

King Saud University

Department of Mathematics

**Final Examination**

ACTU 461 - Mathematics of Finance (2)

(19/4/1441 H, Time 3H)

(3 pages)

**Exercise 1. [5]**

The S&T index pays a continuously compounded dividend rate of  $\delta = 2\%$ . The current value of the index is 1000, and the three-month forward price is 1002.50.

- 1) Find the risk-free continuously interest rate.
- 2) Suppose that the price of a three-month European 1000-strike put on the stock is 38.40. Find the price of a three-month European 1000-strike call.

**Exercise 2. [5]**

You are given the following information:

- The current price to buy one share of XYZ stock is 85.
- The stock does not pay dividends.
- The annual effective rate of interest rate is 6.5%.
- A European call option on one share of XYZ stock with strike price of K that expires in one-year costs 3.33
- A European put option on one share of XYZ stock with a strike price of K that expires in one-year costs 10.38.

- 1) Calculate the strike price K.
- 2) Suppose that  $K=94$ . Describe an arbitrage opportunity on this stock and give the arbitrage profit.

### Exercise 3. [5]

Suppose that you buy a share of a stock for 40 and you also buy a 38-strike put that expires in one year for 1.73. The continuously compound interest rate is 4%.

- What is the name used for your option position?
- If the stock falls to 35 after one year, what is the profit on your combined position?

### Exercise 4. [5]

The current price of a stock is 30. You buy a call with a strike price  $30 - S$  for 3.4431 and sell a call with strike price  $30 + S$  for 0.3421. Both calls expire in three months.

- Give the name of your position
- Suppose that at expiration, the stock price is 35 and you have a profit of 2.8679. You are given that  $S < 4$  and the continuously compound risk-free rate is 4%. Find the strike price of the purchased call.

### Exercise 5. [8]

- Give the definition and the graph of a profit of a **Bear Spread** with call options as a function of the stock price. When an investor should use a strategy of a Bear Spread.
- An investor short-sells a non-dividend paying stock and also writes a collar on this stock consisting of a  $K_1$ -strike European put option and a  $K_2$ -strike European call option with  $K_1 < K_2$ . Both options expire in  $T$ - years with a continuous interest rate  $r$ . We will call these combined positions by **Written Collared Stock**.
  - Give the graph of the profit of a Written Collared Stock
  - When an investor should use a strategy of a Written Collared Stock.
- Prove that:

$$\text{Profit [Written Collared Stock]} = \text{Profit [Bear Spread]}$$

### Exercise 6. [7]

The current price of a stock is 28. The continuously risk-free rate is 3%. The table below gives prices of 6-month put and call options with various exercise prices

Strike Price	Call Option	Put Option
32	0.50	4.26
28	1.77	1.57
24	4.46	0.28

- 1) An investor sells the 28-strike call and the 28-strike put, and buys the 24-strike put and 32-strike call. Give the name of this position, the graph of the profit as a function of the index price and the reason to take this position.
- 2) An investor buys a 32-strike call and 24-strike put, and sells 32-strike put and 24-strike call. Give the name of this position and in this case, the reason to use this position.

### Exercise 7. [5]

A stock index is currently valued at 50. The index pays dividend continuously at a rate proportional to the level of the index. The dividend yield during the first 3 months is 2%, and the dividend yield after 3 months is 3%. The continuously compounded risk-free interest rate is 5%. Calculate the following:

a)  $F_{0,0.8}^P$

b)  $F_{0,0.8}$

c)  $F_{0.3,0.8}^P$  assuming  $S_{0.3} = 48$