *Optimization; Game theory; Multi-criterion game; Equilibria strategy; Energy efficient; Power control; Wireless networks; Communications.*
2010 Mathematics Subject Classification : **60E15; 60J10; 60K25.**

Therefore the optimization problem of a function of energy efficiency is discussed in this paper. This performance metric reflects the average number of bits that can be transmitted reliably through the channel by unit of energy consumed. In the literature, the issue of energy efficiency has been modeled as a non-cooperative game.

The approach who is to consider a single criterion that a terminal wishes to maximize, is often not sufficient to describe the needs and behavior of users. A simplistic approach often used to treat the multiple criteria aspect is to define a single criterion that takes into account several qualities of service. The other approaches more sensitive to each criterion consist to separate criteria and define the concepts of equilibrium that are sensitive to each of them.

Main results In this paper, we consider a wireless network in which N users terminals are transmitting to a common concentration point [1, 2]. Game-theoretic approaches to power control have recently attracted considerable attention. The power control problem is modeled as a non-cooperative game in which users choose their transmit powers in order to maximize their utilities. In this work, the utility is defined as two functions: transmision efficiency and energy consumed. We have modeled the problem as a non-cooperative power control game, who can be expressed as $G = \langle \mathscr{I}, \mathscr{P}_i, u_i \rangle$, where $\mathscr{I} = 1, ..., N$ is the set of users/players, $\mathscr{P}_i$ is the strategy set for the i^{th} user, and u_i is the multi-criterion utility function for the i^{th} user. Each user decides what strategy to choose from its strategy set in order to maximize its own utility.

Using the principe of aggregation method, transforming the bi-criterion game to mono-criterion game. we discretize the interval of powers players $[P_i^{min}, P_i^{max}]$. We have study the change of the power and utilities of the players in equilibrium (Nash equilibrium), by varying the aggregation parameter, the distance between the players and the access point and the number of strategies discretized for each player.

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Image segmentation using particle swarm optimization and level set methods

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Abstract In this paper, a new image segmentation method based on Particle Swarm Optimization (PSO) and outlier rejection caused by the membership function of the kernel fuzzy local information c-menas (KFLICM) algorithm combined with level set is proposed. A traditional approach to segmentation of magnetic resonance (MR) images is the fuzzy c-means (FCM) clustering algorithm. The membership function of this conventional algorithm is sensitive to the outlier and does not integrate the spatial information in the image, thus the algorithm is very sensitive to noise and in-homogeneities in the image, moreover, it depends on cluster centers initialization. To improve the outlier rejection and reduce the noise sensitivity of conventional FCM clustering algorithm, a novel extended FCM algorithm for image segmentation is presented. In general, the FCM algorithm chooses the initial cluster centers randomly, but the use of PSO algorithm gives us a good result for these centers. Our algorithm is also completed by adding into the standard FCM algorithm the spatial neighborhood information. These a priori are used in the cost function to be optimized. The resulting fuzzy clustering is used as the initial level set function. The results confirm the effectiveness of the improved kernel fuzzy local information c-means and outlier rejection (IKFLICMOR) associated with level set for MR image segmentation.

Introduction The Fuzzy C-Means (FCM) algorithm minimizes the objective function :

$$J=\sum_{i=1}^{N}\sum_{k=1}^{c}\mu_{ki}^{m}d^{2}\left(x_i,V_k\right) \tag{153}$$

U represents the membership function matrix, d the distance metric between the element x_i and the cluster center V_k and m the degree of fuzziness (m>1). Stelios Krinidis and Vassilios Chatzis [1] propose the following object function:

$$J=\sum_{i=1}^{N}\sum_{k=1}^{c}\mu_{ki}^{m}d^{2}\left(x_i,V_k\right)+G_{ki} \tag{154}$$

The term G is given as:

$$G_{ki}=\sum_{j\in N}\frac{1}{d_{ij}+1}\left(1-\mu_{kj}\right)^{m}\left\|x_j-V_k\right\|^{2} \tag{155}$$

we propose to modify the original algorithm by considering the fuzzy partition matrix, pixels spatial information and the initialization cluster centers. The proposed algorithm is described in these steps:

1. Replacing Euclidian distance with Mahalanobis.
2. Membership function and cluster centers initialization using PSO algorithm.
3. To improve the membership function of the FLICM algorithm is modified by considering outlier rejection and Gaussian kernel.
4. Using Level set to finalize the segmentation .

