**Chapter 1: Utility theory and insurance.**

1. Definitions:

We start by introducing some essential concepts in the theory of risk.

Risk: In this course we will study the actuarial risk, which means the possibility of exposure to quantified losses in the future. We shall denote these losses by X and then X is a random variable with specified probability distribution.

Insurance policy: the agreed contract between the insured (an individual) and the insurer (insurance company). So that the insurer will compensate the insured for losses he or she may face, against a premium paid by the insured at the beginning of the contract.

1. Premium calculation:

We consider two types of premiums.

2.1- Net premium: the net premium is given by the expected value E(X) of X.

**Example.** Find the net premium for an insurance policy when X is uniformly distributed on the interval (a,b)?

The net premium is given by .

2.2- Premium by utility function:

* The theory of preferences suggests the introduction of a family of utility functions, reflecting the wishes and preferences of decision makers. So an utility function is a function satisfying the following conditions: 1) u is increasing, 2) u is concave, 3) u is twice differentiable, and 4) u(0)=0 and u’(0)=1.
* Five families of utility functions are widely used:

 Linear utility: ,

 Quadratic utility: for ,

 Logarithmic utility: for ,

 Exponential utility: with ,

 Power utility: for and .

* Now we define the premium **P** against losses X and wealth W, as the solution of the equation:

. (\*)

 For an insured’s utility we denote and for an insurer’s utility we denote . So the insured will not accept a premium more than , and the insurer will not accept a premium less than .

**Example 1.** Suppose that an insurer has an exponential utility function with parameter .

1. What is the minimum premium to be asked for a risk X?
2. What is the minimum premium when X is exponentially distributed with parameter ?
3. We recall that the exponential utility function with parameter is given by . So the equation (\*) becomes:

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 We simplify and obtain that:

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 which means that

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 where is the moment generating function of X at .

1. The moment generating function of an exponential distribution with parameter is given by:

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 So the premium is given by

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**Example 2.** Suppose that for a wealth , the insured’s utility function is .

a) What is the maximum premium as a function of W, for an insurance policy against a loss X?

b) Suppose X=1 with probability 0.5 and , what happens to this premium if W increases?

1. The equation (\*) becomes:

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We simplify and obtain that:

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So

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1. We have and , then

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The function P(W) is increasing in W, so the premium will increases when the wealth W increases.

1. Premium approximation:

We provide an approximation of the premium using only the mean and the variance of the losses . By differentiating we get

and

Thanks to the equation (\*), we have

therefore

Where is called the risk aversion coefficient.

1. Reinsurance:

Reinsurance covers part of the claims which are already insured. Stop-Loss reinsurance covers the top part of the claim and the remaining is covered by the insurer. So there is a barrier d called priority or retention such that the reinsurer will pay only

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and the stop-Loss premium ( or stop-loss transform) is given by

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where is the cumulative distribution function of X.

**Example**. Compute the stop-loss transform for losses X uniformly distributed on the interval (2,4).

We recall that the cdf is given by

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So