

**King Saud University**  
**Department of Mathematics**

**Final Examination**

ACTU 461 - Mathematics of Finance (2)

(11/8/1440 H, Time 3H)

(3 pages)

**Exercise 1. [5]**

A stock has current price  $S_0 = 50$ . The annual continuous interest rate is 3%. Semi-annual dividend of 1 will be paid in six month and one year. Suppose that the price of a prepaid forward contract on this stock is 48.

Describe an arbitrage opportunity and find the arbitrage profit.

**Exercise 2. [1+1+2]**

Suppose the S&P 500 index futures price is currently 1135. You wish to purchase 6 futures contracts on margin.

- What is the notional value of your position?
- Assuming 10% initial margin, what is the value of the initial margin?
- Suppose that the maintenance margin is 90% of the initial margin and after one settlement day, the index price declines to 1125. The annual effective interest rate is 3%. Is there a margin call?

**Exercise 3. [2+1+3]**

You are given the following information on 6-month European calls and puts on a non-dividend paying stock:

Strike	Call Premium	Put Premium
58	4.052	3.675
62	2.524	5.913

You enter long positions on two 58-strike put options and one 62-strike call.

- Draw the payoff and the profit diagram of your combined position.
- Calculate the maximum and minimum payoff.
- Calculate the profit if the stock price at expiration is 60.

**Exercise 4. [5]**

You are given:

- The current price of a stock is 60.
- The stock will pay a dividend of 4 dollars six month from now.
- The price of a 1-year European put option on the stock is 1.8 less than that of an otherwise identical call.
- The continuously compounded risk-free interest rate is 5%.

Calculate the strike price of the option.

**Exercise 5. [5]**

- You short an index at  $S_0$  and write a put on this index. Give the name of your combined position, the reason to write a put with index and an equivalent strategy profit (with the proof).
- You short an index and you write a 6-months 100-strike put on this index. Suppose that 100-strike call on this index costs 4 and the risk-free effective interest rate is 2%. What is the profit of your combined position if the index price at expiration is 120?

**Exercise 6. [5]**

The current price of a stock is 40. The continuously compounded risk-free rate and dividend rate are  $r = 0.03$  and  $\delta = 0.01$ . The price of an at-the-money 3-month call is 2.48. An investor buys the at-the-money 3-month call and put.

- Give the name of this combined position and the graph.
- What is the minimum profit?

**Exercise 7. [6]**

The current price for a stock index is 1,000. The following premiums exist for various options to buy or sell the stock index in six months from now:

Strike price	Call Premium	Put Premium
950	120.41	51.78
1,000	93.81	74.20
1,050	71.80	101.21

Strategy 1 is to sell the 950-strike put and to buy 1,050-strike call

Strategy 2 is to buy the 950-strike put and to sell 1,050-strike call

Strategy 3 is to buy the 950-strike call and to sell 1,050-strike call

Strategy 4 is to buy 950-strike call, sell the 1,000 -strike call, sell the 950-strike put and buy the 1,000-strike put.

Determine which, if any, of these strategies will have greater payoffs in six months for higher prices of the stock index than for relatively lower prices.

**Exercise 8. [4]**

The current price of a stock is 43. An investor buys the 1-year 43-strike call for 5.71 and sells the 1-year 43-strike put for 4.44. The stock pays no dividend. At the same time, she buys the 1-year 47-strike put for 6.96 and sells the 1-year 47 strike call for 4.08.

- What the investor creates by these positions? what is the reason?
- Find the annual effective risk-free interest rate (i.e. the risk-free rate of return).