Main results The results of the IKFLICMOR algorithm are presented and compared to those of standard FCM algorithm and other algorithms.

Keywords : *Image segmentation; outlier rejection; FCM; PSO; Spatial fuzzy clustering; Level set methods.*
2010 Mathematics Subject Classification : **62H35; 35Q93**

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Polygonal method for second-derivative Lagrangian

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Abstract We develop a matrix approach to determine the exact propagator related to the Lagrangian having an acceleration term in addition to the usual term. The quantum mechanical system is defined for Euclidean time using the path integral formalism. This technique lead us to a fourth-order differential equation witch it resolution will be via the Green's functions. The propagator is entirely defined by the knowledge of the phase and the normalization constant.

Introduction Every physical system is described by its own Lagrangian. Indeed, knowledge of the latter allows us to study the evolution of the considered system and have a wealth of information on all physical quantities concerning him. Systems that have a quadratic Lagrangian coordinates and velocity are considered the easiest to solve they therefore constitute a first approximation the Lagrangian complicated forms. Moreover, There are systems of Lagrangian that contain an acceleration term which we can meet in many practices cases such as:

1- Polymers on an intermediate distance scale are stiff objects and their energy requires the inclusion of a bending energy which involves the square of the second derivative, x**2(s), where s is the length parameter of the spaces curve.

2- The formation of microemulsion cannot take places without the ampliphilic soap layer between water and oil losing its surface tension.

3- The strings of color electric flux lines, which bind quarks and antiquarks, can lose their tension in a phase transition, in which case they are controlled completely by second-gradient elasticity.

We propose in this, work, to apply the polygonal method in the case of a system described by a Lagrangian of the form [1,2]:

$$L(x,\dot{x},t) = \frac{k}{2}\ddot{x}^2 + \frac{m}{2}\dot{x}^2 - f(t)x \tag{156}$$

where $f(t)$ denote the source term.

Main results The polygonal paths method or matrix method has been applied success in the calculation of the propagator relating to a physical system described by a second-order derivative Lagrangian. The phase of the propagator is calculated with the Green functions. The corresponding normalization constant is determined by the Cramer method.

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Keywords : *Polygonal method; path integral; Green function* .

2010 Mathematics Subject Classification : 13G05; 15A23; 34A05.

On a coupled nonlinear singular thermoelastic system

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Abstract For a coupled nonlinear singular system of thermoelasticity with one space dimension, we consider its initial boundary value problem on an interval. For one of the unknowns a classical condition is replaced by a nonlocal constraint of integral type. Because of the presence of a memory term in one of the equations and the presence of a weighted boundary integral condition the solution requires a delicate set of techniques. We solve a particular case of the given nonlinear problem by using a functional analysis approach. Based on the obtained results and an iteration method we establish the well-posedness of solutions in a weightedSobolev spaces.

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Keywords : *Coupled thermoelastic system; Nonlocal constraint; Strong solution; Weak solution; Iterative method.*
2010 Mathematics Subject Classification : **74B20; 34B10; 35D35; 35D30.**

Solvability problem for fractional dynamic systems and optimal controls

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Abstract In this paper, we study the solvability question of mild solutions for a class of fractional dynamical system in Banach spaces. We use fractional calculus, semigroup theory and fixed point theorem for the main results. In order to generalize previous works in the field, an existence result of optimal control pairs governed by the presented formulated problem is proved. Finally, an example is also given to illustrate the Abstract results.

