King Saud University
College of Sciences
Mathematics Department

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Solution of the first midterm exam Summer ACTU. 462 (25%) (two pages)

July 10, 2018 / Shawwal 26, 1439 (two hours 10–12 PM)

Problem 1. (5 marks)

1. (**2 Marks**) Calculate the net premium for a special fully discrete 20–year term insurance on (30) given the following information:
(i) The death benefit is 1000 during the first ten years and 2000 during the next ten years.
(ii) The net premium is π for each of the first ten years and 2π for each of the next ten years.
(iii) $\ddot{a}_{30:\overline{20}|} = 15.0364$

x	$\ddot{a}_{x:\overline{10}|}$	$1000A^{1}_{x:\overline{10}|}$
30	8.7201	16.66
40	8.6602	32.61

2. (**3 Marks**) Determine the net annual premium for a fully discrete whole life insurance with annual premiums payable for 10 years is issued to (30) given:
(i) The death benefit is equal to 1000 plus the refund of the net level annual premiums paid without interest.
(ii) Premiums are calculated in accordance with the equivalence principle.

Problem 2. (5 marks)

For a special fully continuous whole life insurance on (65):
(i) The death benefit at time t is $b_t = 2000e^{0.05t}$, for $t \geq 0$
(ii) Level premiums are payable for life.
(iii) $\mu_{65+t} = 0.04$, $t \geq 0$ and $\delta = 0.05$

1. **(1 marks)** Find the present value of the future loss, ${}_0L$ and calculate the mean of ${}_0L$,
2. **(2 marks)** Calculate the annual net premium for this life insurance.
3. **(2 marks)** Calculate the premium reserve at the end of year 2.

Problem 3. (5 marks)

For a fully continuous 20–year endowment insurance of 1 on (x): given that
(i) The force of mortality is constant and equals to 0.02 and $i = 0.06$.
(iii) The premium is determined by the equivalence principle.

1. (**3 Marks**) Calculate the net premium reserve at time 10, ${}_{10}V$ using the prospective approach
2. (**2 Marks**) Calculate the net premium reserve at time 10, ${}_{10}V$ using the retrospective approach

Problem 4. (5 marks)

1. Consider a fully discrete whole life insurance of 1000 on (30). We are given $\ell_{30} = 9,501,381$, $\ell_{77} = 4,828,182$, $\ell_{78} = 4,530,360$ and $i = 6\%$.

   (a) **(1 marks)** Find an integer k so that ${}_kq_{30} \leq 0.5 < {}_{k+1}q_{30}$

   (b) **(1 marks)** Find the 50^{th} percentile premium for this insurance.

2. Consider a fully continuous whole life insurance of 1000 on (x), whose future lifetime T_x, has the density function

$$f_x(t) = \begin{cases} \frac{t}{1250}, & 0 \leq t \leq 50 \\ 0, & \text{otherwise.} \end{cases} \quad \text{assume that } \delta = 5\%.$$

   (3 marks) Find the 25^{th} percentile premium for this insurance?

Problem 5. (5 marks)

For a special fully discrete whole life insurance on (40):

(i) The death benefit is 1000 for the first 20 years; 5000 for the next 5 years; 1000 thereafter.

(ii) The annual benefit premium is $1000P_{40}$ for the first 20 years; $5000P_{40}$ for the next 5 years; π thereafter.

(iii) Mortality follows the Illustrative Life Table.

(iv) $i = 0.06$

1. (**3 Marks**) Calculate ${}_{20}V$, the benefit reserve at the end of year 20 for this insurance using retrospective approach.

2. (**2 Marks**) Calculate ${}_{21}V$, the benefit reserve at the end of year 21 for this insurance.

Useful formulas:

The following table summarizes the percentile premiums for n–year term and n–year endowment insurances of S on (x) for fully continuous policies.

Type of plan	$t_\alpha > n$	$t_\alpha \leq n$
Whole life	$\frac{S}{\bar{s}_{\overline{t_\alpha}\rvert}}$	$\frac{S}{\bar{s}_{\overline{t_\alpha}\rvert}}$
n–year term	0	$\frac{S}{\bar{s}_{\overline{t_\alpha}\rvert}}$
n–year endowment	$\frac{S}{\bar{s}_{\overline{n}\rvert}}$	$\frac{S}{\bar{s}_{\overline{t_\alpha}\rvert}}$

The net premium reserve at the end of year h is

$${}_hV = E\left[{}_hL\right] = \sum_{j=0}^{n-h-1} b_{h+j+1}v^{j+1}\ {}_{j|}q_{x+h} + S \times {}_{n-h}E_{x+h} - \sum_{j=0}^{n-h-1} \pi_{h+j}v^j\ {}_jP_{x+h}.$$

The net premium reserve at time h for whole life insurance of 1 on (x), with benefits payable at the moment of death, level premiums are payable at the beginning of each year is given by

$${}_hV = \bar{A}_{x+h} - \pi\ddot{a}_{x+h}.$$

We know that $\bar{A}_{x:\overline{n}\rvert} = \bar{A}^{1}_{x:\overline{n}\rvert} + {}_nE_x$, and $A_{x:\overline{n}\rvert} = A^{1}_{x:\overline{n}\rvert} + {}_nE_x$, ${}_nE_x = v^n{}_nP_x$

Under **UDD** $\bar{A}_x = \frac{i}{\delta}A_x$, and $\bar{A}^{1}_{x:\overline{n}\rvert} = \frac{i}{\delta}A^{1}_{x:\overline{n}\rvert}$.

Under **CRM** $\bar{A}^{1}_{x:\overline{n}\rvert} = \frac{\mu}{\delta+\mu}(1 - {}_nE_x)$ and ${}_nE_x = e^{-(\delta+\mu)n}$

The recursion formula for the net premium reserve: $({}_hV + \pi_h)(1+i) = q_{x+h}\ b_{h+1} + {}_{h-1}V\ p_{x+h}$