

King Saud University

Department of Mathematics

Final Examination

ACTU 361 - Mathematics of Finance (1)

(23/3/1437 H, Time 3H)

(3 pages)

Exercise 1. [9]

We consider an annuity for n periods in which payments are being made continuously at the rate $f(t)$ at exact every moment t and the interest rate is variable with a variable force of interest δ_t .

- 1) Give the formula of the present value of this annuity
- 2) Find the present value for 10-years annuity if $f(t) = e^t$ and $\delta_t = 1$.
- 3) We suppose that $f(t) = t$ and $\delta_t = \delta$ (constant). Prove that the present value of this annuity is equal to:

$$(\bar{I}\bar{a})_n = \frac{\bar{a}_n - nv^n}{\delta}$$

Where \bar{a}_n is the present value of an annuity payable continuously for n periods.

- 4) Deduce that the accumulated value of this annuity is equal to:

$$(\bar{I}\bar{s})_n = \frac{\bar{s}_n - n}{\delta}$$

- 5) Deduce from 3) the present value of the corresponding perpetuity.
- 6) Determine the present value and the accumulated value that pays a rate of $2t + 3$ with a force of interest $\delta = 9\%$ from 0 to 10 years.

Exercise 2. [8]

You have decided to invest in 10 years bond with the following characteristics:

- (i) Face value is 100 and the bond is redeemable at par
- (ii) Coupon rate is 3% p.a. payable semi-annually
- (iii) Yield rate is i

- 1) We suppose that $i = 3.6\%$ p.a. Find the price of the bond.
- 2) Use the linear interpolation method to find the yield rate if the bond is bought for 95.25

Exercise 3. [8]

- 1) Project P requires an investment of 2000 today. The investment pays 1000 one year from today and 2000 two years from today. Calculate the net present value of the project P with an annual effective interest rate of 5%.
- 2) Project Q requires an investment of X one year from today. The investment pays 1000 today and 2000 two years from today. Calculate in term of X the net present value of the project Q with an annual effective interest rate of 5%.
- 3) We suppose that the net present values of the two projects P and Q are equal. Calculate X .

- A) 20,688.63
- B) 20,698.57
- C) 20,966.35

Exercise 4. [8]

You are giving the following information about a loan of L that is to be repaid with a series of 16 annual payments.

- (i) The first payment of 2000 is due one year from now.
- (ii) The next seven payments are each 3% larger than the preceding payment.
- (iii) From the 9th to the 16th payment, each payment will be 3% less than the preceding payment.
- (iv) The loan has an annual effective interest rate of 7%.

- 1) Find the present value of the first eight payment at $t=0$.
- 2) Find the present value of the last eight payments at $t=0$.
- 3) Deduce the total amount L of the loan.

Exercise 5. [7]

- 1) Give the formula of the accumulation function from 0 to t years with the force of interest equals to $\delta(t)$ for 1 unit of capital.
- 2) The annual force of interest credited to saving account is defined by:

$$\delta(t) = \frac{\frac{t^2}{100}}{3 + \frac{t^3}{150}}$$

with t in years. Austin deposits 500 into this account at time 0. Calculate the time in years it will take for the fund to be worth 2000.

- (A) 6.7
- (B) 8.8
- (C) 14.2
- (D) 16.5
- (E) 18.9