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Keywords : *Solvability; fractional calculus; optimal control; Banach space.*
2010 Mathematics Subject Classification : **26A33; 34B10; 49J15; 49J45.**

Determination of the ground state energy of symmetric PT complex potential in the framework of Feynman integrals formalism

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Abstract Feynman Kleinert variational method [1] allows to evaluate, in a satisfactory manner, the energy of the ground state for a given quantum system. To make the method more efficient, Kleinert found it useful to introduce into its starting technique, new corrections, called systematic and whose contribution seemed to be decisive later. In his new method [1], Kleinert combines the perturbative and variational methods to express the classical effective potential in the form of a convergent series. The various terms appearing in the final expression for the energy will be calculated through correlation functions. The bulk of the work consists of using algebraic and variational methods with or without systematic corrections to calculate the energy of the ground state of a physical system subject to a PT-symmetric anharmonic potential. The choice of this family of potential lies in the fact that they have the particularity to be unsealed but admit however, real and positive eigenvalues. We have also shown through the matrix method of Feynman formalism of Feynman that a harmonic oscillator subject to linear and complex potential admits the same energy spectrum as a pure harmonic oscillator. The results obtained in each study are fully consistent with those calculated by other methods.

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Keywords : *variational method; quantum system; systematic; complex potential*
2010 Mathematics Subject Classification : **35A15; 81Q80; 31E05.**

Existence of weak positive solution for class of $\left(p_1, p_2, \ldots, p_n\right)$ -laplacian elliptic system with Different Weights

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Abstract Consider the system

$$\begin{cases} -\Delta_{P_1,p_1} u_1 = \lambda a_1(x)\, u_1^{\alpha_{11}} u_2^{\alpha_{12}} \ldots u_n^{\alpha_{1n}}, \text{in } \Omega, \\ -\Delta_{P_2,p_2} u_2 = \lambda a_2(x)\, u_1^{\alpha_{21}} u_2^{\alpha_{22}} \ldots u_n^{\alpha_{2n}}, \text{in } \Omega, \\ \quad\ldots. \\ -\Delta_{Pn,p_n} u_n = \lambda a_n(x)\, u_1^{\alpha_{n1}} u_2^{\alpha_{n2}} \ldots u_n^{\alpha_{nn}}, \text{in } \Omega, \\ u_i = 0 \text{ on } \partial\Omega, i = 1,2,\ldots n, \end{cases}$$

where Δ_{R_i,r_i} with $r_i > 1$ and $R_i = R_i(x)$ is a weights functions, denotes the weighted r_i-Laplacian defined by $\Delta_{R_i,r_i} u_i = \operatorname{div}\left(R_i(x)\left|\nabla u_i\right|^{r_i-2} \nabla u_i\right)$, $i = 1,2,\ldots n, \lambda$ is a positive parameter,$a_i(x)$, $i = 1,2,\ldots n$, are a weights functions,and Ω is a bounded domain in $\mathbb{R}^N$ $(N > 1)$ with smooth boundary $\partial\Omega$. We prove the existence of a large positive solutions for λ large, we use the method of sub–supersolutions to establish our results.

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Keywords : *Positive solutions; Sub-supersolutions; p-Laplacian systems.*
2010 Mathematics Subject Classification : **05C38; 15A15; 05A15; 15A18.**

An optimal control problem for nonlocal fractional differential equations

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Abstract In this paper, we study an optimal control problem for a class of fractional differential equations in Banach spaces. We use fractional calculus, semigroup theory and fixed point theorem for the main results. In order to generalize previous works in the field, an existence result of optimal control pairs governed by the presented formulated problem is proved. Finally, an example is also given to illustrate the Abstract results.

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Keywords : *Fractional calculus; optimal control; Banach space.*
2010 Mathematics Subject Classification : **26A33; 34B10; 49J15; 49J45.**

Impulsive fractional differential controlled inclusions in Hilbert spaces

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Abstract In this paper, we study a class of impulsive fractional differential inclusion in Hilbert spaces. We use fractional calculus, semigroup theory, multivalued analysis and fixed point theorem for the main results. Finally, an example is also given to illustrate the Abstract results.

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Keywords : *Fractional calculus; multivalued maps; control theory; impulsive condition; Hilbert space.*
2010 Mathematics Subject Classification : **26A33; 35R70; 58E25; 49N25.**

Bilateral contact problem with adhesion and damage for an electro elasticviscoplastic body

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Abstract We study a mathematical problem describing the frictionless adhesive contact between a electro elastic-viscoplastic material with damage and a foundation. The adhesion process is modeled by a bonding field on the contact surface. The contact is bilateral and the tangential shear due to the bonding field is included. We establish a variational formulation for the problem and prove the existence and uniqueness of the solution. The existence of a unique weak solution for the problem is established using arguments of nonlinear evolution equations with monotone operators, a classical existence and uniqueness result for parabolic inequalities, and Banach's fixed point theorem.

