King Saud University
College of Sciences
Mathematics Department

Academic Year (G) 2016–2017
Academic Year (H) 1437–1438
Bachelor AFM: M. Eddahbi

Second midterm exam QMF: ACTU. 468 (25%) (two pages)

Thursday, May 4, 2017 / Sha'ban 8, 1438 (two hours 9–11 AM)

Use ballpoint or ink–jet pens and keep four digits after dot

Problem 1. (9 marks)

1. For $0 \leq t \leq T$ we set

$$d_1(S_t, K, r, T-t, \delta) = \frac{\ln\left(\frac{S_t}{K}\right) + \left(r - \delta + \frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}}$$

and

$$d_2(S_t, K, r, T-t, \delta) = \frac{\ln\left(\frac{S_t}{K}\right) + \left(r - \delta - \frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}}$$

(1 mark) Find in terms of $d_1(S_t, K, r, T-t, \delta)$ and $d_2(S_t, K, r, T-t, \delta)$ the pricing Black–Scholes formula for a European call option on stock paying a dividend yield δ at any time t between 0 and maturity T.

2. **(1 mark)** Find the relation between $d_1(S_t, K, r, T-t, \delta)$ and $d_2(K, S_t, \delta, T-t, r)$ and the relation between $d_1(K, S_t, \delta, T-t, r)$ and $d_2(S_t, K, r, T-t, \delta)$.

3. **(1 mark)** Use the formula $1 - \mathbf{N}(x) = \mathbf{N}(-x)$ to find the relation between $C(S_t, K, r, T-t, \delta)$ and $C(K, S_t, \delta, T-t, r)$.

4. **(1 mark)** Let $S = \$100$, $K = \$90$, $\sigma = 30\%$, $r = 8\%$, $\delta = 5\%$, and $T = 1$. What is the Black–Scholes European call price?

5. **(1 mark)** Find the price of a European put where $S = \$90$, $K = \$100$, $\sigma = 30\%$, $r = 5\%$, $\delta = 8\%$, and $T = 1$.

6. **(1 mark)** What is the link between your answers to 4. and 5. ? Explain your answer ?

7. Consider a stock whose price is given by the Black–Scholes model, with volatility $\sigma = 30\%$ per annum and initial price $S_0 = 100$ euros. Such a stock pays a dividend of one euro in 3 months and of one euro in 9 months. The continuously compounded risk–free rate available on the market is of 4% per annum.
(1 mark) Compute the initial price of a European call option set at the money with maturity of one year.

8. **(1 mark)** Give the call–put parity corresponding to this stock.

9. **(1 mark)** Find the price of the corresponding put option?

Problem 2. (8 marks)

1. **(1 mark)** Recall first the Black–Scholes pricing formula of a European call option involving two currencies.
2. **(1 mark)** Recall the corresponding call–put parity.
3. One euro is currently trading for \$1.0896. The dollar–denominated continuously compounded interest rate is 2% and the euro–denominated continuously compounded interest rate is 0.5%. Volatility is 10%.
   (1 mark) Find the Black–Scholes price of a 1–year dollar–denominated euro call with strike price of \$1.1000/€.
4. **(1 mark)** Find the Black–Scholes price of a 1–year dollar–denominated euro put with strike price of \$1.1000/€.
5. **(1 mark)** What is the price of a 1–year euro–denominated dollar call with strike price of $€\frac{1}{1.1000}/\$$
6. **(1 mark)** What is the price of a 1–year euro–denominated dollar put with strike price of $€\frac{1}{1.1000}/\$$
7. **(1 mark)** What is the link between your answers to 3. and to 5. converted to dollar ?
8. **(1 mark)** What is the link between your answers to 4. and to 6. converted to euro ?

Problem 3. (8 marks)

1. **(1 mark)** Give the formula under the objective probability (P) of a stock whose price at time T is given by the Black–Scholes model.
2. **(1 mark)** Give the formula of the continuous compounding rate of return R of a stock whose price at time T is given by the Black–Scholes model.
3. **(1 mark)** What is the distribution of R, specify the mean and the variance..
4. A stock price is currently 120. Assume that the expected return from the stock is 8% and its volatility is 20%.
   (1 mark) What is the probability distribution for the rate of return (with continuous compounding) earned over a one–year period?
5. **(1 mark)** Find the confidence interval of R at 95%.
6. **(1 mark)** Deduce the confidence interval of the stock price in one year.
7. **(1 mark)** What is the probability that a European call option on the stock with an exercise price of \$110 and a maturity date in one year will be exercised?
8. **(1 mark)** What is the probability that a European put option on the stock with the same exercise price and maturity will be exercised?

Standard Normal Probabilities

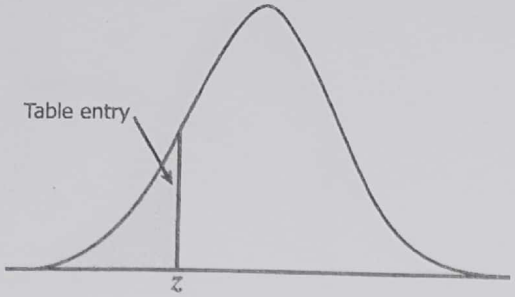

Table entry for z is the area under the standard normal curve to the left of z.

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
−3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
−3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
−3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
−3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
−2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
−2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
−2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
−2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
−2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
−2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
−2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
−1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
−1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
−1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
−1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
−1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
−1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
−1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
−1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
−1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
−1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
−0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
−0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
−0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
−0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
−0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
−0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
−0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
−0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
−0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
−0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

Standard Normal Probabilities

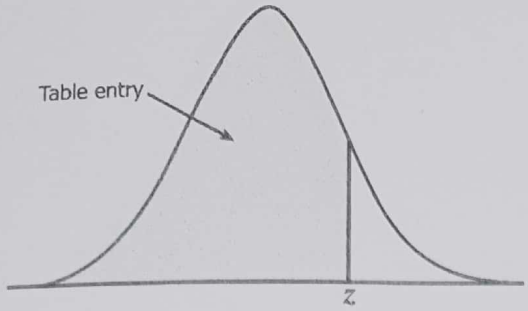

Table entry for z is the area under the standard normal curve to the left of z.

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998