King Saud University

Department of Mathematics

Final Examination

ACTU 363 - Actuarial Mathematics Lab (1)

(10/4/1438 H, Time 3H)

(5 pages)

Exercise 1. [4]

A 30 year annuity is arranged to pay off a loan taken out today at a 5% annual effective interest rate. The first payment of the annuity is due in 10 years in the amount of 1,000. The subsequent payment increase by 500 each year.
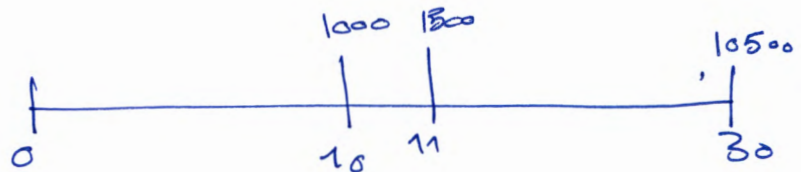
Calculate the amount of the loan.

(A) 58,283
(B) 61,197
(C) 69,211
(D) 64,253 ✓
(E) 64,021

Exercise 2. [4]

At an annual effective interest rate of interest $i > 0$, the present value of a perpetuity paying 10 at the end of each 3 year period, with the first payment at the end of year 3, is 32.

At the same annual effective rate of i, the present value of a perpetuity paying 1 at the end of each 4-month period, with first payment at the end of 4 months is X.

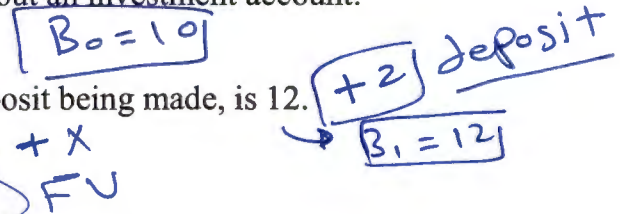Calculate X

(A) 32.6
(B) 31.6
(C) 35.6
(D) 34.6
(E) 33.6

$$\left(1+i\right)^t = \frac{B_7}{B_0} \cdot \frac{B_{\pm 2}}{B_{\frac{1}{4}}+W_{\frac{1}{4}}} * \frac{B_n}{B_{n-1}+W_{n-1}}$$

$$= \frac{12}{10}$$

Exercise 3. [4]

You are given the following information about an investment account:

(i) The value on January 1 is 10 $B_0 = 10$

(ii) The value on July 1, prior to a deposit being made, is 12. $+2$ deposit

(iii) On July 1, a deposit of X is made. $+X$ $B_1 = 12$

(iv) The value on December 31 is X.   FV

Over the year, the time weighted return is 0%, and the money-weighted return is Y.

Calculate. Y

(A) -25%
(B) -10%
(C) 0%
(D) 10%
(E) 25%

$10 \qquad \overset{+2}{\underset{+X}{|}} \qquad X$

$I = \frac{FV - V_0 - \sum c_i}{}$

$= X - 10 - 2 - X = -12$

$Y_0 t - \sum (t - t_j) c_i = 10\left(\frac{12}{12}\right) - 2\left(\frac{6}{12}\right) - X\left(\frac{6}{12}\right)$

$= 10 - 1 - \frac{X}{2} = 9 - \frac{X}{2}$

Exercise 4. [4]

You are given the following information with respect to a bond:
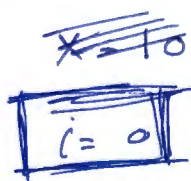
(i) Par value: 1000
(ii) Term to maturity: 3 years
(iii) Annual coupon rate: 6% payable annually

You are also given that the one, two and three year annual rate are 7%, 8% and 9% respectively.