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Keywords : *Dynamic process; viscoelastic material with damage; adhesion; bilateral; frictionless contact, existence and uniqueness; fixed point.*
2010 Mathematics Subject Classification : **74H20; 74H25; 74M15; 74R99.**

Regional stabilization for semilenear parabolic systems

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Abstract In this paper, we shall study the regional stability and the regional stabilization of a constrained feedback control for semilinear parabolic systems defined on a Hilbert state space. Then, we shall show that stabilising such a system reduces stabilising only its projection on a suitable subspace. For this purpose, a new constrained stabilising feedback control that allows a polynomial decay estimate of the stabilised state is given. Also, the robustness of the considered control is discussed. An illustrating example and simulations are presented.

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Keywords : *Semilinear parabolic systems; constrained feedback stabilisation; regionally exponentially stabilization.*
2010 Mathematics Subject Classification : **35K58; 93D15.**

First principle calculations on optoelectronic properties of $BiNbO_4$

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Abstract We have performed an ab- initio calculation using the FPLAW method within local density approximation (LDA) implemented in WIEN2k code for orthorhombic (α) and triclinic (β) phases of $BiNbO_4$. In addition, the modified Becke-Johnson exchange potential (mBJ) + LDA approach is also used to improve the electronic properties. Our calculated lattice constants of both structures using LDA are in good agreement with experimental values. For band structure calculations, mBJ-LDA approach provides good results for band gap value as compared to LDA. The estimated LDA (mBJ) band gap values are 2.89 eV (3.73 eV) and 2.62 eV (3.15 eV) for α and β phases of $BiNbO_4$, respectively. A significant optical anisotropy is clearly observed in the visible-light region. We also calculated and discussed the electron energy loss spectrum for $BiNbO_4$. This is the first quantitative theoretical prediction of optical properties and electron energy loss spectrum, for both orthorhombic and triclinic phases of $BiNbO_4$ compound.

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Keywords : *$BiNbO_4$; FP-LAPW method; Wien2K code; mBJ approximation; structural properties; electronic properties; optical properties*
2010 Mathematics Subject Classification : **37L65; 74G65.**

Theoretical studies of the spectroscopic (FT-IR, NMR), conformational stability, structure and reaction mechanism of 2-hydroxyquinoxaline derivatives and its tautomers

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Abstract In the present work, the structural and electronic properties of 2-hydoroxyquinoxaline and all of its possible tautomer's have been investigate by the application of MM2, MMFF94, AM, PM3, DFT and HF type quantum chemical calculations. According to the results of the calculations, tautomer's B-B' have been find to be the most stable one among all the structures. The calculated density of partial charges by extended Hückel method shows that the O- and N-alkylation are due to the different values of O and N atom's charges. Some important geometrical properties and the calculated IR and NMR spectra of the systems have also been report in the study. The data obtained for are in a good agreement with experimental foundations.

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Keywords : *2-hydroxyquinoxaline; tautomer; MM2; DFT; partial charge; reaction mechanism.*
2010 Mathematics Subject Classification : **37J25; 70B15.**

Nonlinear elliptic problem with singular term at the boundary

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Abstract Let $\Omega \subset \mathbb{R}^N$ be a bounded regular domain of $\mathbb{R}^N$ we consider the following class of elliptic problem

$$\begin{cases} -\Delta u = \frac{u^q}{d^2} & \text{in } \Omega, \\ u > 0 & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega, \end{cases}$$

where $0 < q \leq 2^* - 1$. We investigate the question of existence and nonexistence of positive solutions depending on the range of the exponent q.

