

King Saud University

Department of Mathematics

Second Mid Term Exam

ACTU 361 - Mathematics of Finance (1)

(22/7/1438 H, Time 1H30)

Exercise 1. [2+2+2]

We consider an annual annuity immediate with the following payments for n years:

$$P, P + Q, P + 2Q, P + 3Q, \dots, P + (n - 1)Q$$

- 1) Prove that the present value of this annuity is equal to:

$$PV = Pa_{\overline{n}|i} + Q \left(\frac{a_{\overline{n}|i} - nv^n}{i} \right)$$

with i is the annual effective interest rate.

- 2) Deduce the present value of this annuity if it continuous infinitely.
3) Find the present value of a perpetuity immediate that make the following annual payments

$$1, 3, 5, 7, \dots$$

with an annual effective interest rate of 2%.

Exercise 2. [5]

A 20-year annuity pays 10 at the end of year 1 and increase by 10 each year until the payment is 100 at the end of year 10. The payment remains constant for one year. Payments then decrease by 10 each year until a payment of 10 is paid at the end of year 20. The annual effective rate is 4%.

Find the present value and the accumulated value of this annuity.

Exercise 3. [5]

An annuity due make payments at the beginning of every 2 years. It pays 5 now and each successive payment will increase by 3% during 10 years. Starting from year 10, the payments remain constant forever.

Find the present value of this perpetuity with an annual effective interest rate of 4%.

Exercise 4. [1+2+3]

- 1) Give the formula of the present value of a varying annual continuous annuity that pays a rate of $f(t)$ at a varying force of interest δ_t with $0 \leq t \leq n$.
- 2) We consider a continuous annuity paying $f(t) = 3t + 5$ with a constant rate of interest equal to 7% p.a. and $0 \leq t \leq 10$.
Find the present and the accumulated value of this annuity.
- 3) We consider a continuous annuity paying $f(t) = t^2$ with a constant force of interest equal to 4% p.a. and $0 \leq t \leq 10$.
Find the present value of this annuity.

Exercise 5. [3]

Find the present value of a 3-year annuity that pays 3 at the end year 3 and each annual successive payment increase by 3% with an annual interest rate of 3%.