The bond is sold at a price equal to its value

Calculate the annual effective yield rate for the bond i

(A) 8.5%
(B) 8.7%
(C) 8.3%
(D) 8.9%
(E) 8.1%

$X = 10$

$i = 0$

$B_1 = 12$
$B_0 = 10$
$B_2 = 12 + X$

$1 = \frac{1728 + 144X}{10}$

$X = \frac{1931}{-12}$

$\frac{12}{10} \qquad \frac{12 + X}{12}$

$\frac{144(12 + X)}{10} \qquad \boxed{\frac{1728 + 144X}{10}}$

$=$
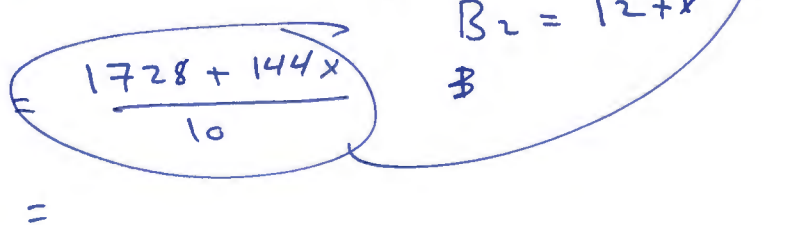
Exercise 5. [4]

An investor purchases a 10-year callable bond with face amount of 1000 for price P. The bond has an annual nominal coupon rate of 10% paid semiannually.

The bond may be called at par by the issuer on every other coupon payment date, beginning with the second coupon payment date.

The investor earns at least an annual nominal yield of 12% compounded semiannually regardless of when the bond is redeemed.

Calculate the largest possible value of P.

(A) 982
(B) 926
(C) 885
(D) 965
(E) 892

Exercise 6. [4]

You are given the following information about a company's liabilities

- Present value: 9697
- Macaulay duration: 15.24 d
- Convexity: 242.47 c

The company decides to create an investment portfolio by making investments into 2 of the following three zero coupon bonds: 5-year, 15-year, and 20-year. The company would like its position to be Redington immunized against small change in yield rate.

The annual effective yield rate for each of the bonds is 7.5%.

Determine which of the following portfolios the company should create.

(A) Invest 3077 for the 5-year bond and 6620 for the 20-year bond.
(B) Invest 6620 for the 5-year bond and 3077 for the 20-year bond.
(C) Invest 465 for the 15-year bond and 9232 for the 20-year bond.
(D) Invest 4165 for the 15-year bond and 5541 for the 20-year bond.
(E) Invest 9232 for the 15-year bond and 465 for the 20-year bond.

$$P \quad 3077(1.075)^{-5} + 6620(1.075)^{-20} - 9697 = 0 \; X$$

Exercise 7. [4]

Cash flows are 40,000 at time 2 (in years), 25,000 at time 3, and 100,000 at time 4. The annual effective yield rate is 7.0%

Calculate the Macaulay duration

(A) 3.1
(B) 3.3
(C) 3.4
(D) 2.2
(E) 2.3

Exercise 8. [4]

You are given the following information about stock XYZ:

(i) The current price of the stock is 35 per share
(ii) The expected continuously compounded annual rate of return is 8%
(iii) The stock pays dividends of 0.32 per share 2 months from now and 6 months after.

The continuously compounded annual risk-free interest rate is 4%.

Calculate the current one-year forward price for stock XYZ.

(A) 34.37
(B) 35.77
(C) 36.43
(D) 37.23
(E) 37.92

Exercise 9. [4]

An investor is analyzing the cost of two-year, European options for aluminum and zinc at a particular strike price.

For each ton of aluminum, the two year forward price is 1400, a call option costs 700 and a put option costs 550.

For each ton of zinc, the two year forward price is 1600 and a put option costs 550. The risk-free annual effective interest rate is constant 6%.

Calculate the cost of a call option per ton of zinc.

(A) 878

(B) 522

(C) 800

(D) 900

(E) 1231

Exercise 10. [4]

The current price for a stock index is 1,000. The following premium exists for various options to buy or sell the stock index six months from now:

Strike Price	Call Premium	Put Premium
950	120.41	51.78
1,000	93.81	74.20
1,050	71.80	101.21

Strategy I is to buy the 1,050-strike call and to sell the 950-strike call

Strategy II is to buy the 1,050-strike put and to sell the 950-strike put.

Strategy III is to buy 950-strike call, sell the 1,000-strike call, sell the 950-strike put and buy the 1,000-strike put.

Assume that the price of the stock index in 6 months will be between 950 and 1,050.

Determine which, if any, of the three strategies will have greater payoffs in six months for lower prices of the stock index than for relatively higher prices

(A) None
(B) I and II only
(C) I and III only
(D) II and III only
(E) The correct answer is not given by (A), (B), (C), or (D).