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Keywords : *Hardy inequality; Nonlinear elliptic problems; singular weight.*
2010 Mathematics Subject Classification : **35D05; 35D10; 35J20; 35J25; 35J70.**

Existence of Solutions for a Quasilinear Elliptic Equation with Nonlocal Boundary Conditions on Time Scales

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Abstract The purpose of this work is the construction upper and lower solutions for a class of second order quasilinear elliptic equation subject to nonlocal boundary conditions. More specifically, we consider the following nonlinear boundary value problem

$$\begin{cases} -\left(\varphi_p\left(u^{\Delta}\right)\right)^{\Delta} = f\left(x,u\right), \text{ in } \left(a,b\right)_T, \\ u\left(a\right) - a_0 u^{\Delta}\left(a\right) = l_0\left(u\right), \\ u\left(\sigma\left(b\right)\right) + a_1 u^{\Delta}\left(\sigma\left(b\right)\right) = l_1\left(u\right), \end{cases} \tag{157}$$

where $p > 1$, $\varphi_p\left(y\right) = \left|y\right|^{p-2} y$, $\left(\varphi_p\left(u^{\Delta}\right)\right)^{\Delta}$ is the one-dimensional $p-$Laplacian, $f : [a,b]_T \times \mathbb{R} \to \mathbb{R}$ is a rd-continuous function, $l_i : C_{rd}\left([a,b]_T\right) \times C_{rd}\left([a,b]_T\right) \to \mathbb{R}$ ($i = 0$ and 1) are rd-continuous and a_0 and a_1 are a positive real numbers.

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Keywords : *Quasilinear elliptic equation; Time-Scale; nonlocal boundary conditions; upper and lower solutions; monotone iterative technique.*
2010 Mathematics Subject Classification : **34B10; 34B15.**

on the use of computers in the accounting tasks

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Abstract The introduction of informatics in accounting has automated a number of tasks that were repetitive operations such as charging and deferral accounting entries in the accounts, centralization and tabulation of the data. The accountant is working on faster data entry. The transmitted data will be more reliable. Informatics must enable firms or accountants to work more efficiently and thus increase profitability.

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Keywords : *Accounting; faster data*

2010 Mathematics Subject Classification : 97R50.

The Modified Gradient-Average Direction of subgradient method

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Abstract The concept of deflecting the subgradient direction has been developed in various algorithms such as the Modified Gradient Technique (MGT) proposed by Camerini, Fratta, and Maffioli (1975) and the Average Direction Strategy (ADS) of Sherali and Ulular (1989). In this paper, we present a new subgradient deection technique called the Modified Gradient-Average Direction.

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Keywords : *subgradient optimization methods, Lagrangian relaxation, Traveling Salesman problem (TSP).*
2010 Mathematics Subject Classification : **35Q93; 53D12.**

Blow-up results of solutions for some weight damped wave partial fractional differential equations

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Abstract Firstly, we use the fixed point theorem to prove existence and uniqueness of local solution theorem and secondly we use the technic of test function to prove the non existence of global solutions theorem.

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Keywords : *Local existence; weight damped wave equation; Blow-up.*
2010 Mathematics Subject Classification : **34A60; 49J24; 49K24.**

Synthesis and characterization of antacids industrial ceramic refractory from algerian kaolins

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Abstract This work aims is to prepare antacid refractory bricks for industrial use by substituting importation clay "KBB" (France origin) by local kaolin ï£¡s and other raw materials: grog, quartz and shard: starting from local Tamazert kaolin's (KT1 and KT2): abundant and cheap composed principally of kaolinite and quartz, origin from the zone of El Milia - Jijel in the North-East of Algeria. The production process is achieved by mixing the ground raw materials (kaolin-Grog-Shard quartz) with optimized proportions and granulometry. The mixture is shaped by applying hydrostatic pressure about 250 bars, then drying at 100 ï£¡C during 24h and sintering at 1300ï£¡C. Quartz is added in various concentrations, in order to increase the silica content which resists to the attacks of the acids better. The raw materials and the products were characterized by various techniques: thermic analyzes (test fusibility, dilatometry), analyzes by diffraction of X-rays (XRD) and X-ray fluorescence. Physical properties of the starting products and the products obtained were also given such as: density, porosity and linear withdrawal of firing, resistance to the attack by the sulfuric acid, compression and flexural strength. The chemical composition of elaborated refractory bricks shows high silica rates (63 - 72). The obtained products are argillaceous refractory with low content alumina (Al2O3 < 35)

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Keywords : *Antacid refractory; Tamazert kaolin's KT1 & KT2; Grog; Quartz.*
2010 Mathematics Subject Classification : **93B50; 20B10.**

The numerical solution of the fractional Algebric-differential equations by fractional complex transform method

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Abstract In this paper, we present a Fractional complex transform method for solving the fractional Algebric-differential equation .The main idea in this method is applying the complex transform method for algebraic-differential equation some examples are present to show of the model. The results confirm that the method is very effective and simple.

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Keywords : *Fractional differential equation; complex transform method; Algebric-differential equation*
2010 Mathematics Subject Classification : **26A33; 03D15.**

Structural, electronic study and QTAIM analysis of host-guest interaction of Warfarin with $\beta-$cyclodextrin and calix[4]arene

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Abstract We investigated, using first principle calculations, the inclusion process of anticoagulant Warfarin (Warf) in two different cavitand supramolecules: $\beta-$Cyclodextrine and para-tert-butyl calix[4]arene. The algorithm of the docking and details of our calculations can be found in references [1, 2, 3] and references therein. We considered all plausible cases of inclusion which differ by the moiety of Warf pointing to Head (H) or Tail (T) part of $\beta-$CD or to upper rim of Calix. Therefore, 'A' and 'B' orientations correspond respectively to phenyl or chromone group entering first in the host cavity. Among the six consequent orientations, 'AH' gave the optimum structure for Warf: $\beta-$CD complex while the most favorable conformer is in 'BH' orientation for Warf:Calix complex. The obtained structures correspond respectively to 'End' and 'Exo' complexes. From electronic computations, we determined Frontier Molecular Orbitals (FMO) from which we inferred global reactivity descriptors. These results suggest that the best stability is of $\beta-$CD complex. Moreover, we used two models to give a deeper understanding of the type and magnitude of intermolecular interactions between guest and host molecules. The first is called Natural Bond Orbital (NBO) analysis which quantifies the donor-acceptor interactions. The second known as Bader's Quantum Theory of Atom In Molecule (QTAIM) uses the topological analysis of the electron charge density. The two approaches led to similar results as far as forces stabilizing structures are concerned.

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Keywords : *Warfarin; $\beta-$CD; p-tert-butyl calix[4]arï£¡ne, ONIOM2, NBO, QTAIM.*
2010 Mathematics Subject Classification : **70J50; 19L64.**

Fully Bayesian Neural Networks for High Dimensional Output Spaces

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Abstract An emulator or metamodel is an approximation of a high-fidelity computer model based on applying machine learning algorithms to the datapoints generated by the model. The main advantage is that it can be estimated cheaply and fast. This is the reason it can be used in applications such as design optimization and uncertainty analysis. Replacing a computer model with an emulator it is possible to predict the behaviour of a system as a function of different parameters a operating conditions. Artificial Neural Networks (ANN) are combined with dimensionality reduction techniques to produce an efficient emulator.

Introduction Emulators apply machine learning algorithms (e.g. Gaussian process modeling, support vector machines) are applied to map input-output data generated from a computer model [2, 6, 5, 4] They have been used in a broad range of problems usually involving the approximation of a few scalar outputs for multiple inputs, and almost invariably based on Gaussian process emulations (GPE) models. When the problem includes a numerical solution of a set of PDEs (generated from a high fidelity computer model) the aim usually is to estimate a spatial or spatio-temporal field as a function of a vector of input parameters. To tackle such high-dimensional problems, Higdon et al. in [1] applied linear principal component analysis to the output space for a multivariate Gaussian process emulator. In cases where the response surface exhibits high curvature, the LPCA-GPR method may, on the other hand, fail, which suggests the replacement of LPCA with a nonlinear dimensionality reduction technique.

This work aims to develop a fully Bayesian Artifical Neural Network (ANN) emulator for high dimensional outputs. A Bayesian treatment marginalizes over the distribution of parameters in order to make predictions. To extract the hyperparameters over the distribution the Hamiltonian Markov Chain Monte Carlo also known as Hybrid Markov Chain Monte Carlo will be used. The random-walk behavior of many Markov Chain Monte Carlo (MCMC) algorithms makes Markov chain convergence to a target stationary distribution ***p(x)*** inefficient, resulting in slow mixing. Hamiltonian/Hybrid Monte Carlo (HMC), is a MCMC method that adopts Hamiltonian dynamics and not a probability distribution to propose future states in the Markov chain. This is an efficient way for the Markov chain to explore the target distribution, resulting in faster convergence.

The use of a Bayesian ANN has several advantages of GPE. Firstly, an ANN can deal with multiple outputs, in contrast to GPE. When reducing the dimensionality of the outputs space, this allows coefficients to be learned simultaneously. Correlations are naturally accounted for, which also means that multiple fields can be emulated without making assumptions.

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Keywords : *Emulator; metamodel; dimensionality reduction; machine learning; artificial neural networks; bayesian neural networks.*
2010 Mathematics Subject Classification : **78M32; 92B20; 68T27.**

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Gaussian process emulation for high-dimensional outputs based on diffusion maps

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Abstract We present a new framework for multivariate output Gaussian process emulation using diffusion maps. The training data is mapped into a low dimensional space that well describs the system through diffusion maps. The GP emulation is applied in that space and the predictions are gained by a pre-image techniques that are nomally used in de-noising probem. Compared with standard GP emulator our method is shown to more robust especially for highly nonlinear complex system.

Introduction Simulator are also known as computer models that describes a real-world system in mathematical forms and runs on a computer to give predictions of reasonable level of accuracy of reality([11]). Rather than real physical experiments, simulators are faster and cheaper. However, Repeated calls to such an complex simulator are often computationally impractical for applications such as design optimization, uncertainty quantification, sensitivity analysis and inverse parameter estimation. For instance standard Monte Carlo-based method of sensitivity analysis require thousands of runs ([1]).

To overcome the problem of high computational cost of a simulator the concept of emulator is introduced. An *emulator* (also referred as surrogate model or metamodel) is an approximation of a high-fidelity computational model (simulator), with the advantage that it can be evaluated very cheaply, ideally in real-time ([11, 6]).

In Gaussian process emulation the uncertainty is fully accounted for by making a predictive distribution. The output of a simulator is modelled as a GP. The GP emulator is trained using data from a simulator at carefully selected design points $\boldsymbol{x}$. GP emulation has been applied successfully to a broad range of problems, usually involving the approximation of either a scalar or a vector in a low-dimensional space ([10]).

Normally there are multiple variables of interest in a physical system. In simulation it is common that some output contains a wealth of information while the others possess irrelevant properties for the system ([8]). It would be beneficial if we can simultaneously consider all outputs, but the joint calibration would be too difficult to tackle. The natural extension is to consider a multivariate Gaussian process prior, but these approach would largely increase the computational complexity. We want to combine all sources of information to be able to reduce the uncertainty using given dataset and to decide a fitting calibration strategy for the simulator. Methods for solving such multivariate problem has been studied by some papers as followed [10, 7, 5, 1, 3]. Recent work by [8] using LMC is proved to be useful; however the computation complexity grows dramatically with an increase in the dimensionality of the output space. [5] proposed the framework by standard PCA is a great progress but fails when dealing with highly complex nonlinear response surfaces.

In this work we present a new framework using diffusion maps for multivariate output emulation by using diffusion maps to exploit patterns in the permissible output space to dramatically reduce the output space dimensionality (by several orders of magnitude) when emulating spatial and spatio-temporal datasets arising from a simulator. The proposed method inherits the advantages of HH but permits a much broader class of response surfaces to be modeled. Follow the framework of HH, out work would as well make use of the basic framework of [**?**] to solve more complex physical process involving spatial, spatio-temporal and temporal data from computer simulations. It is shown that HH fails to provide meaningful results, while manifold learning approaches such as diffusion maps prove to be accurate. The method can be combined with any of the aforementioned multi-variate GP methods ([2, 3, 4, 9]) to efficiently perform emulation of *multiple spatio-temporal fields* simultaneously, e.g., when pressure, temperature and velocity fields are required from a single simulator.

Keywords : *Gaussian process emulation; dimension reduction; manifold learning; diffusion maps*
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